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Testing $f(R)$-Theories by Binary Pulsars

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Abstract

Using the Post-Keplerian parameters to obtain, in the Minkowskian limit we obtain constraints on $f(R)$-theories of gravity from the first time derivative of the orbital period of a sample of binary stars. In the approximation in which the theory is Taylor expandable, we can estimate the parameters of an an analytic $f(R)$-theory, and fulfilling the gap between the General Relativity prediction and the one cames from observation, we show that the theory is not ruled out.

Keywords: gravitation - binary pulsar systems - $f(R)$-theories - gravitational waves.

1 Introduction

Astrophysical systems like Neutron Stars (NS), coalescing binary systems, Black Holes (BHs), and White Dwarfs (WDs), are the most promising to study the gravitational waves (GWs) emision. Indeed, studying the binary system B1913+16, known as the Hulse-Taylor binary pulsar, the first time derivative of the orbital period was measured to be different from zero [13, 18], as predicted by General Relativity (GR) when gravitational radiation is emitted. This measurements was confirmed by study in other relativistic binary systems. The agreement between GR and the observation is at the order of $\sim 1\%$. However, using the Extended Theories of Gravity (ETG) it should be possibile to explain the observational results as shown in [10, 12] where, starting from a class of analytic $f(R)$-theories it is possible evaluate the first time derivative of the orbital period and compare it with the data. This approach permit both to test the ETGs both to explain the gap between observation and the theoretical prediction. This paper was organized as follow: in Sec. 1 we calculate the quadrupole emission for an analytic $f(R)$-Lagrangian using the weak-field limit; in Sec 2 we compare the theoretical prediction with the observed data. Finally in Sec 3 we give our conclusions and remarks.

2 The First Time Derivative of the Orbital Period in the $f(R)$-Theories

The simplest extension to GR is the $f(R)$-gravity, in which, the Lagrangian is an arbitrary function of Ricci scalar [2]. Starting from the field equations in $f(R)$-gravity (for details see [2, 16, 4, 5])

$$f'(R)R_{\mu\nu} - \frac{f(R)}{2}\,g_{\mu\nu} - f'(R)_{;\mu\nu} + \\ +g_{\mu\nu}\Box_g f'(R) \;=\; \frac{\mathcal{X}}{2}T_{\mu\nu}\,, \tag{1}$$

$$3\Box f'(R) + f'(R)R - 2f(R) \;=\; \frac{\mathcal{X}}{2}T\,, \tag{2}$$

where $T_{\mu\nu} = \dfrac{-2}{\sqrt{-g}}\dfrac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$ is the energy momentum tensor of matter (T is the trace), $\mathcal{X} = \dfrac{16\pi G}{c^4}$ is the coupling, $f'(R) = \dfrac{df(R)}{dR}$, $\Box_g = {}_{;\sigma}{}^{;\sigma}$, and $\Box = {}_{,\sigma}{}^{,\sigma}$.

it is possibile, in the Minkowskian approximation of an analytic $f(R)$-Lagrangian[1],

$$f(R) = \sum_n \frac{f^n(R_0)}{n!}(R-R_0)^n \simeq \\ \simeq f_0 + f'_0 R + \frac{f''_0}{2}R^2 + ... \tag{3}$$

to compute the quadrupole emission due to GWs [10, 11]. Furthermore, it is possible calculate the energy

[1] For convenience we will use f instead of $f(R)$. All considerations are developed here in metric formalism. From now on we assume physical units $G = c = 1$.

momentum tensor of gravitational field in $f(R)$-gravity that assumes the following form

$$t^{\lambda}_{\alpha} = \underbrace{f'_0 k^{\lambda} k_{\alpha} \left(\dot{h}^{\rho\sigma}\dot{h}_{\rho\sigma}\right)}_{GR} - \underbrace{\frac{1}{2} f''_0 \delta^{\lambda}_{\alpha} \left(k_{\rho} k_{\sigma} \ddot{h}^{\rho\sigma}\right)^2}_{f(R)} . \tag{4}$$

To be more precise, the first term, depending on the choice of the constant f'_0, is the standard GR term, the second is the $f(R)$ contribution. It is worth noticing that the order of derivative is increased of two degrees consistently to the fact that $f(R)$-gravity is of fourth-order in the metric approach [10].

In this contest, we can write the total average flux of energy due to the GWs integrating over all possible directions as

$$\underbrace{\left\langle \frac{dE}{dt} \right\rangle}_{(total)} = \frac{G}{60} \left\langle \underbrace{f'_0 \left(\dddot{Q}^{ij}\dddot{Q}_{ij}\right)}_{GR} - \underbrace{f''_0 \left(\ddddot{Q}^{ij}\ddddot{Q}_{ij}\right)}_{f(R)} \right\rangle , \tag{5}$$

where we point out that for $f''_0 \to 0$ and $f'_0 \to \frac{4}{3}$, the previous equation becomes

$$\underbrace{\left\langle \frac{dE}{dt} \right\rangle}_{(GR)} = \frac{G}{45} \left\langle \dddot{Q}^{ij} \dddot{Q}_{ij} \right\rangle , \tag{6}$$

that is the prediction of GR [14, 17]. In order to evaluate the above expressions for the flux it is necessary to form explicit expressions for $\left\langle \dddot{Q}^{ij}\dddot{Q}_{ij} \right\rangle$ and $\left\langle \ddddot{Q}^{ij}\ddddot{Q}_{ij} \right\rangle$ for the system under consideration. For our purposes we consider a binary pulsar system. If we assume a Keplerian motion of the stars in the binary system, wherewe m_p is the pulsar mass, m_c the companion mass, and $\mu = \dfrac{m_c m_p}{m_c + m_p}$ is the reduced mass, it is possible to compute the time average of the radiated power computing the first time derivative of the orbital period [11]

$$\dot{P}_b = -\frac{3}{20}\left(\frac{T}{2\pi}\right)^{-\frac{5}{3}} \frac{\mu G^{\frac{5}{3}}(m_c+m_p)^{\frac{2}{3}}}{c^5(1-\epsilon^2)^{\frac{7}{2}}} \times$$
$$\times \left[f'_0 \left(37\epsilon^4 + 292\epsilon^2 + 96\right) - \frac{f''_0 \pi^2 T^{-1}}{2(1+\epsilon^2)^3} \times \right.$$
$$\left. \times \left(891\epsilon^8 + 28016\epsilon^6 + 43520\epsilon^2 + 3072\right) \right] , \tag{7}$$

where ϵ is the orbital eccentricity and T is the orbital period of the binary.

3 Methodology and Data Analysis

Knowing exactly the Lagrangian that describes the system, we can predict the orbital period decay, however,we want understand how well the relativistic binary systems can fix bounds on $f(R)$ parameters using eq. (7), and getting an estimation of the second derivative of the Lagrangian with respect to Ricci scalar, f''_0. We use the following prescription, the difference between the first derivative of the binary observed period variation ($\dot{T}_{b_{Obs}} \pm \delta$) and the theorethical one obtained by GR, $\Delta\dot{T}_b = \dot{T}_{b_{Obs}} - \dot{T}_{GR}$, is fulfilled imposing that:

$$\dot{T}_{b_{Obs}} - \dot{T}_{GR} - f''_0 \dot{T}_{b_{f(R)}} = 0, \tag{8}$$

$$\dot{T}_{b_{Obs}} \pm \delta - \dot{T}_{GR} - f''_{0_{\pm\delta}} \dot{T}_{b_{f(R)}} = 0, \tag{9}$$

where δ is the experimental error, that we propagate on the $\dot{T}_{b_{Obs}}$, into an uncertainty on $f''_{0_{\pm\delta}}$. In this way, the extra contribution to the loss of energy due to the emission of GWs radiation in the ETGs regime can provide to fill the difference between theory and observations. We select a sample of Observed Relativistic Binary Pulsars (see their references reported in Tab. 1 of [11]) for which we compute the correction $\dot{T}_{b_{f(R)}}$, the difference $\Delta\dot{T}_{GR}$ between $\dot{T}_{b_{Obs}}$ and $\dot{T}_{GR}$ (equal to the correction $-f''_0\dot{T}_{b_{f(R)}}$), the corresponding f''_0 solution of (8), the interval centered on f''_0 and finally, the interval centered on f''_0 and computed from the difference: $\frac{f''_{0_{+\delta}} - f''_{0_{-\delta}}}{2}$, all results are reported in Tab. 1. In Fig. 1 we show, for sake of convenience, in logarithmic scale, the absolute values of f''_0 reported in Tab.1 versus the ratio $\frac{\dot{T}_{b_{Obs}}}{\dot{T}_{GR}}$. There are six binaries in tables, for which the ETGs are not ruled out $0.04 \leq f''_0 \leq 38$, getting $0.5 \leq \frac{\dot{T}_{b_{Obs}}}{\dot{T}_{GR}} \leq 1.5$. For those systems the difference between $\dot{T}_{GR}$ and $\dot{T}_{b_{Obs}}$ can be explained adding an extra contribution that comes out from the $f(R)$-thoery. Instead for most of binaries we have f''_0 values that can surely rule out the theory, since taking account of the weak field assumption we obtain $38 \leq f''_0 \leq 4 \times 10^7$. From this last values to the first ones, there is a jump of about four up to five order of magnitude on f''_0. The origin of these strong discrepancies, perhaps, is due to the extreme assumption we made, to justify the difference between the observed $\dot{T}_{b_{Obs}}$ and the predicted $\dot{T}_{GR}$ using the ETGs.

Table 1: Upper Limits of f_0'' correction to $\dot{T}_{GR}$ of binary relativistic pulsars assuming that all the loss of energy is caused by Gravitational Wave emission. We reported the J-Name of the system,the difference $\Delta\dot{T}_{GR}$ between $\dot{T}_{b_{Obs}}$ and $\dot{T}_{GR}$ equal to the correction $-f_0''\dot{T}_{b_{f(R)}}$, the correction $\dot{T}_{b_{f(R)}}$, the corresponding f_0'' solution of (8), the interval centered on f_0'' and computed from the difference $\frac{f''_{0+\delta}-f''_{0-\delta}}{2}$,where $f''_{0\pm\delta}$,are the corresponding solutions of (8) taking account of the experimental errors $\pm\delta$ on the observed orbital period variation $\dot{T}_{b_{Obs}}$.

Name	$\Delta\dot{T}_{GR}$	$\dot{T}_{b_{f(R)}}$	f_0''	$\pm\Delta f_0''$
J2129+1210C	-2.17E-14	6.01E-13	3.61E-02	8.32E-02
J1915+1606	-2.04E-14	2.10E-13	9.74E-02	4.77E-03
J0737-3039A	-4.23E-15	1.86E-14	2.28E-01	9.15E-02
J1141-6545	-1.65E-14	3.88E-15	4.25E+00	6.44E+00
J1537+1155	5.39E-14	1.42E-15	-3.79E+01	7.03E-02
J1738+0333	-1.56E-15	1.06E-16	-1.47E+01	2.92E+01
J0751+1807	1.41E-13	8.98E-16	-15.7E+01	1.002E+01
J0024-7204J	-5.22E-13	3.13E-16	1.67E+03	4.15E+02
J1701-3006B	-5.03E-12	8.81E-16	5.71E+03	7.04E+01
J2051-0827	-1.55E-11	4.77E-16	3.24E+04	1.68E+03
J1909-3744	-5.47E-13	2.62E-18	2.09E+05	1.14E+04
J1518+4904	2.41E-13	3.42E-19	-7.05E+05	6.43E+03
J1959+2048	1.47E-11	1.07E-17	-1.38E+06	7.51E+04
J2145-0750	4.01E-13	1.00E-19	-4.00E+06	2.99E+06
J0437-4715	1.59E-13	1.04E-19	-1.57E+06	2.73E+06
J0045-7319	3.02E-07	1.11E-16	2.74E+9	8.13E+07
J2019+2425	-3.00E-11	1.11E-22	2.71E+11	5.41E+11
J1623-2631	4.00E-10	2.02E-23	-1.98E+13	2.97E+13

4 Discussion and Remarks

In this paper, we develop expressions for quadrupole gravitational radiation in $f(R)$-gravity theory using the weak field technique and apply these results, which are applicable in general, to a sample of a binary pulsars, though their orbits are eccentric. Here, we seen that, where the GR theory is not enough to explain the gap between the data and the theoretical estimation of the orbital decay, there is the possibility to extend the GR theory with a generic $f(R)$- theory to cover the gap. According to eq. (7),we have selected a sample of relativistic binary systems for which the first derivative of the orbital period is observed, we have computed the theoretical quadrupole radiation rate, and finally we have compared it to binary system observations. From Tab. 1, it is seen that the first five systems have masses determined in a manner quite reliable, while for the remaining sample, masses are estimated by requiring that the mass of the pulsar is $1.4M_\odot$ and, assuming for the orbital inclination one of the usual statistical values ($i = 60°$ or $i = 90°$), and from here comes then the estimate of the mass of the companion star. So a primary cause of major discrepancies, not only for the ETGs, but also for the GR theory, between the variation of the observed orbital period and the predicted effect of emission of gravitational waves, could be a mistake in the estimation of the masses of the system. In addition, other causes may be attributable to the evolutionary state of the system, which, for instance, if it does not consist of two neutron stars may transfer mass from companion to the neutron star. In our sample, there are only five double NS that can be used to test GR and ETGs. Taking into account of the strong hypothesis we made, the ETG correction to $\dot{T}_{GR}$ can also include the galactic acceleration term correction ([7], [8]). Here, we give a preliminary result about the energy loss from binary systems and we show that, when the nature of the binary systems can exclude energy losses due to trade or loss of matter, then, we can explain the gap between the first time derivative of the observed orbital period and the theoretical one predicted by GR, using an analytical $f(R)$-theory of gravity.

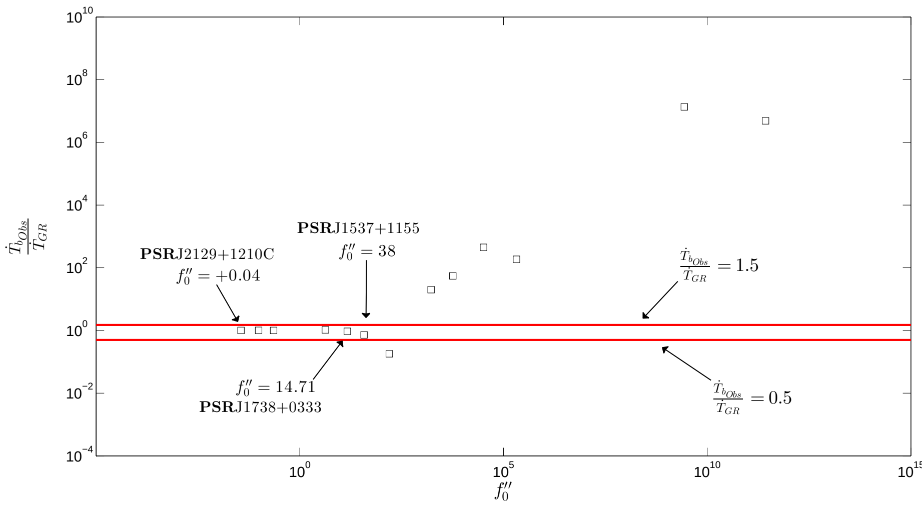

Figure 1: In figure there are shown, for sake of convenience, in logaritmic scale, the absolute values of f_0'' reported in Tab. 1 versus the ratio $\frac{\dot{T}_{b_{Obs}}}{\dot{T}_{GR}}$. We must note that for five binaries the ETGs we are probing is not ruled out $0.04 \leq f_0'' \leq\approx 38$, for those systems the difference between $\dot{T}_{GR}$ and $\dot{T}_{b_{Obs}}$ is tiny, indeed we get $0.5 \leq \frac{\dot{T}_{b_{Obs}}}{\dot{T}_{GR}} \leq 1.5$. Instead for most of binaries we have f_0'' values that can surely rule out the theory, since taking account of the weak field assumption we obtain $38 \leq f_0'' \leq 4 \times 10^7$. From this last values to the first ones, there is a jump of about four up to five order of magnitude on f_0''.

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