ACTA IMEKO
ISSN: 2221-870X
June 2023, Volume 12, Number 2, 1 - 6

A theoretical model for uncertainty sources identification in tip-timing measurement systems

Giulio Tribbiani[1], Lorenzo Capponi[2], Tommaso Tocci[3], Tiberio Truffarelli[3], Roberto Marsili[3], Gianluca Rossi[3]

[1] *CISAS G. Colombo, University of Padova, Via Venezia 15, 35131 Padova, Italy*
[2] *Department of Aerospace Engineering, University of Illinois Urbana-Champaign, 104 S. Wright Street, 61801 Urbana, USA*
[3] *Department of Engineering, University of Perugia, Via G. Duranti 93, 06125 Perugia, Italy*

ABSTRACT
This paper presents a theoretical analysis of uncertainty sources in measurement techniques used to determine vibrations of turbomachinery blades using stationary sensors mounted on the casing of the turbomachine. A mathematical model based on fundamental physical principles is proposed, and two different measurement set-ups are evaluated. One set-up uses a reference sensor to measure the passage of an undeformed part of the blades (blade base), while the other set-up does not involve the use of a reference sensor, with both sensors facing the blades tip (deformed part). The intrinsic uncertainty of these methods and the performance of the complete measurement chain are defined. The analysis of the measurement technique leads to conclusions about the practical set-up and possible performances of these measurement techniques.

Section: RESEARCH PAPER

Keywords: vibrations, blade, turbomachinery, contactless measurement

Citation: Giulio Tribbiani, Lorenzo Capponi, Tommaso Tocci, Tiberio Truffarelli, Roberto Marsili, Gianluca Rossi, A theoretical model for uncertainty sources identification in tip-timing measurement systems, Acta IMEKO, vol. 12, no. 2, article 28, June 2023, identifier: IMEKO-ACTA-12 (2023)-02-28

Section Editor: Laura Fabbiano, Politecnico di Bari, Italy

Received March 14, 2023; **In final form** April 13, 2023; **Published** June 2023

Corresponding author: Giulio Tribbiani, e-mail: giulio.tribbiani@phd.unipd.it

1. INTRODUCTION

In operating conditions, monitoring turbomachinery blades vibrations is necessary for improving structural health techniques and for validating the dynamics of the system [1]. In fact, uncontrolled vibrations at or close to natural frequencies, combined with high thermo-mechanical loads, can lead to damage for the machinery and increase the risk of unexpected failures [2]. Traditionally, dynamical analysis and measurement of vibrations of rotating blade have been performed using strain gauge sensors [3]–[5]. However, this technique can have impact on the monitoring procedures [6]. In fact, although strain gauges have high accuracy and their usage is well established, they have relatively limited lifetime in high-temperature conditions, and, as intrusive technique, their installation on rotating systems and their data transmission are complex steps to achieve [5]. To overcome these problematics, non-contact and non-intrusive blade vibration measurement techniques have been developed in the past years, based on vibration, temperature [7] and ultra-sound approaches [5], [8]–[12]. One of the most successful and promising on-site techniques for axial turbomachinery blade dynamics measurements is called Blade Tip-Timing (BTT) [13], [14]. BTT is based on the measurement of instantaneous blade tip deflection by detecting advances or delays in the Time of Arrival (TOA) of the blade tip by means of sensors installed on the casing at fixed angular positions [15]. BTT can use different types of sensing probes, such as laser probes [16], [17], microwave sensors [4], [18], capacitive sensors [19], magneto resistive sensors [20], [21], or optical probes [6], [22]–[24]. While the measurement principle is the same for all different sensors, the optical probes are usually preferred for their performances in accuracy and resolution [6], [25]. The blade-sensor interaction generates electrical pulse signals. In ideal conditions, where no blade vibration is considered (i.e., rigid blade), the TOA is determined a priori given the geometry and the dynamic of the system [26], [27]. On the other side, considering blades as vibrating flexible structures, blade deflections lead to delays or advances of the blade tip with respect to the expected TOA [13], [28], [29]. These shifts in the TOA are extracted and used for determining the amplitude of deflection of each blade [30]. For this reason, a thorough identification and definition of sources of uncertainties for tip timing measurement systems (i.e., uncertainty on the measurement of the TOA) is essential for obtaining a detailed information on the dynamics of the system [31]. This will lead to a reliable structural health monitoring, giving insights on instrumentation best practices and information for validating numerical models [32].

Different studies have been carried out on BTT measurement systems, focusing on the uncertainty of specific cases technique [33]–[36].

This study proposes a complete overview of the uncertainty sources of a generic BTT system, and their influence on the estimation of the TOA. With regard to this, two different probes configuration are analysed, typically employed in BTT measurement systems, either with or without a reference sensor.

2. MATERIALS AND METHODS

A typical BTT measurement system allows the sampling of the relative displacement law *s(t)*, supposed periodic, between two points of the blade, expressed in a reference system rotating with the blade itself, using a non-contact sensor installed on the machine casing. The sensors are usually paired in couple, with one being the reference sensor and the other the sensor used to measure the blade deformation. The first one is typically installed at the base of the blade, where there is no deformation, the other is placed at the blade tip. By analysing the resulting signals, it is possible to obtain the instants in which the blade passes in front of the sensors and measure its deformation. If the tip deflects from the base, there will be a Δt, in delay or in advance, between the signals obtained by the reference and the non-reference sensor. In addition to this, the pulse width of the passage signal is representative of the duration of the passage itself. In this way, a displacement and velocity value can be measured for each transition of the blade in front a pair of sensors.

The problem lies in the fact that the displacement *s(t)*, as well as the velocity *v(t)*, *i.e.* its first derivative, have generally higher frequency components than the passage frequency in front of the sensors. Hence, *s(t)* and *v(t)* are sub-sampled. Nevertheless, it is possible, under particular conditions discussed in [37], to calculate the harmonics of *s(t)*, even if the sampling is performed in such conditions. A method is described that allows to obtain the discrete spectrums of the blade displacement and velocity, *s(t)* and *v(t)*, by solving a system of $2M+1$ non-linear equations, with M being the number of harmonics to be evaluated.

3. VELOCITY MEASUREMENT PRINCIPLE AND DISPLACEMENT USING REFERENCE SENSOR

In this section it will be described the working principle of a BTT measuring system using a reference sensor.

Considering a blade on a rotating drum subject to bending deformations as shown in Figure 1. Let ω be the angular rotation velocity of the disk.

Two reference systems are considered: a fixed one (O', s') and a second rotating one (O, s). Both s and s' are curvilinear abscissae defined on the circumference containing the blade tips, of radius R. In this reference systems, s identifies the position of the undeformed blade tip, hence *s(t)* represents the circumferential component of the blade tip displacement over time. It is assumed that this displacement changes in time according to a periodic law:

$$s(t) = S \cdot f(t), \tag{1}$$

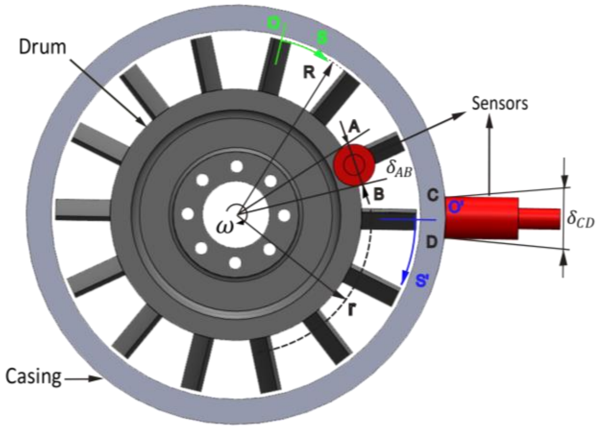

Figure 1. Proposed measuring set-up using a reference sensor (facing the base of the blade) to measure the deformation.

where $f(t)$ is a periodic function of period T, fundamental frequency $f_0 = 1/T$ and unitary amplitude, multiplied by a scalar S, greater than 0. The function *s(t)* is the measurand, sampled by the BTT measurement technique. Deriving the displacement *s(t)* the velocity *v(t)* is obtained. Considering the relative motion between drum and casing:

$$v(t) = v'(t) - v_0'(t) \tag{2}$$

with $v'(t)$ being the velocity in the rotating system and $v_0'(t)$ the absolute velocity due to drum rotation.

The blade deflection can be measured by analysing the signals generated by the two sensors arranged as shown in Figure 1. Figure 2 shows the qualitive representation of these two signals. The blade passage is detected when the signal exceeds a threshold value S_g. In conditions of undeflected blade, there will be a fixed time delay between the threshold crossing of signals from sensor 1 and 2. If the blade is deformed, the time delay Δt_{AC} will differ from the one obtained in the previous condition. Hence, it is possible to measure the blade deformation by measuring Δt_{AC}.

This method is valid if some key hypotheses are met; these assumptions will inevitably lead to an increase in measurement uncertainty.

First, it is considered that the time interval Δt_{AC} does not vary due to the irregularity of the rotation, hence the angular speed ω of the drum remains strictly constant. Second, the Δt_{AC} in the undeformed configuration is equal to the mean $\overline{\Delta t_{AC}}$ when the blades are vibrating:

$$\overline{\Delta t_{AC}} = \frac{1}{N} \sum_{i=1}^{N} \Delta t_{AC}(i)\,. \tag{3}$$

With these hypotheses, the time delays due to the oscillation *s(t)* can be calculated from:

$$\delta(\Delta t_{AC}) = \Delta t_{AC} - \overline{\Delta t_{AC}}\,. \tag{4}$$

Associating the times intervals Δt_{AB} and Δt_{CD} respectively to the distance δ_{AB} and δ_{CD} along which the sensors see the blade tip, defined as distances corresponding to the crossing of the threshold S_g as illustrated in Figure 2, it is possible to compute the average velocity respectively between A and B and between C and D by the following two equations:

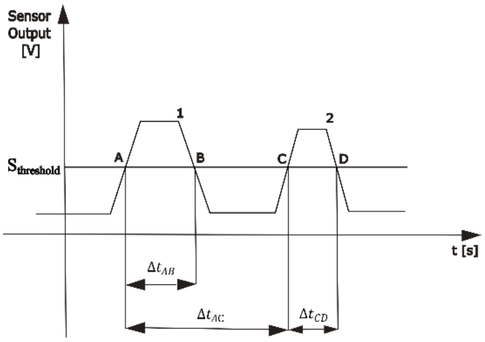

Figure 2. Time history of the outputs of sensors 1 (reference on the base) and 2 (on the tip) typically in Volt: the blade is passing when the output exceeds the threshold value $S_{\text{threshold}}$.

$$\underline{v_0'} = \frac{\delta_{\text{AB}}}{\Delta t_{\text{AB}}} \cdot \frac{R}{r} \tag{5}$$

$$\underline{v'} = \frac{\delta_{\text{CD}}}{\Delta t_{\text{CD}}}. \tag{6}$$

Being the duration of these signals very short, such quantities can be reasonably considered as the instantaneous velocity of the blade tip and base. Assuming that both s and the blade tip velocity remain constant during the passage occurred in Δt_{CD}, the displacement can be calculated as follows:

$$s(i) = \frac{\delta_{\text{CD}}}{\Delta t_{\text{CD}}} \cdot \delta(\Delta t_{\text{AC}}) \tag{7}$$

The value of v, assuming that $v_o'(t)$ remains constant during Δt_{AC} using (2) can be expressed as:

$$v(i) = \frac{\delta_{\text{CD}}}{\Delta t_{\text{CD}}} - \frac{\delta_{\text{AB}}}{\Delta t_{\text{AB}}} \cdot \frac{R}{r} \tag{8}$$

The usage in (7) and (8) of the instantaneous velocities of rotation allows a considerable extension of the applicability of the BTT measurement methods on machines with high degrees of irregularity compared to the ones presented in [15], [30], [38], where velocity is calculated as ω R, hence assumed constant during the entire revolution. In the method proposed here, v_o' remains constant only in the Δt_{AC} period.

In fact, by using (7) and (8) at each instant of passage of the blade in front of a couple of fixed sensors, $s(i)$ and $v(i)$ values are calculated.

4. METHOD WITHOUT REFERENCE SENSOR

In many applications, it is not possible to install a reference sensor in a position useful to detect the passage of a not deformed part of the blade, *i.e.,* the blade base. In these cases, it is possible to install the sensors as shown in Figure 3, facing only the blade tip. It is assumed once again that the time interval does not change due to the irregularity of the rotation between the two pulses and that the Δt, relative to the passage of the blade in the undeformed configuration, is equal to the average value of Δt measured over a certain number of revolutions. The time delays $\delta(\Delta t(i))$ are therefore calculated with respect to these average values. Assuming a strictly-constant tangential velocity equal to ωR, the measured blade deflection S_i, as shown in Figure 4, can be calculated by the following relation:

$$S_i = \omega \cdot R \cdot \delta(\Delta t(i)) \tag{9}$$

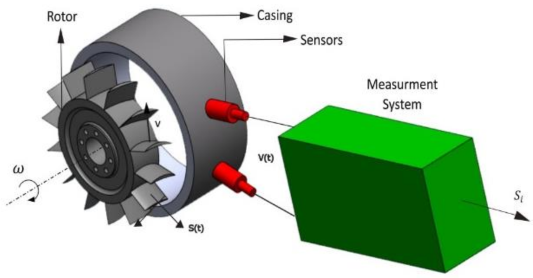

Figure 3. Measuring technique without reference sensor.

Hence, for each couple of pulses, we can have one pair of displacement samples s and s'. Average displacement can be estimated by $(s + s')/2$ and a velocity value v can be estimated by $(s'-s)/t_d$, where t_d is the pulse width.

To estimate the vibration frequency, we can use the following considerations. If the displacement of the blade is:

$$s = A \cdot \cos(\omega\, t). \tag{10}$$

Deriving it, the blad tip velocity v is obtained and can be expressed by:

$$v = -A \cdot \omega \cdot \sin(\omega\, t). \tag{11}$$

By computing the ratio between the minimum and maximum value of s and v, it is possible to estimate the vibration frequency:

$$f = \frac{v_{\max} - v_{\min}}{s_{\max} - s_{\min}}. \tag{12}$$

5. VIBRATION HARMONICS CALCULATION

The sampling of blade tip vibration by BTT technique produces a series of displacement and velocity samples on the rotating reference system: one for each passage of the rotating blade in front of the couple of the sensors. At each instant of

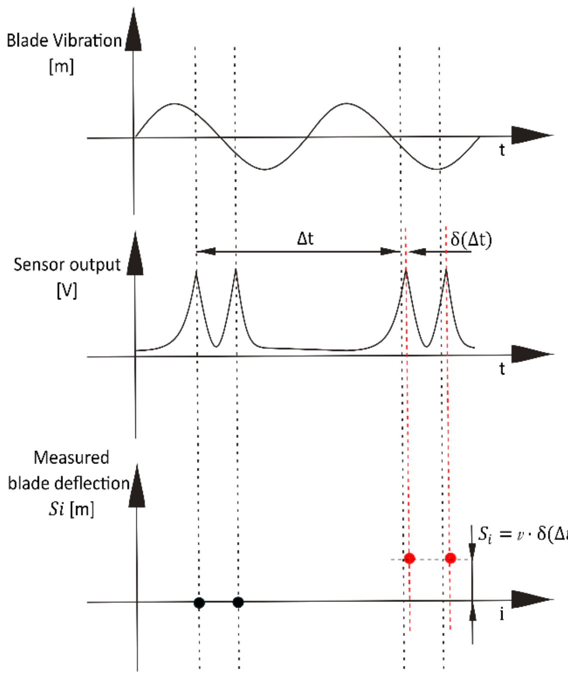

Figure 4. Blade Vibration and Sensor Output over time; Measured blade deflection at the *i-th* passage, resulting from the variation *δ(Δt(i)).*

passage, *i.e.*, at time $t(i)$, a deflection value $s(i)$ and a velocity value $v(i)$ are calculated.

The relative motion of the blade tip can be in general described by a periodic function over time. As vibrations, this periodic motion can be effectively described by a relatively small number of harmonics, with the energy being almost completely contained in the first harmonics.

The two functions $s(t)$ and $v(t)$ can be in general approximated by a Fourier series limited to M harmonics:

$$s(t) = \sum_{j=1}^{M} A_j \cdot \cos(2\,\pi\,v_0\,j\,t) + B_j \cdot \sin(2\,\pi\,v_0\,j\,t) \tag{13}$$

$$v(t) = 2\,\pi\,v_0 \cdot \cdot \sum_{j=1}^{M} -A_j \cdot \sin(2\,\pi\,v_0\,j\,t) + B_j \cdot \cos(2\,\pi\,v_0\,j\,t). \tag{14}$$

The values of $s(t)$ and $v(t)$ are measured at the instants $t(i)$, not necessarily equally distributed in time, as the sensors could be placed on the turbine casting at not equi-spaced angles. These values $s(i)$ and $v(i)$ can be used to write a system of nonlinear equations with unknowns that are the fundamental frequency and the $2\,M$ coefficients A_j and B_j of the harmonics. Thus, $(2\,M + 1)$ equations can be written; to solve this system, it is necessary to get enough samples of $s(t)$ and $v(t)$, at least $(2\,M + 1)$ or more. So, in principle, by the acquisitions of the $(2\,M + 1)$ $s(i)$ and $v(i)$ samples and the solution of a non-linear system of equations, it is possible to estimate the discrete spectrum of the harmonics of the vibration. With more than $(2\,M + 1)$ samples of $s(i)$ and $v(i)$, a least square approach is also possible. The limits of these ideas have been discussed in [37], where a complete theoretical approach is illustrated.

6. UNCERTAINTY SOURCES

The hypotheses made for the description of the BTT measurement principle of velocity and displacements samples of a rotating blade tip, limit the applicability of the methods. The main parameters of the rotating disk with a vibrating blade are ω, R, i, v, and s. A first uncertainty source can be identified in the variation of the Δt_{AC} due to the irregularity rotation, expressed by:

$$i = \frac{\omega_{\max} - \omega_{\min}}{\omega_{\mathrm{average}}}, \tag{15}$$

with ω being the angular speed of the rotating drum. As a first linear approximation, this uncertainty source can be estimated as follows:

$$E_i = i \cdot \Delta t_{\mathrm{AC}} \tag{16}$$

It was previously assumed that $s(t)$ and $v(t)$ do not change in the time period Δt_{AC}. A linear estimation of changes of $s(t)$ and $v(t)$ during Δt_{AC} can be expressed by the following relations:

$$E_s = \left[\frac{ds}{dt}\right] \cdot \Delta t_{\mathrm{AC}} \tag{17}$$

$$E_v = \left[\frac{dv}{dt}\right] \cdot \Delta t_{\mathrm{AC}}\,. \tag{18}$$

Assuming the time history of $s(t)$ is a simple sinusoidal vibration:

$$s(t) = S \cdot \sin(2\,\pi\,v\,t) \tag{19}$$

the following relations are obtained:

$$\left[\frac{ds}{dt}\right]_{\max} = 2\,\pi \cdot v \cdot s \tag{20}$$

$$\left[\frac{dv}{dt}\right]_{\max} = 4\,\pi^2 \cdot v^2 \cdot s\,. \tag{21}$$

By replacing these values in (17) and (18), it is possible to estimate the maximum effect on uncertainty due to changing of s and v over Δt_{AC}:

$$E_s = 2\,\pi \cdot v \cdot s \cdot \Delta t_{\mathrm{AC}} \tag{22}$$

$$E_v = 4\,\pi^2 \cdot v^2 \cdot s \cdot \Delta t_{\mathrm{AC}}\,. \tag{23}$$

Considering that the maximum of Δt_{AC} is obtained when the absolute speed of the blade tip is at minimum, (22) and (23) become:

$$E_s = 2\,\pi \cdot v \cdot s \cdot \frac{\delta_{\mathrm{AC}}}{\omega\,R - 2\,\pi\,v\,s} \tag{24}$$

$$E_v = 4\,\pi^2 \cdot v^2 \cdot s \cdot \frac{\delta_{\mathrm{AC}}}{\omega\,R - 2\,\pi\,v\,s}\,. \tag{25}$$

(16), (24) and (25) estimate the uncertainty amplitude, therefore can be used to identify the conditions of applicability of the measurement methods previously described.
Knowing this, it is possible to change the parameters of the measurement system, in order to properly choose and install the sensors in convenient locations, as well as find the optimal acquisition set up.

The resolution Δs of the displacement measurement is given by the $\delta(\Delta t_{\mathrm{AC}})$ resolution. Starting from:

$$R_s = \frac{\Delta s}{S} \tag{26}$$

and being t_{rs} the resolution in the time measurements, *i.e.,* sampling time, we can write:

$$t_{rs} = R_s\,(\delta(\Delta t_{\mathrm{AC}}))_{\max} \tag{27}$$

and being:

$$(\delta(\Delta t_{\mathrm{AC}}))_{\max} = \frac{S}{\omega \cdot R}\,. \tag{28}$$

t_{rs} strictly affects the resolution on the measurement of the displacement s. A useful equation can be defined to relate t_{rs} to R_s. This relation becomes fundamental when choosing the resolution of the time measurement, in order to achieve the target resolution of the blade tip displacement s

$$t_{rs} = R_s\,\frac{S}{\omega \cdot R}\,. \tag{29}$$

The resolution on blade tip velocity measurement depends essentially on $\delta(\Delta t_{\mathrm{CD}})$ measurement resolution. As for the displacement measurements, the following equation is obtained:

$$t_{rv} = R_v \cdot (\delta(\Delta t_{\mathrm{CD}}))_{\max} \tag{30}$$

and being

$$(\Delta t_{\mathrm{CD}})_{\min} = \frac{\delta_{\mathrm{CD}}}{\omega + 2\,\pi \cdot v \cdot s} \tag{31}$$

$$(\Delta t_{\mathrm{CD}})_{\max} = \frac{\delta_{\mathrm{CD}}}{\omega - 2\,\pi \cdot v \cdot s} \tag{32}$$

Hence

$$(\delta(\Delta t_{\mathrm{CD}}))_{\max} = \frac{(\Delta t_{\mathrm{CD}})_{\max} - (\Delta t_{\mathrm{CD}})_{\min}}{2}\,. \tag{33}$$

Therefore, from (33), it is possible to define another useful formula to choose sampling time of sensor signals in order to have a sufficient resolution for tip time velocity measurement:

$$t_{rv} = 2\,\pi \cdot S \cdot \delta_{\mathrm{CD}} \cdot \frac{R_v}{\omega^2 \cdot R^2 - 4\,\pi^2 \cdot v^2 \cdot s^2} \tag{34}$$

Further causes of uncertainty can also be due to: variations of δ_{AB} and δ_{CD}; definition of the threshold S_{g}; relative radial motion between sensor and blade tip; noise in sensors signals. This analysis has been developed in [39].

7. CONCLUSION

Some basic models for blade tip timing measurement systems have been defined: the first using one of the two sensors as a reference; the second with both sensors facing the blade tip.
Although the second approach is more versatile and easier to install, using one sensor as a reference is needed when the analysed machineries have higher degrees of irregularity.

Uncertainty on the parameters of these models has been theoretically analysed. Some simple formulas to relate the uncertainty and resolution obtained with reference to measurement system chosen parameters and sensor installation have been proposed. These formulas could be very useful to make proper choices over measurement system components, in relation to blade tips expected vibration characteristics and turbomachine parameters. So, this work can be used for blade tip timing measurement system design and installation guidelines.

REFERENCES

[1] S. L. Dixon, Fluid Mechanics, Thermodynamics of Turbomachinery, Elsevier, 2010, ISBN 978-1-85617-793-1.

[2] R. I. Lewis, Turbomachinery performance analysis. Arnold, 1996.

[3] P. Russhard, The rise and fall of the rotor blade strain gauge, Mechanisms and Machine Science, vol. 23, 2015, pp. 27–37. DOI: 10.1007/978-3-319-09918-7_2

[4] H. Guo, F. Duan, J. Zhang, Blade resonance parameter identification based on tip-timing method without the once-per revolution sensor, Mech Syst Signal Process 66-67 (2016), pp. 625–639. DOI: 10.1016/J.YMSSP.2015.06.016.

[5] B. Kestner, T. Lieuwen, C. Hill, L. Angello, J. Barron, C. A. Perullo, Correlation analysis of multiple sensors for industrial gas turbine compressor blade health monitoring, J Eng Gas Turbine Power 137(11) (2015). DOI: 10.1115/1.4030350

[6] F. Mevissen, M. Meo, A Review of NDT/Structural Health Monitoring Techniques for Hot Gas Components in Gas Turbines, Sensors 2019, Vol. 19, Page 711, vol. 19, no. 3, p. 711, Feb. 2019 DOI: 10.3390/s19030711

[7] G. Allevi, P. Castellini, P. Chiariotti, F. Docchio, R. Marsili, R. Montanini, S. Pasinetti, A. Quattrocchi, R. Rossetti, G. Rossi, G. Sansoni, E. P. Tomasini, Qualification of additive manufactured trabecular structures using a multi-instrumental approach, Proc. of the 2019 IEEE Int. Instrumentation and Measurement Technology Conf. (I2MTC), Auckland, New Zealand, 20-23 May 2019. DOI: 10.1109/I2MTC.2019.8826969

[8] S. Bornassi, T. M. Berruti, C. M. Firrone, G. Battiato, Vibration parameters identification of turbomachinery rotor blades under transient condition using Blade Tip-Timing measurements, Measurement 183 (2021) art. No. 109861. DOI: 10.1016/J.MEASUREMENT.2021.109861

[9] J. P. Feist, S. Berthier, Precision Temperature Detection Using a Phosphorescence Sensor Coating System on a Rolls-Royce Viper Engine, Proc. of the ASME Turbo Expo 2012: Turbine Technical Conference and Exposition. Volume 1: Aircraft Engine; Ceramics; Coal, Biomass and Alternative Fuels; Controls, Diagnostics and Instrumentation. Copenhagen, Denmark. June 11–15, 2012, pp. 917-926. DOI: 10.1115/GT2012-69779

[10] T. Tagashira, N. Sugiyama, Y. Matsuda, M. Matsuki, Measurement of blade tip clearance using an ultrasonic sensor, 35th Aerospace Sciences Meeting and Exhibit, 1997. DOI: 10.2514/6.1997-165

[11] F. Cannella, A. Garinei, R. Marsili, E. Speranzini, Dynamic mechanical analysis and thermoelasticity for investigating composite structural elements made with additive manufacturing, Composite Structures 185 (2018), pp. 466-473. DOI: 10.1016/j.compstruct.2017.11.029

[12] M. Becchetti, R. Flori, R. Marsili, G. L. Rossi, Measurement of stress and strain by a thermocamera, Proc. of the SEM annual conference, Albuquerque, New Mexico, June 1-4 2009. Online [Accessed 24 April 2023] https://www.researchgate.net/profile/Roberto-Marsili-2/publication/267236205_Measurement_of_stress_and_strain_by_a_thermocamera/links/576cf00908aedb18f3ec5bac/Measurement-of-stress-and-strain-by-a-thermocamera.pdf

[13] L. Andrenelli, N. Paone, G. Rossi, E. P. Tomasini, Non-Intrusive Measurement of Blade Tip Vibration in Turbomachines, Proceedings of the ASME Turbo Expo 5 (2015). DOI: 10.1115/91-GT-301

[14] L. J. Kiraly, Digital system for dynamic turbine engine blade displacement measurements, Proceedings of Measurement Methods in rotating Components of Turbomachinery, New Orleans, LA, 1979. Online [Accessed 24 April 2023]. https://ntrs.nasa.gov/api/citations/19800005857/downloads/19800005857.pdf

[15] W. B. Watkins, W. W. Robinson, R. M. Chi, Noncontact engine blade vibration measurements and analysis, AIAA Paper, 1985. DOI: 10.2514/6.1985-1473

[16] S. Heath, M. Inregun, An improved single-parameter tip-timing method for turbomachinery blade vibration measurements using optical laser probes, Journal of Mechanical Sciences 38(10) (1996), pp. 1047-1058. DOI: 10.1016/0020-7403(95)00116-6

[17] L. Andrenelli, N. Paone, G. Rossi, Large-bandwidth reflection fiber-optic sensors for turbomachinery rotor blade diagnostics, Sensors and Actuators A-physical 32 (1992), pp. 539-542. DOI: 10.1016/0924-4247%2892%2980040-A

[18] J. Zhang, F. Duan, G. Niu, J. Jiang, J. Li, A blade tip timing method based on a microwave sensor, Sensors 17(5) (1997) art. No. 1097. DOI: 10.3390/s17051097

[19] C. Huang, M. Hou, Technology for measurement of blade tip clearance in an aeroengine, Measurement and Control Technology, Measurement and Control Technology 27(3) (2011) pp. 27-32.

[20] E. Cardelli, A. Faba, R. Marsili, G. Rossi, R. Tomassini, Magnetic nondestructive testing of rotor blade tips, J Appl Phys 117(17) (2005) art. No. 17A705. DOI: 10.1063/1.4907180

[21] R. Tomassini, G. Rossi, J. F. Brouckaert, On the development of a magnetoresistive sensor for blade tip timing and blade tip clearance measurement systems, Review of Scientific Instrument

87(10) (2016) art. No. 20003.
DOI: 10.1063/1.4964858

[22] J. M. Gil-García, I. García, J. Zubia, G. Aranguren, Measurement of blade tip clearance and time of arrival in turbines using an optic sensor, Proceedings of 2015 Int. Conf. on Applied Electronics (AE), 08-09 September 2015. Online [Accessed 24 April 2023] https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7301053

[23] R. Reinhardt, D. Lancelle, O. Hagendorf, M. Schultalbers, O. Magnor; P. Duenow, Improved reference system for high precision blade tip timing on axial compressors, Proceedings of 2017 25th Optical Fiber Sensors Conference (OFS), Jeju, South Korea, 24-28 April 2017.
DOI: 10.1117/12.2263295

[24] I. García, J. Beloki, J. Zubia, G. Aldabaldetreku, M. Asunción Illarramendi, F. Jiménez, An optical fiber bundle sensor for tip clearance and tip timing measurements in a turbine rig, Sensors 13(6) (2013), pp. 7385-7398.
DOI: 10.3390/S130607385

[25] D. Ye, F. Duan, J. Jiang, G. Niu, Z. Liu, F. Li, Identification of vibration events in rotating blades using a fiber optical tip timing sensor, Sensors 19(7) (2019) art. No. 1482.
DOI: 10.3390/S19071482

[26] G. Rossi, J. Brouckaert, Design of blade tip timing measurements systems based on uncertainty analysis (2012) pdfs.semanticscholar.org. Online [Accessed 24 April 2023] https://pdfs.semanticscholar.org/6c78/2f97a0b8c181c29aeb9ac7f15d9fff110522.pdf

[27] M. Pan, Y. Yang, F. Guan, H. Hu, H. Xu, Sensors, and undefined 2017, Sparse representation based frequency detection and uncertainty reduction in blade tip timing measurement for multi-mode blade vibration monitoring, Sensors 17(8) (2017), art. No. 1745.
DOI: 10.3390/s17081745

[28] J. M. Gil-García, A. Solís, G. Aranguren, J. Zubia, An Architecture for On-Line Measurement of the Tip Clearance and Time of Arrival of a Bladed Disk of an Aircraft Engine 17(10) (2017) art. No. 2162.
DOI: 10.3390/S17102162

[29] J. Bouckaert, Tip timing and tip clearance problem in turbomachines, Von Karman Institute Lecture Series, 2(5) (2007).

[30] H. Roth, Vibration and clearance measurements on rotating blades using stationary probes, Von Karman Inst. for Fluid Dyn. Meas. Tech. in Turbomachines, 1981.

[31] S. Chatterton, L. Capponi, T. Tocci, M. Marrazzo, R. Marsili, and G. Rossi, Experimental Investigation on Hardware and Triggering Effect in Tip-Timing Measurement Uncertainty, Sensors 23(3) (2023), art. no. 1129.
DOI: 10.3390/S23031129

[32] T. Tocci, L. Capponi, G. Rossi, R. Marsili, M. Marrazzo, State-Space Model for Arrival Time Simulations and Methodology for Offline Blade Tip-Timing Software Characterization, Sensors 23(5) (2023), art. No. 2600.
DOI: 10.3390/S23052600

[33] C. Zhou, H. Hu, F. Guan, Y. Yang, Modelling and simulation of blade tip timing uncertainty from rotational speed fluctuation, 2017 Prognostics and System Health Management Conference, PHM-Harbin 2017 - Proceedings, Oct. 2017.
DOI: 10.1109/PHM.2017.8079252

[34] Russhard and Pete, Blade tip timing (BTT) uncertainties, AIP Conference Proceedings 1740(1) (2016), art. No. 020003.
DOI: 10.1063/1.4952657

[35] M. E. Mohamed et al., Experimental validation of FEM-computed stress to tip deflection ratios of aero-engine compressor blade vibration modes and quantification of associated uncertainties, Mechanical Systems and Signal Processing 178 (2022), art. No. 109257.
DOI: 10.1016/j.ymssp.2022.109257

[36] S. Catalucci, R. Marsili, M. Moretti, G. Rossi, Measurement, and undefined, Comparison between point cloud processing techniques, Measurement 127 (2018), pp. 221-226.
DOI: 10.1016/j.measurement.2018.05.111

[37] A. J. Jerri, The Shannon sampling theorem—Its various extensions and applications: A tutorial review, Proceedings of the IEEE 65(11) (1977), pp. 1565–1596.
DOI: 10.1109/PROC.1977.10771

[38] P. E. McCarty, J. W. Thompson, R. S. Ballard, Development of a Noninterference Technique for Measurement of Turbine Engine Compressor Blade Stress, Proc. of 16th Joint Propulsion Conf., Hartford, CT, USA, 30 June 1980 – 02 July 1980.
DOI: 10.2514/6.1980-1141

[39] P. Nava, N. Paone, G. L. Rossi, E. P. Tomasini, Design and experimental characterization of a nonintrusive measurement system of rotating blade vibration, Journal of Engineering for Gas Turbines and Power 116(3) (1994); pp. 657-662.
DOI: 10.1115/1.2906870