Analysis of Gear Wheel-shaft Joint Characterized by Comparable Pitch Diameter and Mounting Diameter

J. Ryś, H. Sanecki, A. Trojnacki

This paper presents the design procedure for a gear wheel-shaft direct frictional joint. The small difference between the operating pitch diameter of the gear and the mounting diameter of the frictional joint is the key feature of the connection. The contact surface of the frictional joint must be placed outside the bottom land of the gear, and the geometry of the joint is limited to the specific type of solutions.

The strength analysis is based on the relation between the torque and statistical load intensity of the gear transmission. Several dimensionless parameters are introduced to simplify the calculations. Stress-strain verifying analysis with respect to combined loading, the condition of appropriate load-carrying capacity of the frictional joint and the fatigue strength of the shaft are applied to obtain the relations between the dimensions of the joint and other parameters. The final engineering solution may then be suggested. The approach is illustrated by a numerical example.

The proposed procedure can be useful in design projects for small, high-powered modern reducers and new-generation geared motors, in particular when manufactured in various series of types.

Keywords: geared motor, gear wheel, frictional joint, strength analysis, fatigue.

1 Introduction

Users expect modern geared motors and reducers to have high reduction ratios and small overall dimensions and total weight as well as a closed, rigid one-part housing containing the toothed elements mounted on rolling bearings, as shown in [1]. The above requirements can be satisfied for fine diametral pitch of gears made of high-quality toughening or carburizing alloy steel. The transmission ratio of the speed reducer is a function of the number of teeth, in particular in the pinion. The assumed high ratio forces the small number of teeth in the pinion. The fine diametral pitch and the small number of teeth cause the operating pitch diameter of the gear wheel often to be comparable to the output shaft diameter of the applied electric motor. This results in serious difficulties connected with the mounting of the gear.

This paper deals with the strength analysis of one specific version of the gear wheel-shaft connection, and the tapered self-locking frictional joint is considered. Such a connection is preferred in application to lot production of geared motors, manufactured in various series of types.

The strength analysis of the joint is based on the relation between the torque and statistical load intensity of the gear transmission. Several geometric, strength and engineering dimensionless parameters are introduced to simplify the calculations and to generalize the approach. The procedure requires initial selection of the permissible range of the parameters. Stress-strain analysis with respect to combined bending and torsion of the circular shaft, the condition of right contact pressure distribution in the frictional joint and fatigue strength investigations of the shaft lead to the relations between the fundamental dimension of the joint and other parameters. The final acceptable engineering solution may then be suggested and verified. The results of a numerical example illustrate the influence of the considered parameters of the gear wheel and the joint on its functional dimensions.

2 Engineering solution of a gear wheel-shaft joint

The connection between the gear wheel and the shaft is considered in the case when the operating pitch diameter d of the gear is comparable to the mounting diameter D. The connection is realized by means of the tapered self-locking frictional joint. The geometry of the joint is presented in Fig. 1 for the cylindrical helical gear. The gear wheel unit consists of the pinion and the sleeve of external diameter D_s and internal conical hole of taper C. The output shaft of nominal diameter D of the electric motor is executed with the same taper. The length of contact of the coupled elements is l and the relation between the cone angle φ and the taper C is $\varphi = \text{arc tg}(C/2)$. The bolted joint with a slotted nut as a locking device between the pinion and the shaft is applied to produce the effective axial force on the subassembly in the frictional connection.

The transitory zone between the pinion and the sleeve (Details B and C in Fig. 1) must be carefully designed. Both production technology requirements and operational requirements must be satisfied and these parts should also be optimally designed with respect to fatigue strength. The main purpose is to obtain dimension e as small as possible, because this leads to a lower fundamental dimension L of the connection and consequently the bending stress produced by the teeth forces is less.

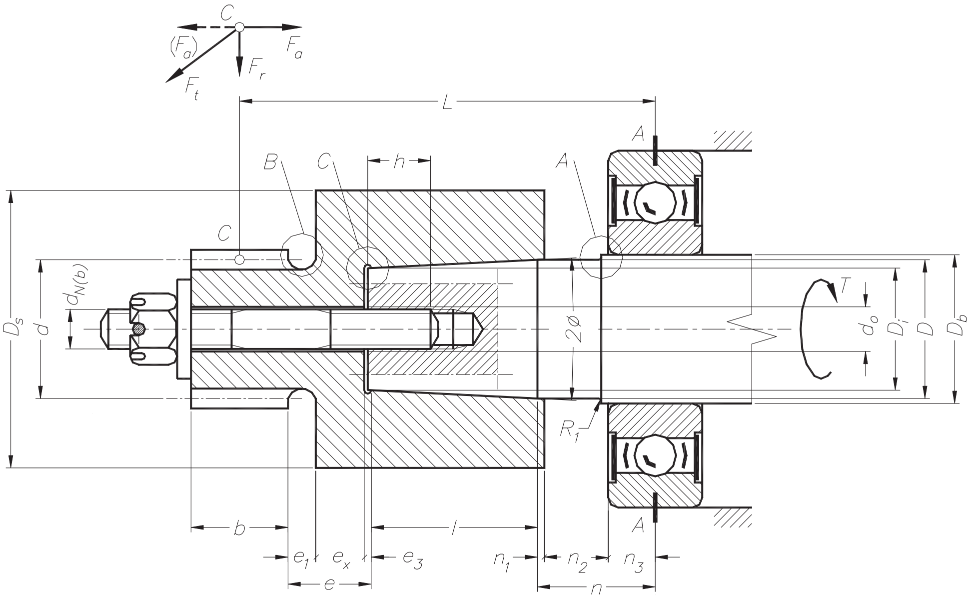

Fig. 1: Geometry of the gear wheel-shaft joint

3 Strength analysis of frictional joint

3.1 Combined strength of the shaft with respect to bending and torsion

The analysis is based on the relation between the rated torque T and the statistical load intensity Q_u of the gear transmission, introduced by Müller [2]

$$T = \frac{bd^2}{2}\frac{u}{u+1}Q_u\,, \tag{1}$$

and on the strength condition for the circular solid shaft of diameter D with respect to combined repeated and reversed bending, and repeated one-direction torsion

$$\frac{M_{bE}}{S} \le s_{bn}^{r}\,, \tag{2}$$

where b is the width of the gear face, d is the operating pitch diameter, $u = z_2/z_1$ denotes the ratio of the first stage, S is the section modulus of the cross-section of the shaft, and s_{bn}^{r} stands for endurance strength in repeated and reversed bending. The equivalent bending moment in Eq. (2) can be expressed as $M_{bE} = \sqrt{M_b^2 + (\chi\, M_t/2)^2}$, where $\chi = s_{bn}^{r}/s_{tn}^{o}$, and s_{tn}^{o} is the endurance strength in repeated one-direction torsion. The distance from the midpoint C of the face to the cross-section A–A at the first bearing of the electric motor (where the moment M_b reaches its maximum) is L. The moments M_b and M_t are produced by the force between the teeth, the components of which are

$$F_t = \frac{2T}{d}\,,$$
$$F_r = F_t\frac{\mathrm{tg}\alpha}{\cos\beta} = \frac{2T}{d}\frac{\mathrm{tg}\alpha}{\cos\beta}\,, \tag{3}$$
$$F_a = \pm F_t\,\mathrm{tg}\beta = \pm\frac{2T}{d}\mathrm{tg}\beta$$

and may be expressed as functions of the statistical load intensity Q_u as

$$M_b = \psi\delta^2\sqrt{\left(\frac{2\lambda\,\mathrm{tg}\alpha \mp \delta\sin\beta}{2\cos\beta}\right)^2 + \lambda^2}\,D^3\frac{u}{u+1}Q_u\,,$$
$$M_t = T = \frac{\psi\delta^3}{2}D^3\frac{u}{u+1}Q_u\,, \tag{4}$$

where α and β stand for the pressure angle and helix angle, respectively, and several dimensionless parameters are introduced as follows: $\psi = b/d$, $\delta = d/D$, $\lambda = L/D$. The negative sign in Eq. (4_1) must be taken if the directions of rotation and the helix angle are designed in such a way that the component force F_a between the teeth has the same sense as the total force F^* in the frictional joint (as depicted in Fig. 1 by the solid line) and causes an increase of loading in the joint during service. The positive sign should be applied in the opposite case.

Combining Eqs. (2) and (4) enables the fundamental dimensionless parameter λ of the frictional joint to be determined

$$\left[\left(\frac{\mathrm{tg}\alpha}{\cos\beta}\right)^2 + 1\right]\lambda^2 \mp \left(\delta\frac{\mathrm{tg}\alpha}{\cos\beta}\mathrm{tg}\beta\right)\lambda + \left(\frac{\delta}{2}\right)^2\left[\mathrm{tg}^2\beta + \left(\frac{\chi}{2}\right)^2\right] - \left(\frac{\pi s_{bn}^{r}}{32\psi\delta^2}\frac{u+1}{u}\frac{1}{Q_u}\right)^2 \le 0 \tag{5}$$

in terms of other parameters of gear transmission (u, ψ, δ, α, β), strength properties of the material of the shaft (s_{bn}^{r}, χ), and statistical load intensity Q_u of gear transmission.

3.2 Load-carrying capacity of the frictional joint

Under the assumption that in the tapered self-locking frictional joint under study the distribution of the contact pressure is constant ($p = \mathrm{const}$) and additionally taking the coefficient of friction the same over the total surface of contact ($\mu = \mathrm{const}$), the torque T expressed by Eq. (1) may be carried by the joint if the contact pressure is

$$p \geq \frac{6\psi\,\delta^3 \sin\varphi}{\pi\mu\left[1-\left(1-2\xi\,\mathrm{tg}\,\varphi\right)^3\right]}\frac{u}{u+1}Q_u\,, \tag{6}$$

where $\xi = l/D$ is the dimensionless length of the frictional joint. The above condition is satisfied for the longitudinal force acting in the joint

$$F \geq \frac{3\psi\,\delta^3}{2\mu}\left(\sin\varphi + \mu\cos\varphi\right)\frac{1-\left(1-2\xi\,\mathrm{tg}\,\varphi\right)^2}{1-\left(1-2\xi\,\mathrm{tg}\,\varphi\right)^3}D^2\frac{u}{u+1}Q_u\,. \tag{7}$$

The contact pressure in the joint is caused by the force $F^* = F_{(b)} \pm \Delta F_a^{(2)}$ (Fig. 2), where $F_{(b)}$ is produced in the bolted joint on the subassembly, and $\Delta F_a^{(2)}$ stands for the portion of the resultant force $F_a = \Delta F_a^{(1)} + \Delta F_a^{(2)}$ that is transmitted to the surface of the frictional joint generating its additional loading or unloading.

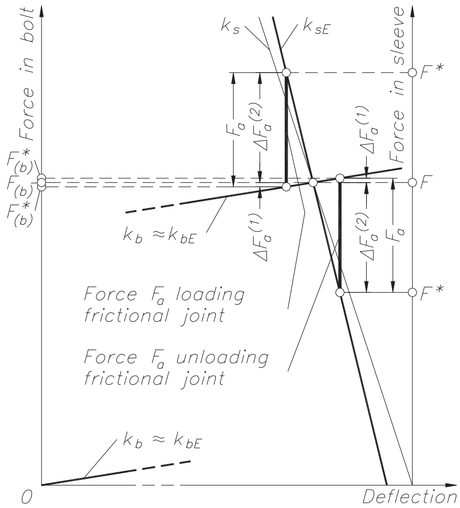

Fig. 2: Diagram of the forces acting at the surface of the frictional joint

The strength conditions of the bolted joint must be satisfied with respect to tension for the minor diameter $d_{m(b)}$ of the bolt and with respect to the contact pressure at the thread surface along portion h, which lead to the equations, respectively

$$d_{m(b)} \geq \sqrt{\frac{4\left(F_{(b)} \mp \Delta F_a^{(1)}\right)}{\pi\, s_{t(b)}}}\,,$$
$$h \geq \frac{4P}{\pi\left(d_{(b)}^2 - D_1^2\right)p_{(b)}}\left(F_{(b)} \mp \Delta F_a^{(1)}\right), \tag{8}$$

where P is the pitch of the bolt thread, $d_{(b)}$ stands for the major bolt diameter, D_1 is the minor nut diameter (in the sleeve), and $s_{t(b)}$ and $p_{(b)}$ denote the tensile strength and permissible contact pressure of the bolt, respectively.

An analysis of the frictional joint under initial loading $F_{(b)}$ and additional loading F_a is applied to determine the components $\Delta F_a^{(1)}$ and $\Delta F_a^{(2)}$. The force F_a facing as shown in Fig. 3 unloads the portion of the pinion of length $b/2$, the spring rate of which is k_{p1}, and the portion of the bolt of length $b + e_1 + e_x + e_3/2$ and spring rate k_b – Fig. 3. At the same time the remaining part of the pinion of length $b/2 + e_1$ and spring rate k_{p2} is loaded as well as a part of the sleeve of length $e_x + e_3 + l/2$, the spring rate of which is denoted by k_s. By applying well-known calculations, the components of force F_a can be found

$$\Delta F_a^{(1)} = \frac{k_{bE}}{k_{bE} + k_{sE}}F_a\,,$$
$$\Delta F_a^{(2)} = \frac{k_{sE}}{k_{bE} + k_{sE}}F_a\,, \tag{9}$$

where k_{bE} denotes the equivalent spring rate of k_{p1} and k_b, and k_{sE} is the equivalent spring rate of k_{p2} and k_s.

The frictional joint is simultaneously loaded with the bending moment M_{bp}, which disturbs the contact pressure p which is initially assumed to be uniform (Fig. 4). The assembly pressure and the pressure produced by the bending moment lead to the non-uniform distribution of the resultant contact pressure in radial direction. The values of the resultant contact pressure as well as the effective friction torque carried by the joint with the contribution of M_{bp}, are practically the same. Nevertheless, the bending moment M_{bp} must be limited to prevent the minimal pressure on the acceptable value $p_{\min}$. It is usually assumed that the resultant contact pressure changes

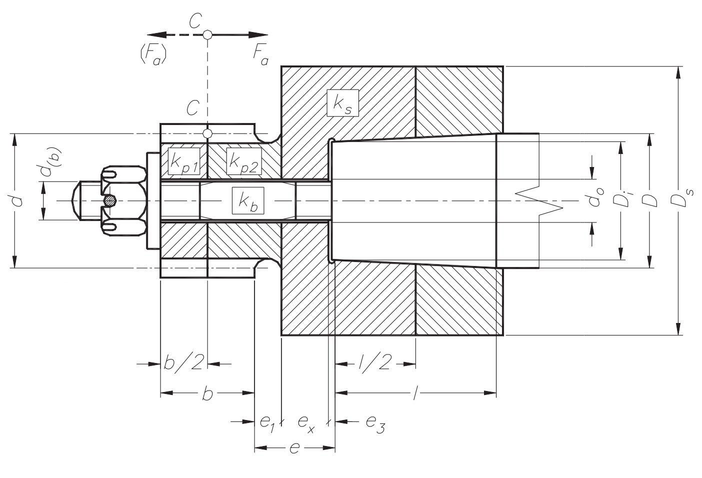

Fig. 3: Definition of spring rates of the considered portions of the frictional joint

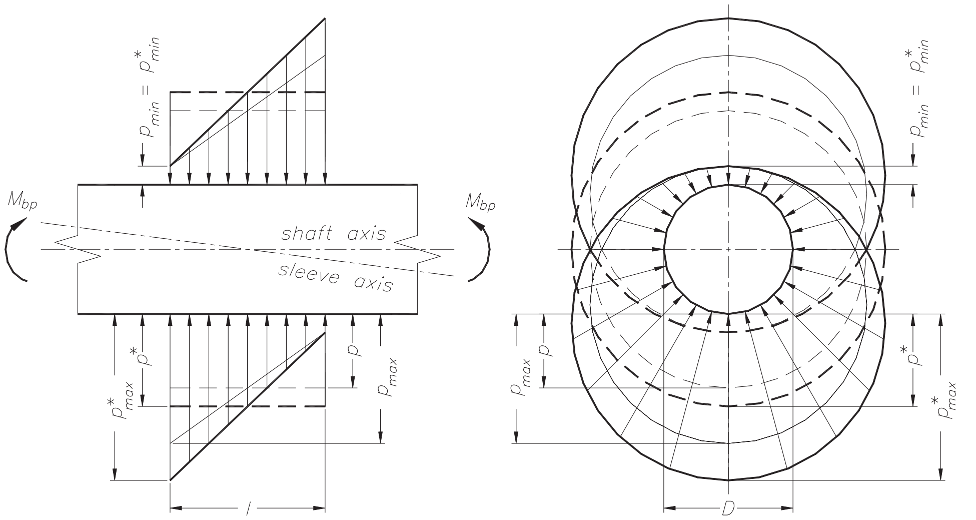

Fig. 4: Distribution of resultant contact pressure in the frictional joint loaded with the bending moment

linearly along the length l of the joint and must exceed the minimum value defined as $cp(c \cong 0.25)$, which leads to the condition

$$p_{\min} \geq cp\,. \tag{10}$$

The maximum value of tensile force $F_{(b)}$ which can be applied on assembly to the bolt depends on its nominal minor diameter $d_{mN(b)}$ and the mechanical properties of the bolt material $s_{t(b)}$

$$F_{(b)} \leq \frac{\pi\, d^2_{mN(b)}\, s_{t(b)}}{4} - \Delta F_a^{(1)}. \tag{11}$$

The forces $F_{(b)}$ and $\Delta F_a^{(2)}$ produce the resultant contact pressure at the conical surface of the joint

$$p^* \leq \frac{\sin\varphi}{(\sin\varphi + \mu\cos\varphi)\left[1-(1-2\xi\,\mathrm{tg}\,\varphi)^2\right]}\cdot \left(v^2 s_{t(b)} + \frac{-k_{bE} \pm k_{sE}}{k_{bE}+k_{sE}}\,\frac{4\,\psi\,\delta^2}{\pi}\,\mathrm{tg}\,\beta\,\frac{u}{u+1}Q_u\right), \tag{12}$$

where $v = d_{mN(b)}/D$. In this case condition (10) takes the form $p^*_{\min} \geq cp$.

The well-based assumption that the taper C of the conical joint is small and it can be replaced by the cylindrical joint in the above considerations leads to the limiting condition for the bending moment in the frictional joint

$$M_{bp} \leq \frac{\psi\,\delta^3\xi^2(1-c)\sin\varphi}{2\mu\left[1-(1-2\xi\,\mathrm{tg}\,\varphi)^3\right]}D^3\,\frac{u}{u+1}Q_u\,. \tag{13}$$

Bending moment M_{bp} is produced by the forces acting between the teeth. The influence of M_{bp} on the distribution of contact pressure in the joint is derived for the forces F_t and F_r acting on the arm $L_p = L - (l/2 + n)$, i.e., at the midpoint of the frictional joint. After substituting $\lambda_p = L_p/D$ and applying Eq. (13) one parameter of the joint (consequently λ_p) can be expressed in terms of the other parameters

$$\left[\left(\frac{\mathrm{tg}\,\alpha}{\cos\beta}\right)^2+1\right]\lambda_p^2 \mp \left(\delta\frac{\mathrm{tg}\,\alpha}{\cos\beta}\,\mathrm{tg}\,\beta\right)\lambda_p + \left(\frac{\delta}{2}\right)^2\left\langle \mathrm{tg}^2\beta - \left\{\frac{\xi(\omega-c)\cos\varphi}{2\mu\left[4(\xi\,\mathrm{tg}\,\varphi)^2-6\xi\,\mathrm{tg}\,\varphi+3\right]}\right\}^2\right\rangle \leq 0, \tag{14}$$

where

$$w = \frac{p^*}{p} = \mu\,\frac{1-(1-2\xi\,\mathrm{tg}\varphi)^3}{(\sin\varphi+\mu\cos\varphi)\left[1-(1-2\xi\,\mathrm{tg}\varphi)^2\right]}\cdot \left(\frac{\pi v^2 s_{t(b)}}{6\psi\delta^3}\,\frac{u+1}{u}\,\frac{1}{Q_u} + \frac{-k_{bE}\pm k_{sE}}{k_{bE}+k_{sE}}\,\frac{2}{3\delta}\,\mathrm{tg}\beta\right). \tag{15}$$

In Eqs. (12) and (15) positive signs should be used when force F_a faces as shown in Fig. 3 and negative signs in the opposite case. The maximum value $p^*_{\max} = (2\omega - c)p$ of the contact pressure distribution changed by the bending moment M_{bp} must be limited to permissible compressive loading p_c for both members of the frictional joint.

3.3 Fatigue strength of the shaft

The frictional joint must be evaluated with respect to fatigue strength. The usual procedure is to estimate the fatigue factor of safety FS. The members of the joint are subjected to combined loading: bending and torsion, in which case the effective factor of safety is described by Niezgodzinski et al [3] as

$$FS = \frac{FS_b FS_t}{\sqrt{FS_b^2 + FS_t^2}}\,, \tag{16}$$

where the component fatigue factors of safety in reversed bending and one–direction torsion are, respectively

$$FS_b = \frac{S^r_{bn}}{(\beta\gamma)_b\,\sigma_a}\,, \qquad FS_t = \frac{S^r_{tn}}{\tau_m\left(2\dfrac{S^r_{tn}}{S^o_{tn}}-1\right)+(\beta\gamma)_t\,\tau_a}\,. \tag{17}$$

In Eqs. (17), $(\beta\gamma)_b$ and $(\beta\gamma)_t$ denote the products of partial fatigue factors, calculated for the considered part of the shaft in pure bending separately, and in pure torsion separately, and S^r_{bn}, S^r_{tn}, S^o_{tn} stand for the endurance stresses in reversed bending, reversed torsion, and one-direction torsion, respectively.

The fatigue calculations of the shaft were carried out for a cross-section with two diameters D_b and D joined by a fillet of radius R_1 (Detail A in Fig. 1), located at a distance $L_f = L - n_3$ from the midpoint of the face. This cross-section was recognized as the weakest plane with respect to stress concentration.

In the solid shaft of diameter D subjected to repeated and reversed bending with the moment M_{bf} and also subjected to repeated one-direction torsion with torque M_t, the amplitude of the bending stress is $\sigma_a = 32 M_{bf} / \pi D^3$, and the amplitude of the shear stress equals the mean shear stress $\tau_a = \tau_m = 8 M_t / \pi D^3$. The above relations may be combined with (4) and (17), and introduced into (16) after substituting $\lambda_f = L_f / D$. Assuming that $FS \ge FS_w$, where FS_w denotes the working fatigue factor of safety introduced for the considered plane of the shaft, equation (16) can be rearranged and written in the form

$$\left[\left(\frac{\operatorname{tg}\alpha}{\cos\beta}\right)^2 + 1\right]\lambda_f^2 \mp \left(\delta \frac{\operatorname{tg}\alpha}{\cos\beta}\operatorname{tg}\beta\right)\lambda_f +$$
$$+\left(\frac{\delta}{2}\right)^2 \left\{\operatorname{tg}^2\beta + \frac{1}{16(\beta\gamma)_b^2}\left[2\frac{S^r_{tn}}{S^o_{tn}} + (\beta\gamma)_t - 1\right]^2 \left(\frac{S^r_{bn}}{S^r_{tn}}\right)^2\right\} + \qquad (18)$$
$$-\left(\frac{\pi}{32\,\psi\,\delta^2}\,\frac{u+1}{u}\,\frac{1}{Q_u}\right)^2 \left[\frac{S^r_{bn}}{(\beta\gamma)_b FS_w}\right]^2 \le 0.$$

Following the approach presented by Niezgodzinski at al in [3], FS_w may be estimated by applying the so called required fatigue factor of safety defined as

$$FS_r = x_1\, x_2\, x_3\, x_4\,, \qquad (19)$$

where x_1 is the factor for the reliability of the assumptions, x_2 is the factor for the importance of a machine part, x_3 is the factor of material homogeneity, and x_4 stands for the factor of preservation of dimensions.

4 Acceptable range of dimensionless parameters

All resulting equations in the per are derived in dimensionless quantities related to diameter D. In some cases, however, the geometry of the frictional joint had to be specified and $D = 28$ [mm] was assumed. As shown by Krukowski et al in [4], frictional joints are usually designed assuming $\mu = 0.10 \div 0.20$ as for polished clean parts, and $c = 0.25$. Conical self–locking connections are executed with the taper between 1:5 ÷ 1:100 and with the length $l = (1 \div 2)D$, i.e., $\xi = 1.0 \div 2.0$. The dimensionless diameter of the joint was assumed $\delta = 0.8 \div 1.2$ and the outer diameter of the sleeve $D_s = 2D$.

The detailed calculations were carried out for a cylindrical helical pinion with the usual pressure angle $\alpha = 20$ [°] and he helix angle $\beta = 25$ [°]. On the basis of literature recommendations the dimensionless width of the face was assumed $\psi = 0.5 \div 0.9$ and the ratio of the first stage of transmission $u = 5 \div 9$. statistical load intensity Q_u depends on the strength properties of the material applied for the pinion and its heat treatment. The gears of high-powered gear units for general engineering are usually made of carburizing or nitriding alloy steel, e.g., 18HGT or toughening alloy steel, e.g., 45HN, for which $Q_u = 2 \div 5$ [MPa].

Three grades of steel for the shaft were taken into account: toughening quality carbon steels 35 and 45, and toughening alloy steel 45HN. Some strength properties of these materials necessary for the calculations were introduced after Ciszewski et al [5] and Lysakowski [6].

The specific values of several dimensions must be introduced in the fatigue calculations of the shaft. Well-known relations adopted in shafting (Dabrowski [7]) for the diameter reduction ratio $D_b / D \le 1.2$ and for the fillet radius $R_1 \ge 0.25(D_b - D)$ should be satisfied – Detail A in Fig. 1. In the flange-type electric motor SKg112M (made by TAMEL, Poland), the end of the output shaft of diameter $D = 28$ [mm]

Table 1: Results of fatigue calculations

Quantity	Steel 35	Steel 45	45 HN
$R_{m\,\min}$ [MPa]	580	660	1030
$R_{e\,\min}$ [MPa]	365	410	835
S^r_{bn} [MPa]	255	310	475
S^r_{tn} [MPa]	152	183	285
S^o_{tn} [MPa]	300	365	520
s^r_{bn} [MPa]	64	78	119
s^o_{tn} [MPa]	75	95	130
p_c [MPa]	87	98	120
ρ_m [mm]	0.62	0.57	0.36
ρ [mm]	1.12	1.07	0.86
ρ/D	0.0400	0.0382	0.0307
η	0.83	0.86	0.95
$(\alpha_k)_b$	1.81	1.82	1.93
$(\alpha_k)_t$	1.30	1.32	1.36
$(\beta_k)_b$	1.6723	1.7052	1.8835
$(\beta_k)_t$	1.2490	1.2752	1.3420
β_p	1.07	1.08	1.13
$(\beta)_b$	1.7894	1.8416	2.1284
$(\beta)_t$	1.3364	1.3772	1.5165
$(\gamma)_b$	1.24	1.26	1.28
$(\gamma)_t$	1.16	1.18	1.20
$(\beta\gamma)_b$	2.2189	2.3204	2.7244
$(\beta\gamma)_t$	1.5502	1.6251	1.8198

is supported on the ball bearing type 6306ZZ, the bore diameter of which is $D_b = 30$ [mm], i.e., $D_b/D = 1.0714$. The minimum (i.e, the worst with respect to fatigue) fillet radius for the above data equals $R_1 = 0.5$ [mm], and this was applied in the fatigue calculations. The effect of surface conditions on the stress concentration was introduced as for mechanical polishing, after which the surface quality of roughness number $R_a = 0.32$ [μm] may be obtained. The partial fatigue factors β and γ in reversed bending and one-direction torsion were arrived at through usual considerations, in which the appropriate equations, tables and diagrams presented by Niezgodzinski et al in [3] were applied. Several results are gathered in Table 1. The working factor of safety $FS_w = 1.5246$ was estimated employing the proposed criterion in which the partial factors of the required factor of safety FS_r were equal $x_1 = 1.1$, $x_2 = 1.2$, $x_3 = 1.1$, $x_4 = 1.05$, respectively.

The Polish Standard [8] recommends application of the M8 screw at the end of a conical shaft of diameter $D = 28$ [mm]. It was assumed that the screw is made of ordinary carbon steel St5.

Dimensions e_1, e_x, e_3, n_1, n_2 and n_3 must be initially estimated to evaluate parameter λ. They were related to the diameter D, as $\varepsilon_1 = e_1/D$, $\varepsilon_x = e_x/D$, $\varepsilon_3 = e_3/D$, $\eta_1 = n_1/D$, $\eta_2 = n_2/D$ and $\eta_3 = n_3/D$. The fundamental parameter λ of the frictional joint may be expressed as a sum

$$\lambda = 0.5\psi\,\delta + \varepsilon_1 + \varepsilon_x + \varepsilon_3 + \xi + \eta_1 + \eta_2 + \eta_3\,. \qquad (20)$$

The extreme values of each component in Eq. (20) were estimated with respect to the production and operational requirements: $\varepsilon_1 = 0.2 \div 0.4$, $\varepsilon_x = 0.35 \div 0.7$, $\varepsilon_3 = 0.05 \div 0.07$, $\eta_1 = 0.05 \div 0.07$, $\eta_2 = 0.4 \div 1.2$ and $\eta_3 = 0.34$ (as for the electric motor SKg112M). The acceptable range of parameter λ was obtained employing Eq. (20), and its extreme values are $\lambda_{min} = 2.59$, $\lambda_{max} = 4.82$, and the mean value $\lambda_m = 3.71$. The solution of the problem must then satisfy the inequality $\lambda_{min} \le \lambda \le \lambda_{max}$.

5 Numerical example

Parameter λ was chosen as the fundamental quantity of the joint, and it was subjected to investigation in the design process. The explicit influence of other parameters on the parameter λ excludes the standard optimization procedure with imposed appropriate production technology, operational and strength constraints. The set of resulting conditions (5), (14) and (18) was rearranged and uniformly presented as $\lambda \le f\left(u, \psi, \delta, \xi, C, \mu, \text{material properties}, Q_u\right)$, introducing the substitutions $\lambda_p = \lambda - \left(\xi/2 + \eta\right)$ and $\lambda_f = \lambda - \eta_3$. The influence of each single variable parameter on the total dimensionless length l of the joint was examined individually, while the other parameters were set as the mean values in the acceptable ranges (suggested in Sect. 4), i.e.: $u = 7$, $\psi = 0.7$, $\delta = 1.0$, $\xi = 1.5$, $C = 1{:}20$ (Morse taper), $\mu = 0.15$, $Q_u = 3.5$ [MPa] and steel 45 applied for the shaft.

A special comment should be made on the relation between the component forces $\Delta F_a^{(1)}$ and $\Delta F_a^{(2)}$. The detailed calculations carried out for the mean values of the parameters indicate that the equivalent spring rate k_{sE} is much greater than the equivalent spring rate $k_{bE} - k_{sE}/k_{bE} \cong 26$. The portion of the resultant force F_a between the teeth transmitted to the bolt is then $\Delta F_a^{(1)} \cong 0.035\,F_a$, and the portion transmitted to the surface of the frictional joint is $\Delta F_a^{(2)} \cong 0.965\,F_a$.

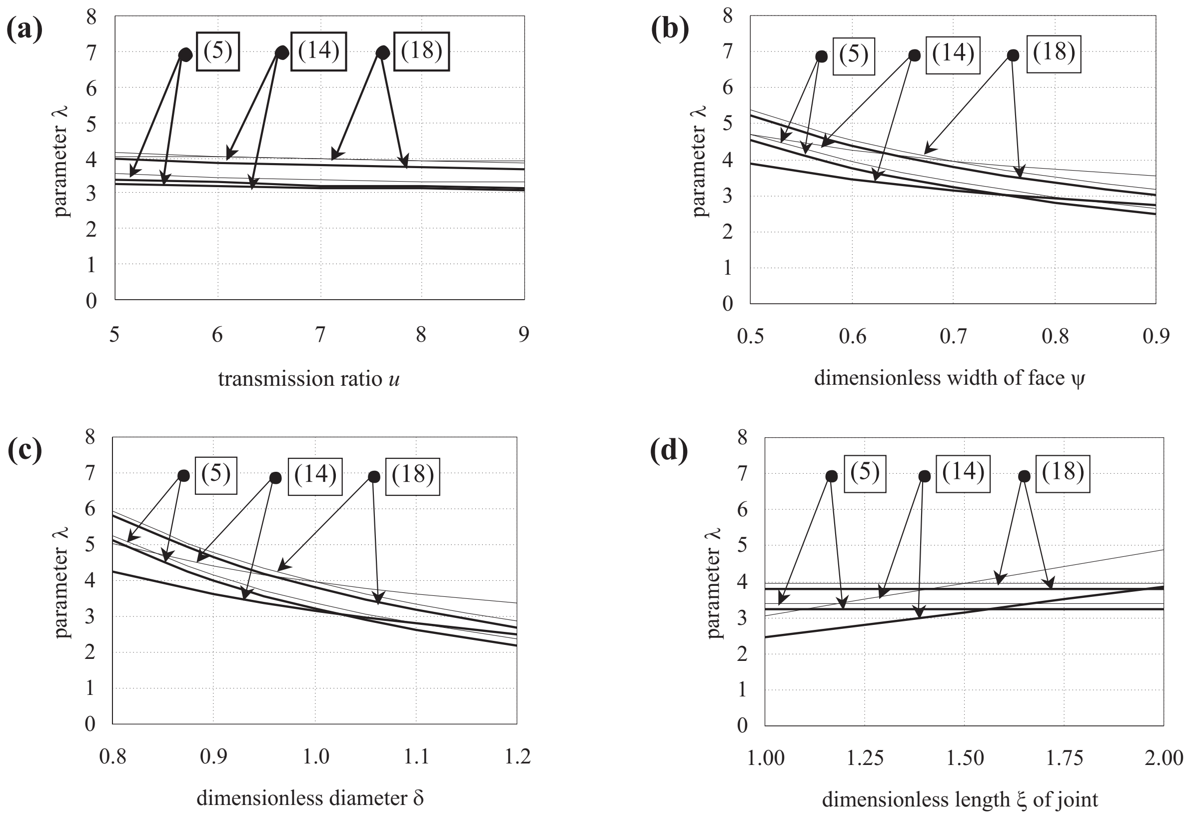

Fig. 5: Parameter λ versus: (a) – transmission ratio u, (b) – width of face ψ, (c) – operating pitch diameter δ, and (d) – length ξ of the joint. Other parameters are fixed.

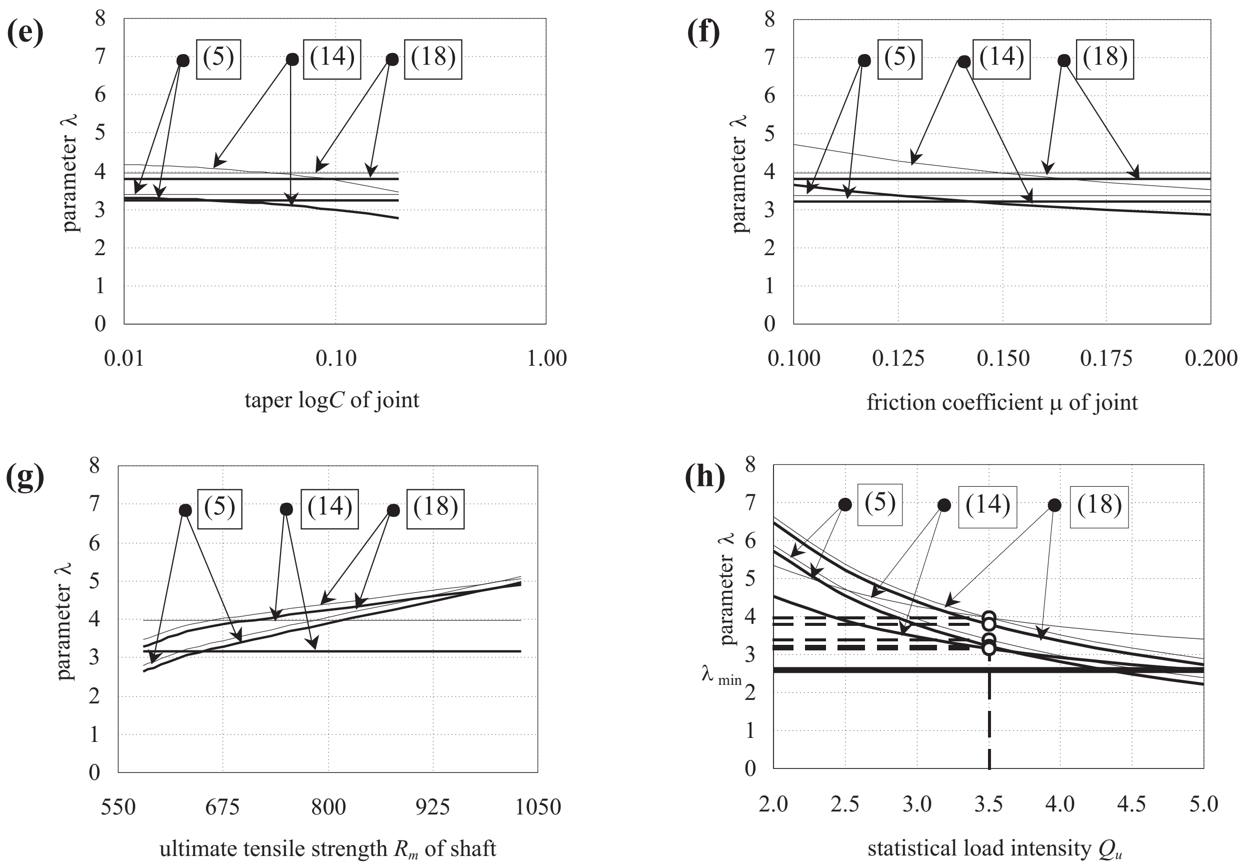

Fig. 6: Parameter λ versus: (e) – taper logC of the joint, (f) – friction coefficient μ of the joint, (g) – ultimate tensile strength R_m of the shaft, and (h) – statistical load intensity Q_u. Other parameters are fixed.

The well-founded assumption can be introduced that the frictional joint is additionally loaded with the entire force $F_a\left(F^* = F_{(b)} \pm F_a\right)$, while the force $F^*_{(b)}$ in the bolt is the same during service ($F^*_{(b)} = F_{(b)}$).

The results of the numerical calculations are presented in Figs. 5 and 6, where the fine lines correspond to the same senses of the forces F_a and F^*, and the bold lines are applied in the opposite case. The curve labels indicate the appropriate condition. It appears that parameter λ depends most strongly on the width b of the gear face (Fig. 5b), on the operating pitch diameter d (Fig. 5c) and on the length l of the frictional joint (Fig. 5d). It is clear, because the variation of these parameters results in significant changes of the bending moment loading the shaft and the frictional joint. The influence of the transmission ratio u (Fig. 5a), taper C (Fig. 6e) and friction coefficient μ (Fig. 6f) is relatively small. The relation between the parameter λ and the strength properties R_m of the shaft, depicted in Fig. 6g, leads to the conclusion that high-quality toughened steel should be used for a shaft, if the gears in the transmission are made of high-strength materials of statistical load intensity $Q_u > 2$ [MPa].

It should be noted, however, that none of conditions (5), (14) and (18) is of primary importance; for different ranges of analyzed parameters different conditions impose the largest reduction on the total length L of the joint.

The final calculations were carried out for the mean values of all parameters and for the shaft made of toughened steel 45. Regarding the estimation suggested in Sect. 4 the working factor of safety was assumed $FS_w = 1.6$, which seems to be well based from the engineering point of view. The inequalities (5), (14) and (18) calculated for $Q_u = 3.5$ [MPa] and the same senses of forces F_a and F^* give $\lambda \le 3.38$, 3.96 and 3.96, respectively (Fig. 6h). For different senses of the forces the appropriate values decrease and the result is $\lambda \le 3.22$, 3.15 and 3.80. All above values are greater than $\lambda_{min} = 2.59$, and on the whole they are close to the mean value of the parameter $\lambda_m = 3.71$.

The minimum value $\lambda_{min} = 3.15$ should be finally accepted for the assumed data and applied in the further dimensioning of the joint. In particular dimensions e and n must be selected with great care.

6 Conclusions

In the presented procedure for analysis and design of the frictional joint initial data connected with the strength and geometry of the first stage of transmission is required. Moreover, several parameters of the joint have to be introduced as well as the production requirements. The strength, load-carrying capacity and fatigue of the joint are then verified employing equations (5), (14) and (18) derived in the paper. The final values of the parameters may be corrected under the assumption that the resulting length of the gear unit must satisfy appropriate conditions.

The numerical calculations carried out for an exemplary set of data demonstrate that the suggested approach is of practical meaning and may be useful in the design process of small, compact reducers and geared motors.

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Prof. Ing. Jan Ryś, DrSc.
phone: +48 126 489 879
fax: +48 126 484 531
e-mail: szymon@mech.pk.edu.pl

Ing. Henryk Sanecki, Ph.D.
phone:+48 126 283 385
e-mail: hsa@mech.pk.edu.pl

Ing. Andrzej Trojnacki, Ph.D.
phone: +48 126 283 306
e-mail: atroj@mech.pk.edu.pl

Department of Mechanical Engineering
Cracow University of Technology
ul. Warszawska 24
31–155 Kraków, Poland