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PARTIAL DIFFERENTIATION OF AIR DENSITY IN MASS METROLOGY

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Abstract:
There are four major uncertainty components to be considered when performing mass comparisons. They are uncertainties of weighing process, reference weight used, air buoyancy, and mass comparator. The systematic effect of air buoyancy can be greatly reduced if the air density and the densities of the test and reference weights are known. This paper will emphasis on the uncertainty due to air buoyancy correction only. To calculate the uncertainty of air density correction, partial derivatives of temperature, barometric pressure and humidity must be performed. In this paper, two methods for partial differentiation of air density components are discussed.

Keywords: air buoyancy; air density; partial differentiation of air density; August-Roche-Magnus approximation equation

1. INTRODUCTION

In Mass Metrology, there are many important influence quantities have to be taken into account. These include thermal stabilisation time, magnetic susceptibility, density of standard and test weights, environmental conditions, weighing instrument, weighing cycles etc. According to OIML R111-1[1, equation C.6.5-1], the combined standard uncertainty for mass comparison process is calculated using equation (1).

$$u_\mathrm{c}(m_\mathrm{ct}) = \sqrt{u_\mathrm{w}^2(\overline{\Delta m_\mathrm{c}}) + u^2(m_\mathrm{cr}) + u_\mathrm{b}^2 + u_\mathrm{ba}^2} \quad (1)$$

where:
$u_\mathrm{c}(m_\mathrm{ct})$ = combined standard uncertainty of the conventional mass of the test weight
$u_\mathrm{w}(\overline{\Delta m_\mathrm{c}})$ = uncertainty of the weighing process
$u(m_\mathrm{cr})$ = uncertainty of the reference weight
u_b = uncertainty of the air buoyancy correction
u_ba = uncertainty of the balance

2. THE SYSTEMATIC EFFECTS OF AIR DENSITY

When highly accurate mass comparisons are performed under the laboratory conditions, buoyant force that acts upon weights, depending on their volume and the air density, can cause significant systematic error to the measurement results. The systematic effects of air buoyancy can be corrected if the air density is known. The air density is a function of various influence quantities such as temperature, barometric pressure and humidity. To correct for the effect of air density, OIML R111-1 provides the following equation [1, Eq. 10.2-1]:

$$m_\mathrm{ct} = m_\mathrm{cr}(1 + C) + \Delta\overline{m_\mathrm{c}} \quad (2)$$

with [1, Eq. 10.2-2]:

$$C = (\rho_\mathrm{a} - \rho_0)\left[\frac{\rho_\mathrm{t} - \rho_\mathrm{r}}{\rho_\mathrm{r}\rho_\mathrm{t}}\right] \quad (3)$$

where:
m_ct = conventional mass of the test weight
m_cr = conventional mass of the reference weight
$\Delta\overline{m_\mathrm{c}}$ = average weighing difference observed between test and reference weight
C = correction factor for air buoyancy
ρ_a = density of moist air
ρ_0 = density of air as a reference value equal to 1.2 kg·m^{-3}
ρ_t = density of the test weight
ρ_r = density of the reference weight

The uncertainty of air buoyancy correction is calculated according to OIML R111-1, as follows [1, equation C.6.3-1]:

$$u_\mathrm{b}^2 = \left[m_\mathrm{cr}\frac{\rho_\mathrm{r} - \rho_\mathrm{t}}{\rho_\mathrm{r}\rho_\mathrm{t}}u(\rho_\mathrm{a})\right]^2 + [m_\mathrm{cr}(\rho_\mathrm{a} - \rho_0)]^2\frac{u^2(\rho_\mathrm{t})}{\rho_\mathrm{t}^4} + m_\mathrm{cr}^2(\rho_\mathrm{a} - \rho_0)[(\rho_\mathrm{a} - \rho_0) - 2(\rho_\mathrm{al} - \rho_0)]\frac{u^2(\rho_\mathrm{r})}{\rho_\mathrm{r}^4} \quad (4)$$

where:
u_b = uncertainty of the air buoyancy correction
m_cr = conventional mass of the reference weight
ρ_r = density of the reference weight

ρ_t = density of the test weight
$u(\rho_a)$ = uncertainty of density of moist air
ρ_a = density of moist air
ρ_0 = density of the air as a reference value equal to 1.2 kg·m^{-3}
$u(\rho_t)$ = uncertainty of density of the test weight
ρ_{al} = air density during the (previous) calibration of the reference weight
$u(\rho_r)$ = uncertainty of density of reference weight

3. UNCERTAINTY OF AIR DENSITY

For air density ρ_a, OIML R111 provides an approximation formula [1, equation E.3-1] given as equation (5).

$$\rho_a = \frac{0.34848\,p - 0.009\,hr \times \exp(0.061\,t)}{273.15 + t} \quad (5)$$

where:
ρ_a = air density, in kg·m^{-3}
p = air pressure, in mbar or hPa
hr = relative humidity, in %
t = air temperature, in °C

Equation (5) has a relative uncertainty of 2 × 10^{-4} in the ranges 900 hPa < p < 1100 hPa, 10 °C < t < 30 °C and hr < 80 %. Under reference conditions of p = 1013.25 hPa, t = 20 °C, and hr = 50 %, equation (5) gives an air density of 1.199 294 kg·m^{-3}.

The variance of air density is obtained from equation (6) [1, equation C.6.3-3].

$$u^2(\rho_a) = u_F^2 + \left[\frac{\partial \rho_a}{\partial p} u_p\right]^2 + \left[\frac{\partial \rho_a}{\partial t} u_t\right]^2 + \left[\frac{\partial \rho_a}{\partial hr} u_{hr}\right]^2 \quad (6)$$

Under the same reference conditions, the following numerical values for sensitivity coefficients are given:

- u_F = [uncertainty of the formula used] (for CIPM formula u_F = 10^{-4} ρ_a)
- $\frac{\partial \rho_a}{\partial p} = 10^{-5} \rho_a\ \mathrm{Pa}^{-1}$
- $\frac{\partial \rho_a}{\partial t} = -3.4 \times 10^{-3} \rho_a\ \mathrm{K}^{-1}$
- $\frac{\partial \rho_a}{\partial hr} = -10^{-2} \rho_a$

However, if we calculate the three sensitivity coefficients by partial differentiation of equation (5), we obtain:

$$\frac{\partial \rho_a}{\partial p} = \frac{0.34848}{273.15 + t}\,\rho_a = \frac{0.34848}{293.15}\,\rho_a = 10^{-5}\,\rho_a\ \mathrm{Pa}^{-1} \quad (7)$$

$$\frac{\partial \rho_a}{\partial hr} = \frac{-0.009 \times \exp(0.061 t)}{273.15 + t}\rho_a = \frac{-0.009 \times \exp(0.061 \times 20)}{273.15 + 20}\rho_a = -10^{-2}\,\rho_a \quad (8)$$

$$\frac{\partial \rho_a}{\partial t} = \frac{-0.34848\,p}{(273.15 + t)^2} - \frac{0.009 hr\left[(273.15 + t)\frac{d\,e^{0.061t}}{dt} - e^{0.061t}\frac{d^{(273.15+t)}}{dt}\right]}{(273.15 + t)^2}\rho_a$$
$$= \frac{-0.34848\,p}{(273.15 + t)^2} - \frac{0.009(50)e^{0.061t}[293.15 \times 0.061 - 1]}{(273.15 + t)^2}\rho_a \quad (9)$$
$$= \frac{-0.34848 \times 1013.15}{(273.15 + 20)^2} - \frac{0.009(50)e^{0.061 \times 20}[293.15 \times 0.061 - 1]}{(273.15 + 20)^2}\rho_a$$
$$= -4.4 \times 10^{-3} \rho_a\ \mathrm{K}^{-1}$$

From the above calculations, we found that sensitivity coefficient $\frac{\partial \rho_a}{\partial p}$ and $\frac{\partial \rho_a}{\partial hr}$ values are exactly the same as OIML R111 values, but not the value of $\frac{\partial \rho_a}{\partial t}$. Therefore, we try to consider how to obtain the same $\frac{\partial \rho_a}{\partial t}$ value as provided by OIML R111.

According to the August-Roche-Magnus approximation equation [4, equation 4],

$$RH = 100 \times \frac{e^{\frac{17.625\,TD}{243.04 + TD}}}{e^{\frac{17.625\,t}{243.04 + t}}} \quad (10)$$

The changes in temperature t will also cause the relative humidity RH to change, if the dew point temperature TD is kept constant. In this case, we need to consider not only the changes in temperature but also the correlation between temperature and relative humidity according to August-Roche-Magnus approximation equation. In this equation, there are three variables, namely temperature t, relative humidity RH and dew point temperature TD.

The dew point temperature can be calculated by using August-Roche-Magnus approximation equation [4, equation 5] as follows:

$$TD = 243.04 \frac{\ln\left(\frac{RH}{100}\right) + \left(\frac{17.625\,t}{243.04 + t}\right)}{\left(17.625 - \ln\left(\frac{RH}{100}\right) - \frac{17.625\,t}{(243.04 + t)}\right)} \quad (11)$$

Substituting in values of RH = 50 %, t = 20 °C and p = 101 325 Pa, we get:

$$TD = 9.261\ \text{°C} \qquad (12)$$

If we keep TD constant at 9.261 °C while the temperature changes from; t = 20 °C to t = 21 °C, from equation (11), we get:

For t = 20 °C:

$$RH = 100\ \times \frac{e^{\frac{17.625\times9.261}{243.04+9.261}}}{e^{\frac{17.625\times20}{243.04+20}}} = 50\ \% \qquad (13)$$

For t = 21 °C:

$$RH = 100\ \times \frac{e^{\frac{17.625\times9.261}{243.04+9.261}}}{e^{\frac{17.625\times21}{243.04+21}}} = 47\ \% \qquad (14)$$

From the above examples, the change of temperature from 20 °C to 21 °C can cause the relative humidity to change from 50 %RH to 47 %RH. Thus the air density also changes to:

$$\rho_{a1} = \frac{0.34848\times1013.25 - 0.009(47)\mathrm{e}^{0.061\times21}}{273.15+21} \qquad (15)$$

$$= 1.195\ 221\ \mathrm{kg\cdot m^{-3}}$$

And thus, from equations (5) and (15):

$$\frac{\partial\rho_a}{\partial t} = \frac{\rho_{a1} - \rho_0}{\rho_0} = \frac{1.195221 - 1.199294}{1.199294} \qquad (16)$$

$$= -3.4\times10^{-3}\rho_a\ \mathrm{K^{-1}}$$

This value of $\frac{\partial\rho_a}{\partial t}$ is now exactly the same as the value provided in OIML R111. Therefore, we need to consider the correlation between temperature and humidity when we perform partial differentiation of temperature against the air density.

Note: In the August-Roche-Magnus approximation equation, there are three variables, namely temperature t, relative humidity RH and dew point temperature TD. For our purposes, dew point temperature TD is considered constant while the temperature t is changing. In this case any changes in temperature will also cause the relative humidity to change in the opposite direction. Otherwise, we would not get the same value of $\frac{\partial\rho_a}{\partial t}$ as that provided in OIML R111.

4. DISCUSSION

Relative humidity is the ratio, usually expressed in %, of the partial pressure of water vapour to the saturation (equilibrium) vapour pressure of water at a given temperature.

Relative humidity depends on the temperature and pressure of the system of interest. As the air's temperature increases, it can hold more water molecules, decreasing its relative humidity. When temperatures drop, relative humidity increases. This is because colder air does not require as much moisture to become saturated as warmer air. 100 % relative humidity of the air occurs when the air temperature is the same as the dew point value.

Table 1 shows the relationship between temperature and relative humidity. The slope and correlation between these two variables are shown in Figure 1. In this figure, it shows the correlation coefficient close to -1.0.

Correlation coefficient is used to measure the strength of the relationship between two variables. The range of values for correlation coefficient is -1.0 to 1.0. In other words, the values cannot exceed 1.0 nor be less than -1.0.

Table 1: Relationship between temperature and relative humidity

Temperature / °C	**Relative Humidity / %RH**
16	64.3
17	60.3
18	56.6
19	53.2
20	50.0
21	47.0
22	44.2
23	41.6
24	39.2

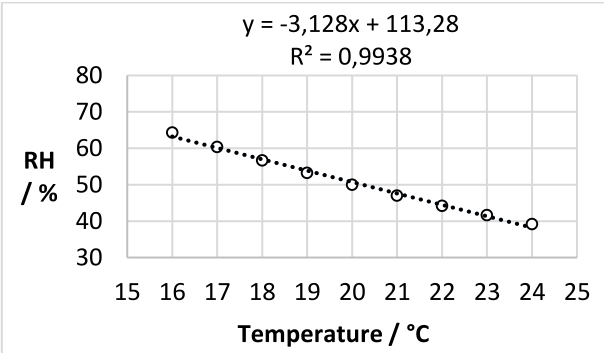

Figure 1: Correlation between temperature and relative humidity

5. SUMMARY

In mass comparison uncertainty evaluation, we need to consider all contributions that are of significant. Among them, one of the important uncertainty sources is air buoyancy correction. For air buoyancy correction, air density is the important parameter. Air density depends on the environmental conditions such as temperature, barometric pressure and humidity. The uncertainty of air density $u(\rho_\mathrm{a})$ is calculated from the standard uncertainty of air pressure $u(p)$, temperature $u(t)$, and relative humidity $u(rh)$, together with their sensitivity coefficients, the partial derivatives of $\frac{\partial \rho_a}{\partial p}$, $\frac{\partial \rho_a}{\partial t}$, and $\frac{\partial \rho_a}{\partial hr}$. However, when we use the ordinary partial differentiation, the derived value of $\frac{\partial \rho_a}{\partial t}$ is not same as the value given in OIML R111. So, we need to consider the correlation between temperature and humidity when we perform partial differentiation of temperature against the air density.

We use August-Roche-Magnus approximation equation to determine the correlation between temperature and relative humidity. Finally, we can obtain the same value for $\frac{\partial \rho_a}{\partial t}$ as given by OIML R111.

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