Logical Structural Models with Multiplexors

J. Bokr, V. Jáneš

The paper deals with the use of multiplexors in designing logical structural models.. The applications can be preferably used in designing morphology on EFGA chips and other programmable structures. Illustrative examples are included.

Keywords: Boolean function, Artjuchov–Shalyto extension, Shannon extension, Boolean function decomposition, multiplexor.

1 Introduction

The procedure in the design of logical structural models with multiplexors might seem to be complete. It appears, however, that the Artjuchov-Shalyto extension of the Boolean function, which models the performance of a multiplexor, leads to its mere "setting", and the generalised model of the multiplexor performance makes it possible to design structural models with multiplexors according to the disjoint decomposition of the given Boolean function.

2 Boolean function

Let the ***Boolean function***

$f:\{0,1\}^m \to \{0,1\}:\langle x_1, x_2, \ldots, x_m\rangle \mapsto y$

be given. If we denote the set $\{x_i\}_{i=1}^m$ of the function f arguments by the symbol X, we can write $f(X)$ instead of $f(x_1, x_2, \ldots, x_m)$. Let us also write $f(x_i = \sigma_i)$ instead of $f(x_1, x_2, \ldots, x_{i-1}, \sigma_i, x_{i+1}, \ldots, x_m)$, where $\sigma_i \in \{0, 1\}$. We require the function $f(X)$to be minimal with respect to the number of arguments, i.e., not to contain fictive arguments; the argument x_i is called fictive if $f(x_i = 0) = f(x_i = 1)$. The term ***Hamming weight w_H*** of the function f – $w_H f$ – denotes the value of the arithmetic expression

$$w_H f(X) = \sum_{\langle \sigma_1, \sigma_2, \ldots, \sigma_m\rangle \in \{0,1\}^m} f(\sigma_1, \sigma_2, \ldots, \sigma_m)$$

Let $x^\sigma = x\sigma \vee \bar{x}\bar{\sigma}(\sigma \in \{0,1\})$; each Boolean function $f(X)$ can be expressed by means of a canonic normal disjunctive formula – *cndf* $f(X)$

$$f(X) = \bigvee_{\langle \sigma_1, \sigma_2, \ldots, \sigma_m\rangle \in \{0,1\}^m} x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\sigma_m} \cdot f(\sigma_1, \sigma_2, \ldots, \sigma_m).$$

Then, if $w_H f < 2^m/2$ or $w_H f > 2^m/2$, or if $w_H f = 2^m/2$, it is preferable to write down the respective *cndf* $f(X)$ or *cndf* $\bar{f}(X)$, or to apply the Artjuchov-Shalyto extension of the function $f(X)$ [1]

$$f(X) = x_i \oplus \left(\bar{x}_i f(x_i = 0) \vee x_i \bar{f}(x_i = 1)\right)$$
$$f(X) = \bar{x}_i \oplus \left(\bar{x}_i \bar{f}(x_i = 0) \vee x_i f(x_i = 1)\right)$$

the validity of which can be easily confirmed by supplying 0 or 1 for x_i.

Let a dichotomy $\{X_1, X_0\}$be given on a set of X arguments $x_1, x_2, , x_m$ without loss of generality, such that $X_1 = \{x_1, x_2, \ldots, x_n\}$ and $X_0 = \{x_{n+1}, x_{n+2}, \ldots, x_m\}$, where $n < m$. Let the ***simple k-multiple (k < n) disjoint decomposition of the function*** $f(X)$ be called the composition

$f(X) = \varphi(\varphi_1(X_1), \varphi_2(X_1), \ldots, \varphi_k(X_1), X_0)$.

The construction of the simple k-multiple ($k < n$) disjoint decomposition of function $f(X)$ can be easily done by means of a decomposition by map[2, 3].

3 Multiplexor

The term multiplexor (*MX*) [2, 4, 5] denotes a logical object modeled both in a parametrical and an algebraic way. See Fig.1, where A_r ($r = 1, 2, \ldots, k$) and d_j ($j = 0, 1, \ldots, 2^k - 1$) are the respective adjustable address-, and data-input ports:

$$y = MX\left(\varphi_1(X_1), \varphi_2(X_1), \ldots, \varphi_k(X_1) \,\middle|\, g_0(X_0), g_1(X_0), \ldots, g_{2^k-1}(X_0)\right) =$$
$$= \bigvee_{j=0}^{2^k-1} \varphi_1^{\sigma_1}(X_1)\, \varphi_2^{\sigma_2}(X_1) \ldots \varphi_k^{\sigma_k}(X_1)\, g_j(X_0)$$

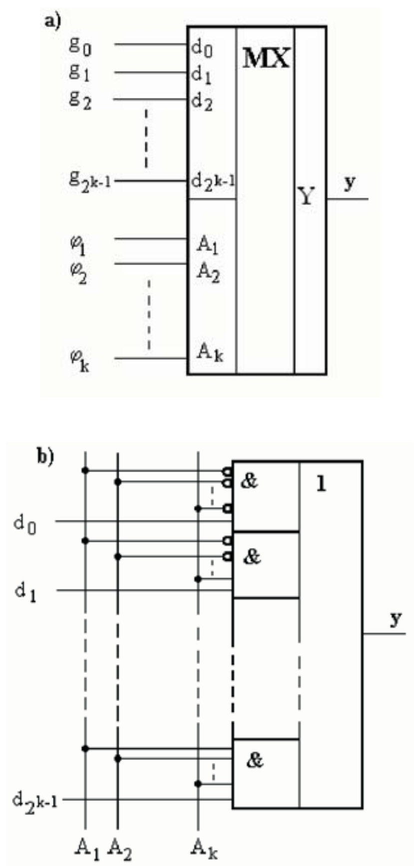

Fig. 1: a) Schematic diagram of a multiplexor, b) structural model of a multiplexor

where

$$j = \sigma_1 \sigma_2 \ldots \sigma_k = \sum_{r=1}^{k} \sigma_r 2^{k-r}.$$

4 Multiplexor and the Boolean function

Let us provide the output port MX with an element of anticoincidence such that $y = Y \oplus z$, where $z \in \{0, 1, \bar{x}_i, x_i\}$ – see Fig. 2. Let us design a multiplexor modelled by the function $f(X)$:

$$y = z \oplus MX\left(x_1, x_2, \ldots, x_m \middle| g_0, g_1, \ldots, g_{2^m - 1}\right) =$$

$$= \bigvee_{\langle \sigma_1, \sigma_2, \ldots, \sigma_m \rangle \in \{0,1\}^m} x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_m^{\sigma_m} f(\sigma_1, \sigma_2, \ldots, \sigma_m).$$

Hence $g_j = f(\sigma_1, \sigma_2, \ldots, \sigma_m)$, where

$$j = \sigma_1, \sigma_2, \ldots, \sigma_m = \sum_{i=1}^{m} \sigma_i 2^{m-i}.$$

If it is more suitable to construct $\boldsymbol{y} = cndf\ f(X)$ or $\boldsymbol{y} = cndf\ \bar{f}(X)$, – see Par. 2 –, then the respective $z = 0$ or $z = 1$, for $\boldsymbol{y} = f(X) \oplus 0$ or $\boldsymbol{y} = f(X) \oplus 1$. If one cannot decide whether to construct $f(X)$ or $\bar{f}(X)$, then if

$$w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \langle 2^m/2} < w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \rangle 2^m/2}$$

or

$$w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \langle 2^m/2} > w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \rangle 2^m/2}$$

then $z = x_1$ and

$$\boldsymbol{y} = \bar{x}_1 f(x_1 = 0) \vee x_1 \bar{f}(x_1 = 1)$$

or $z = \bar{x}_1$ and

$$\boldsymbol{y} = \bar{x}_1 \bar{f}(x_1 = 0) \vee x_1 f(x_1 = 1)$$

respectively.

Example 1: Construct a multiplexor realizing the function $f(x_1, x_2, x_3) = 0001\ 0111$. Since $w_H\ f = 4 = 2^3/2$, as well as

$$w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \langle 4} = 1 < w_H f(X)\big|_{\sigma_1, \sigma_2, \ldots, \sigma_m \rangle 4} = 3,$$

we obtain $z = x_1$ and since

$$f(x_1, x_2, x_3) = MX(x_1, x_2, x_3 | 0, 0, 0, 1, 0, 1, 1, 1) =$$

$$= x_1 \oplus MX(x_1, x_2, x_3 | d_0, d_1, d_2, d_3, d_4, d_5, d_6, d_7)$$

we obtain

$d_0 = d_1 = d_2 = d_5 = d_6 = d_7 = 0$ and $d_3 = d_4 = 1$, i.e.

$\boldsymbol{y} = MX(x_1, x_2, x_3 | 0, 0, 0, 1, 1, 0, 0, 0)$,

which is certainly a simpler setting of MX than that of

$d_0 = d_1 = d_2 = d_4 = 0$ and $d_3 = d_5 = d_6 = d_7 = 1$.

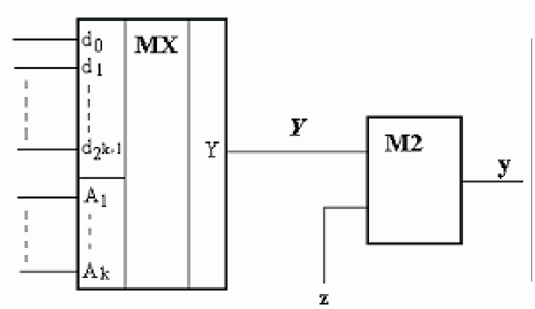

Fig. 2: Multiplexor and the element of the sum modulo 2 – M2

5 Multiplexor and the simple k-multiple disjoint decomposition

Let $f(x_1, x_2, \ldots, x_m) = \varphi(\varphi_1(X_1), \varphi_2(X_1), \ldots, \varphi_k(X_1), X_0)$ where

$X_1 = \{x_1, x_2, \ldots, x_n\}$ and $X_0 = \{x_{n+1}, x_{n+2}, \ldots, x_m\}$,

be a simple k-multiple $(k < n)$ disjoint decomposition of the function $f(x_1, x_2, \ldots, x_m)$.

Let there be $\varphi_r = x_r$ $(r = 1, 2, \ldots, k)$ with $k = n$ and let us construct the Shannon extension of the given function according to the arguments $x_1, x_2, \ldots, x_n$, without loss of generality

$$f(x_1, x_2, \ldots, x_m) = \bigvee_{\langle \sigma_1, \sigma_2, \ldots, \sigma_n \rangle} x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_n^{\sigma_n} \cdot$$

$$\cdot f(\sigma_1, \sigma_2, \ldots, \sigma_n, x_{n+1}, \ldots, x_m).$$

And further let

$$f(x_1, x_2, \ldots, x_m) = MX\left(x_1, x_2, \ldots, x_n \middle| g_0, g_1, \ldots, g_{2^n - 1}\right) =$$

$$= \bigvee_{j=0}^{2^n - 1} x_1^{\sigma_1} x_2^{\sigma_2} \ldots x_n^{\sigma_n} g_j ;$$

hence $g_j = f(\sigma_1, \sigma_2, \ldots, \sigma_n, x_{n+1}, x_{n+2}, \ldots, x_m)$,

where $j = \sigma_1 \sigma_2 \ldots \sigma_n = \sum_{i=1}^{n} \sigma_i 2^{n-i}$.

Note that the selection of arguments according to which the Shannon extension of the given function $f(x_1, x_2, \ldots, x_m)$ is done depends completely on the view of the designer, and there is no reason to distinguish the development qualitatively according to the 'left-side' arguments $x_1, x_2, \ldots, x_n$ or 'right-side' arguments $x_{n+1}, x_{n+2}, \ldots, x_m$ of the function f, as stated in [1].

Example 2: Let the function $\boldsymbol{y} = \vee (5, 6, 7, 10, 11, 19, 21, 23, 26, 27, 30, 31)$ be given; design a structural model with MX according to the Shannon development extension of the given function both according to arguments x_1, x_2, x_3 and according to arguments x_4, x_5, i.e., according to

$$\boldsymbol{y} = MX\left(x_1, x_2, x_3 \middle| h_0, h_1, \ldots, h_7\right) = MX\left(x_4, x_5 \middle| g_0, g_1, g_2, g_3\right).$$

Thus, let us construct decomposition maps (Fig. 3.) hence

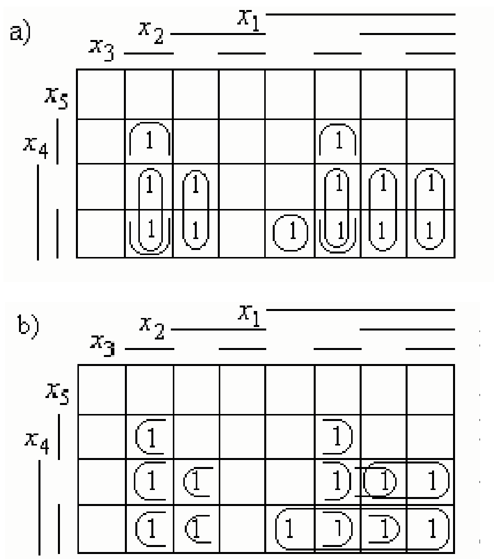

Fig. 3: Decomposition maps of the function from Example 2

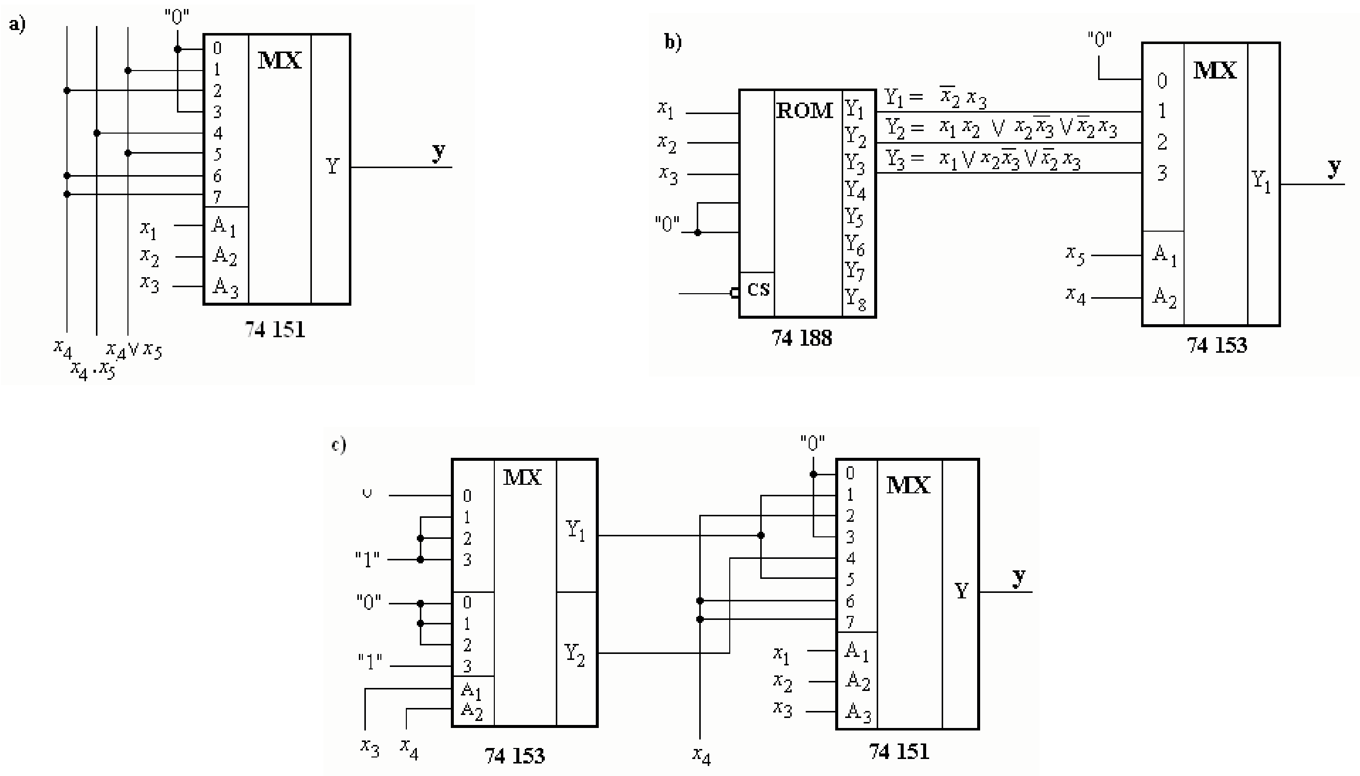

Fig. 4: Structural model with *MX* from Example 2

$$y = \bar{x}_1\bar{x}_2\bar{x}_3(0) \vee \bar{x}_1\bar{x}_2x_3(x_4 \vee x_5) \vee \bar{x}_1x_2\bar{x}_3(x_4) \vee$$
$$\vee \bar{x}_1x_2x_3(0) \vee x_1\bar{x}_2\bar{x}_3(x_4x_5) \vee x_1\bar{x}_2x_3(x_4 \vee x_5) \vee$$
$$\vee x_1x_2\bar{x}_3(x_4) \vee x_1x_2x_3(x_4)$$

as well as

$$y = \bar{x}_4\bar{x}_5(0) \vee \bar{x}_4x_5(\bar{x}_2x_3) \vee x_4\bar{x}_5 \cdot (x_1x_2 \vee x_2\bar{x}_3 \vee \bar{x}_2x_3) \vee$$
$$\vee x_4x_5(x_1 \vee x_2\bar{x}_3 \vee \bar{x}_2x_3).$$

Hence the structural models from Fig. 4. Note that in Fig. 4b) a ROM module is suggested and in Fig. 4c) the structure is realized only with multiplexer modules.

Let a simple k-multiple disjoint decomposition

$$f(x_1, x_2, \dots, x_m) = \varphi(\varphi_1(X_1), \varphi_2(X_1), \dots, \varphi_k(X_1), X_0)$$

be given, where φ will be termed an outer function and the functions $\varphi 1$ ($i = 1, 2, \dots, k$) will be denoted inner functions. And, further, let

$$f(x_1, x_2, \dots, x_m) =$$
$$= MX\left(\varphi_1(X_1), \varphi_2(X_1), \dots, \varphi_k(X_1) \middle| g_0, g_1, \dots, g_{2^k-1}\right)$$

Hence

$$g_j = (\varphi_1(X_1), \varphi_2(X_1), \dots, \varphi_k(X_1), X_0),$$

where

$$j = \sigma_1, \sigma_2, \dots, \sigma_n = \sum_{i=1}^{n} \sigma_i 2^{n-i}.$$

Example 3: Construct a structural model with *MX* according to the decomposition

$$y = \varphi(\varphi_1(x_1, x_2, x_3), \varphi_2(x_1, x_2, x_3), x_4, x_5)$$

of the function y from Example 2. According to the decomposition map (Fig. 5) we obtain

$$\varphi_1(x_1, x_2, x_3) = x_1\bar{x}_2 \vee \bar{x}_2x_3$$
$$\varphi_2(x_1, x_2, x_3) = \bar{x}_1x_2x_3 \vee \bar{x}_2\bar{x}_3$$

for the inner functions.

Since

$$y = MX(\varphi_1, \varphi_2 | g_0, g_1, g_2, g_3)$$

we obtain

$$y = \bar{\varphi}_1\bar{\varphi}_2(x_4) \vee \bar{\varphi}_1\varphi_2(0) \vee \varphi_1\bar{\varphi}_2(x_4 \vee x_5) \vee \varphi_1\varphi_2(x_4x_5)$$

and hence the structural model in Fig. 6.

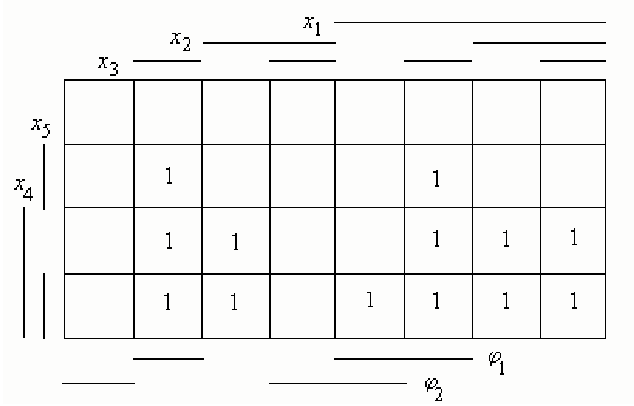

Fig. 5: Decomposition map of the function from Example 3

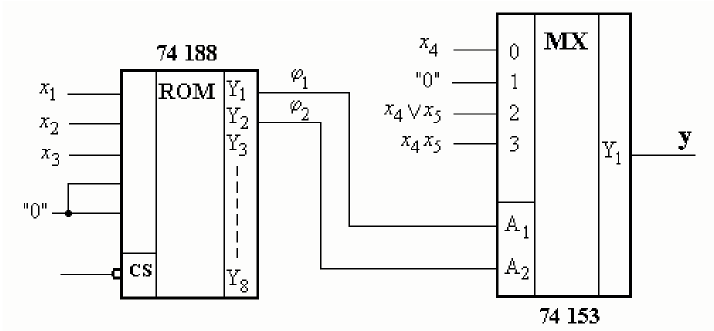

Fig. 6: Structural model with MX prescribed by the decomposition from Example 3

6 Conclusions

The multiplexer appears to be a very helpful MSI module. The design of structural models is sufficiently simple and suitable also for the implementation of logical functions on chips provided with FPGA or FPD.

References

[1] Šalyto, A. A.: *Metody apparatnoj i programmnoj realizacii algoritmov*. Nauka, Sankt-Peterburg 2000.

[2] Bokr, J., Jáneš,V.: *Logické systémy*. Vydavatelství ČVUT, Praha 1999.

[3] Bokr.J.: "Sovmestnaja dekompozicija sistem bulevych funkcij." *Avtomatika i vyčislitelnaja technika*, 2000, No. 2, p. 38–44.

[4] Frištacký, N. at al..: *Logické systémy*. Bratislava/Praha, ALFA/SNTL 1986.

[5] Liebig, H., Thome, S.: *Logischer Entwurf digitaler Systeme*. Springer, Berlin – Tokio 1996.

Doc. Ing. Josef Bokr, CSc.
e–mail: bokr@kiv.zcu.cz

Department of Information and Computer Science

University of West Bohemia
Faculty of Applied Sciences
Universitní 22
306 14 Pilsen, Czech Republic

Doc. Ing. Vlastimil Jáneš, CSc.
e–mail: janes@fd.cvut.cz

Department of Control and Telematics

Czech Technical University in Prague
Faculty of Transportation Sciences
Konviktská 20
110 00 Prague 1, Czech Republic

