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The Mutual Interaction effects between Array Antenna Parameters and Receiving Signals Bandwidth

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Abstract

The presence of a single complex adaptive weight in each element channel of an adaptive array antenna is sufficient for processing of narrowband signals. The ability of an adaptive array antenna to null interference deteriorates rapidly as the interference bandwidth increases. The performance of narrowband adaptive array antenna with LMCV Beamforming algorithm is examined. The interaction effects between received signal angle of arrival and array parameters like the interelement spacing and the number of array element and the received signal bandwidth were studied. The output Signal to Interference plus Noise Ratio (SINR) and Interference to Noise Ratio (INR) are used as performance parameters for evaluation of these effects. It is found that the amount of degradation in the output SINR is increased significantly with the increase of array interelement spacing, number of array elements and when the angle of arrival of received signals are closet to end fire.

Keywords: *Array parameters, recived signal bandwidth, narrowband adaptive array antenna.*

1. Introduction

Adaptive array antennas are playing an important role in many applications such as radar and communication system. Adaptive processor can perform filtering in both space and frequency domain. By adjusting the amplitude and phase of the wavefront in each sensor, it is possible to electronically steer the main beam towards the look direction (desired signal), while suppressing any undesired signal [1- 3].

The function of any adaptive array is to minimize the received power from one or more interference sources. The ability of the adaptive array antenna system to perform this function depends on many factors, some of which include the number of adaptive array elements, antenna aperture size, nulling bandwidth, number of interference sources, strength of signals, and spatial distribution of these sources [4].

In general, adaptive antenna array system is composed from M-element and adaptive processor as shown in Fig (1). Each sensor is followed by complex weight. This weight is calculated according to applied algorithm. The received signal vector **X** is multiplied by weight vector **W** and then summed to perform the output y(t). Adaptive processor subdivided into two ports, the signal processing and adaptive algorithm [5, 6]. The processer hold the information of received signal and processed them according to the operating algorithm to produce the optimum weights which optimize the performance of the adaptive system. Array antenna field pattern depends on the phase delay between array elements due to interelement spacing calculating on the operating frequency f_o , so the wide bandwidth signals will cause a sever distortion on the final field pattern due to the phase.

The amount of degradation in the adaptive array performance due to signal bandwidth does not generate a high attention from researchers, in this paper we will try to investigate interaction effects between array parameters and receiving signal bandwidth. In Section II, we formulate the equations needed to calculate the received signal covariance matrix for an array with M elements. In Section III, the LCMV beamformer is briefly reviewed to calculated output SINR and INR. Section IV, contains our simulations and results cases. Finally, Section V contains conclusions .

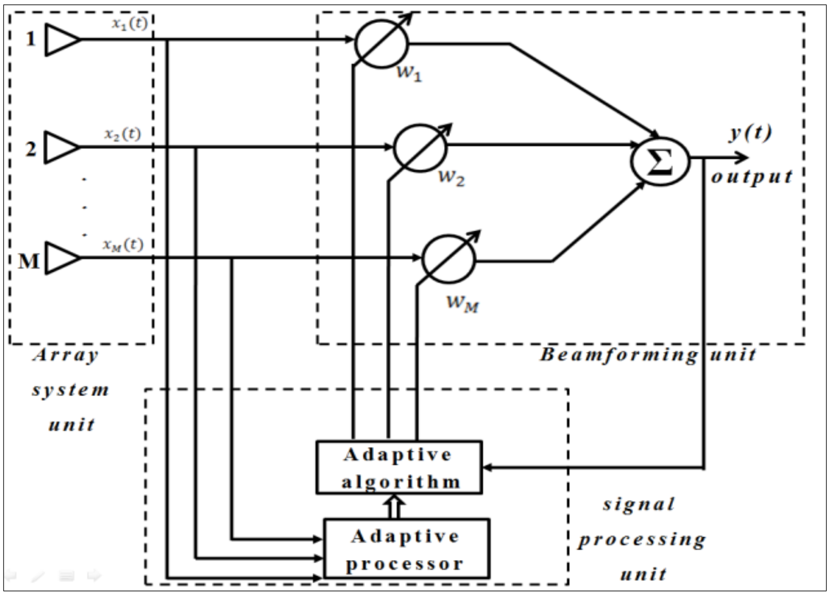

Fig. 1. M-elements adaptive array system.

2. Mathematical Formulation

M-isotropic linear array antenna elements are considered to be uniformly distributed with inter element spacing (d) at the carrier frequency ω_o. The received signal by jth element can be written as

$$x_j(t) = x_{dj}(t) + x_{ij}(t) + n_j(t) \qquad \dots (1)$$

Equation (1) in a vector form is

$$\boldsymbol{X}(t) = \boldsymbol{X}_d(t) + \boldsymbol{X}_i(t) + \boldsymbol{X}_n(t) \qquad \dots (2)$$

where $\boldsymbol{X}_d(t)$, $\boldsymbol{X}_i(t)$ *and* $\boldsymbol{X}_n(t)$ are vectors $(M \times 1)$ of dimension for desired, interference and thermal noise, respectively.

If the desired signal is incident from angle (θ_d), then the desired signal vector is

$$\boldsymbol{X}_d(t) = x_d(t - [m-1]T_d) \qquad \dots (3)$$

where m=1, 2, …, M and T_d is desired signal spatial propagation delay between element

$$T_d = (d\,/\,v)\sin\,\theta_d \qquad \dots (4)$$

If the interference signal is incident from angle (θ_i), then the interference signal vector is

$$\boldsymbol{X}_i(t) = x_i(t - [m-1]T_i) \qquad \dots (5)$$

where T_i is the interference signal spatial propagation delay between element

$$T_i = (d\,/\,v)\sin\,\theta_i \qquad \dots (6)$$

The thermal noise voltage of the jth array element is a random signal with zero mean and σ^2 variance. The thermal noise components of channels are considered to be statistically independent.

The output signal of array can be written as

$$y(t) = \sum_{j=1}^{M} w_j\, x_j(t) \qquad \dots (7)$$

In vector form Eq. (7) is

$$y(t) = \boldsymbol{W}^T\boldsymbol{X} = \boldsymbol{X^T W} \qquad \dots (8)$$

where the weight vector (**W**) and the received signal vector (**X**) are vectors $(1 \times M)$ dimension and given by

$$\boldsymbol{W}^T = [w_1, w_2, \dots, w_M] \qquad \dots (9)$$

$$\boldsymbol{X}^T = [x_1, x_2, \dots, x_M] \quad \dots (10)$$

The covariance matrix of a received signal vector is defined as

$$cov[\boldsymbol{XX}^*] = E([\boldsymbol{X} - E(\boldsymbol{X})]^* [\boldsymbol{X} - E(\boldsymbol{X})]^T \quad \dots (11)$$

where "E" is the expected value of random variable

Since **X** (t) is a deterministic signal it has zero mean and it is considered as a stationary process, these assumptions leads to

$$cov[\boldsymbol{XX}^*] = E[\boldsymbol{X}^* \boldsymbol{X}^T] = \mathcal{F}^{-1}\, \boldsymbol{\Phi}_{xx} \quad \dots (12)$$

where $\boldsymbol{\Phi}_{xx}$ is $(M \times M)$ autocorrelation matrix of received signals vector **X**(t).

The arriving signal has a flat power spectral density with $(2\pi\, P)/\Delta\omega$ amplitude over a bandwidth $\Delta\omega$ centered at frequency ω_o as shown in Fig(2), then the Fourier inverse of this signal is [7]

$$\boldsymbol{R_{xx}}(\tau) = \mathcal{F}^{-1}\, \boldsymbol{\Phi_{xx}}(\omega) \quad \dots (13)$$

For single array element the auto correlation function is

$$\boldsymbol{r}(\tau) = P\ \operatorname{sinc}\left(\frac{\mathrm{B}}{2} \times {}_{\mathrm{o}}\right) e^{j\omega_o \tau} \quad \dots (14)$$

where B is signal relative bandwidth

$$\mathrm{B} = \Delta\omega/\omega_{\mathrm{o}} \quad \dots (15)$$

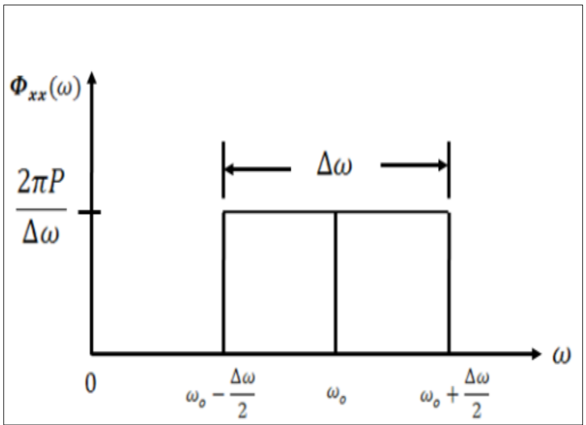

Fig. 2. The power spectral density of received signal.

The autocorrelation matrix $\boldsymbol{R}_{xx}$ of all received signal is Hermitian (i.e. $\boldsymbol{R}_{xx} = \boldsymbol{R}_{xx}^{*\,T}$) expressed as

$$\boldsymbol{R}_{xx} = \boldsymbol{R}_{dd} + \boldsymbol{R}_{ii} + \boldsymbol{R}_{nn} \quad \dots (16)$$

where $\boldsymbol{R}_{dd}$, $\boldsymbol{R}_{ii}$ *and* $\boldsymbol{R}_{nn}$ are desired, interference and thermal noise $(M \times M)$ autocorrelation matrix respectively.

The correlation matrix $\boldsymbol{R}_{dd}$ of desired signal is

$$\boldsymbol{R}_{dd}(m,n) = \boldsymbol{r}_d[(m-n)T_d] \quad \dots (17)$$

where m=n=1, 2, …, M and $\boldsymbol{r}_d[(m-n)T_d]$ are the matrix inverse of covariance matrix $\boldsymbol{\Phi}_{dd}$.

Substituting Eq.(14) into Eq. (17) gives

$$\boldsymbol{R}_{dd}(m,n) = P_d * \operatorname{sinc}\left(\frac{\mathrm{B_d}}{2} * \omega_{\mathrm{o}}(-n)T_d\right) e^{j\omega_o(m-n)T_d} \quad \dots (18)$$

where P_d is the power of desired signal and $\mathrm{B_d} = (\Delta\omega_{\mathrm{d}})/\omega_{\mathrm{o}}$ is the desired signal relative bandwidth.

$$\boldsymbol{R}_{dd}(m,n) = P_d * \operatorname{sinc}\left\{\frac{B_d}{2} * (m-n)\emptyset_d\right\} * e^{j(m-n)\emptyset_d} \quad \dots (19)$$

where $\emptyset_d$ is interelement phase shift due to desired signal

$$\emptyset_d = \omega_{\mathrm{o}} T_d = \beta d \sin(\theta_d) \quad \dots (20)$$

The correlation matrix $\boldsymbol{R}_{ii}$ of interference signal is found to be

$$\boldsymbol{R}_{ii}(m,n) = \boldsymbol{r}_i[(m-n)T_i] \quad \dots (21)$$

Substituting Eq.(14) into Eq. (21) gives

$$\boldsymbol{R}_{ii}(m,n) = P_i * \operatorname{sinc}\left(\frac{\mathrm{B_i}}{2} * \omega_{\mathrm{o}}(m-n)T_i\right) e^{j\omega_o(m-n)T_i} \quad \dots (22)$$

where P_i is the power of interference signal and $\mathrm{B_i} = (\Delta\omega_{\mathrm{i}})/\omega_{\mathrm{o}}$ is the interference signal relative bandwidth.

$$\boldsymbol{R}_{ii}(m,n) = P_i * \operatorname{sinc}\left\{\frac{B_i}{2}(m-n)\emptyset_i\right\} * e^{j(m-n)\emptyset_i} \quad \dots (23)$$

where $\emptyset_i$ is interelement phase shift due to interference signal

$$\emptyset_i = \omega_{\mathrm{o}} T_i = \beta d \sin(\theta_i) \quad \dots (24)$$

The correlation matrix $\boldsymbol{R}_{nn}$ of noise components are independent and equal to

$$\boldsymbol{R}_{nn} = \sigma^2 \boldsymbol{I} \quad \dots (25)$$

where σ^2 is the variance of thermal noise components.

3. LCMV Beamforming

The basic idea behind the Linearly Constrained Minimum Variance (LCMV) beamforming is to constrain the response of the beamformer, so signals from the direction of interest are passed with specified gain and phase. The weights are chosen to minimize output variance or power subject to the response constraint. This has the effect of preserving the desired signal while minimizing contributions to the output due to interfering signals and noise arriving from directions other than the direction of interest [8, 9].

$$\min_w \boldsymbol{W}^{\dagger}\boldsymbol{R}_{xx}\boldsymbol{W} \quad \text{Subject to } \boldsymbol{C}^{\boldsymbol{T}}\,\boldsymbol{W}=\boldsymbol{I} \quad \ldots(26)$$

where $\boldsymbol{R}_{xx}$ is the autocorrelation matrix $(M \times M)$, **C** is a $(M \times 1)$ constraint matrix for look direction (desired direction).

$$\boldsymbol{C} = P_d * \operatorname{sinc}\left(\frac{\mathrm{B_d}}{2} * \omega_{\mathrm{o}}(m-1)T_d\right)\, e^{j\omega_o(m-1)T_d} \quad \ldots(27)$$

The optimum steady state weight vector $[\,\boldsymbol{W_{opt}}]$ can be accomplished by method of Lagrange multipliers

$$W_{opt} = {R_x}^{-1}C\left[C^T {R_x}^{-1}C\right]^{-1}I \quad \ldots(28)$$

With the optimum weight vector given in Eq. (28), the output powers due to desired, interference and noise are given by

$$P_d(\omega) = E|y_d(k)|^2 = \boldsymbol{W}_{opt}^{\dagger}\,\boldsymbol{R}_{dd}\boldsymbol{W_{opt}} \quad \ldots(29)$$

$$P_i(\omega) = E|y_i(k)|^2 = \boldsymbol{W}_{opt}^{\dagger}\,\boldsymbol{R}_{ii}\boldsymbol{W_{opt}} \quad \ldots(30)$$

$$P_n(\omega) = E|y_n(k)|^2 = \boldsymbol{W}_{opt}^{\dagger}\,\boldsymbol{R}_{nn}\boldsymbol{W_{opt}} \quad \ldots(31)$$

Then the output SINR is given by

$$SINR(\omega) = \frac{P_d(\omega)}{P_i(\omega)+P_n(\omega)} \quad \ldots(32)$$

and the output INR is given by

$$INR(\omega) = \frac{P_i(\omega)}{P_n(\omega)} \quad \ldots(33)$$

4. Simulation and Results

The simulation cases programes are written by MATLAB 8.2 and the following assumptions are considered

- Isotropic linear array antenn uniformaly distribution.
- Desired signal input power is 0dB and interference input power is 20dB.
- Interference sourece angle of arrival is from endfire ($\theta_i = 0°$).

CASE 1

Ten isotripoic array elements with $0.5\lambda_o$ ($\lambda_o = 2\pi c\backslash\omega_o$) inter element spacing is considered. A desired signal angles of arrival are assumed to be from ($\theta_d = 80°,\ 60°,\ 40°$).

Fig (3) shows that the output SINR is degraded with the increase of interference bandwidth for all desired signal angles of arrival.The degradation is increased when the desired signal angle of arrival moves towards end fire (decreased), this is due to the increase in element phase delay ($\emptyset_i = \beta d \sin(\theta_i)$) and this in turn leads to an increase in the interference relative bandwidth as given in Eq.(23). Fig. (4) shows that the output INR is increased when the interference bandwidth is increased which means more interference power will appear at the output of the system due to limation of narrowband array processor.

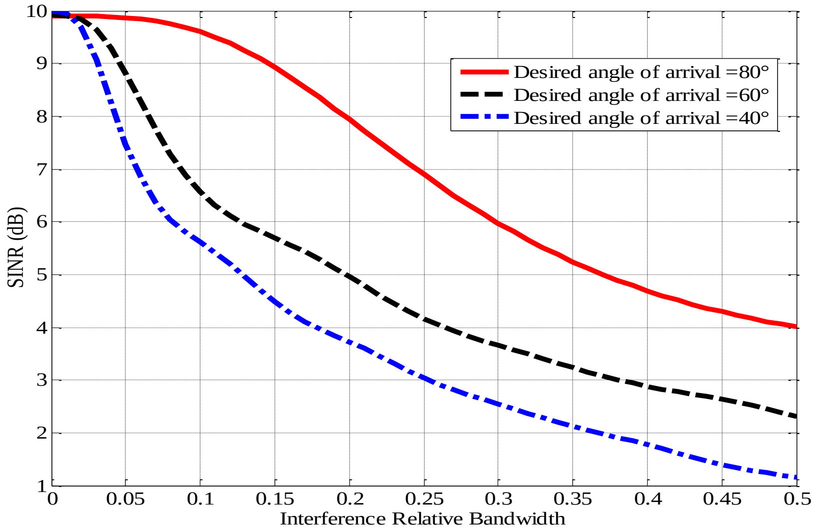

Fig. 3. Output SINR plot versus interference relative bandwidth for different desired signals angle of arrival.

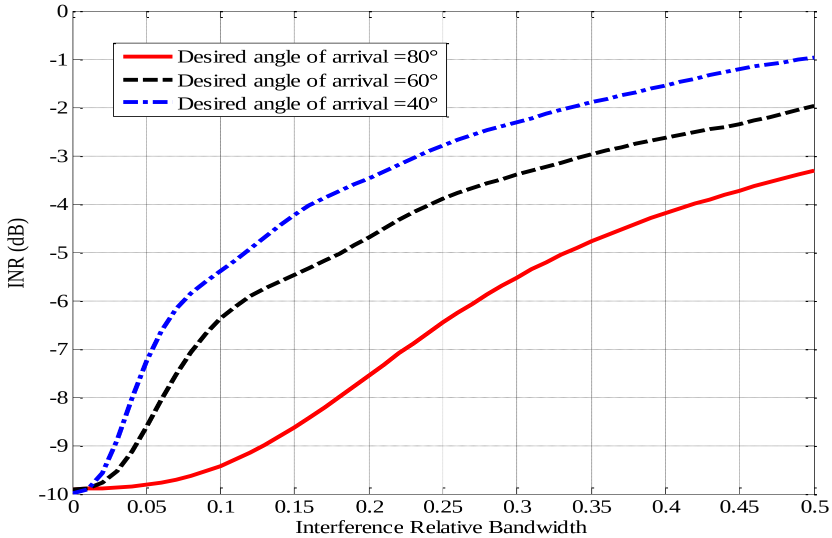

Fig. 4. Output INR plot versus interference relative bandwidth for different desired signals angle of arrival.

CASE 2

Ten isotripoic array elements with desired signal angle of arrival from 60° and for different interelment spacings (d= 0.25, 0.5, 0.75 λ_o).

Fig. (5) shows that the output SINR is degraded when the interference bandwidth is increase for all inter element spacing (d= 0.25, 0.5 and 0.75 λ_o). The increase in the interelement spacing from $0.25\lambda_o$ to $0.75\lambda_o$ cause an increase in the effect of interference relative bandwidth as given in Eq.(23), so it causes a more degradation in the outpt SINR. Fig. (6) shows that the output INR is increased with the increase in the interferences bandwidth and interelment spacing this is due to the increase in the effect of interference bandwidth effect on a narrowband processor.

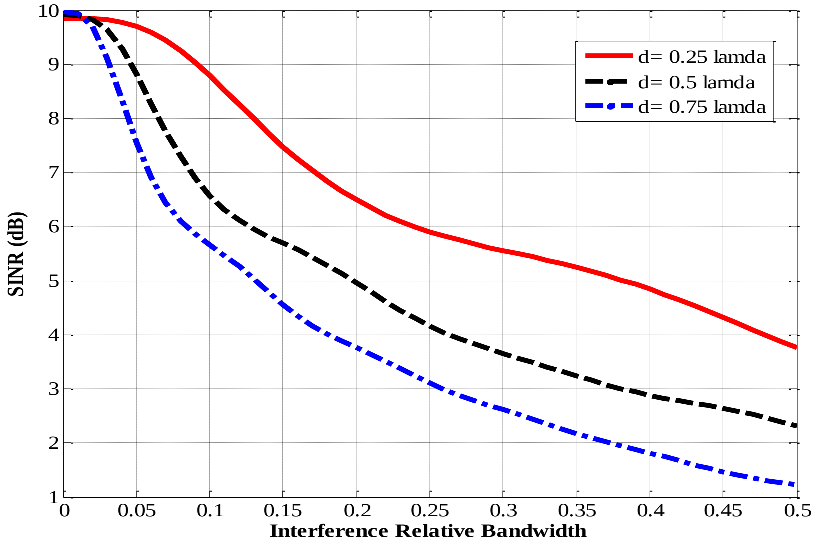

Fig. 5. SINR plot versus interference relative bandwidth for different interelements spacing

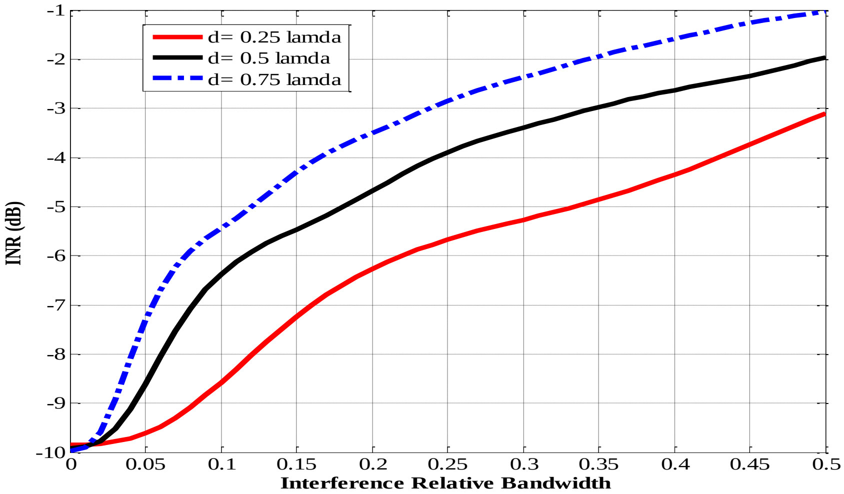

Fig. 6. INR plot versus interference relative bandwidth for different interelements spacing

CASE 3

Two array sets with nine and four elements with $0.5\lambda_o$ inter element spacing are considered. A desired signal angle of arrival is from 60° with relative bandwidth between 0 to 0.5.

Fig. (7) shows that the system exhibit more degradation in the output SINR when the number of array antenna is increased from 4 to 9 (i.e. when the array system has wide operatere). This is due to the increase in the effect of interference bandwidth with the increase of array aperture (Array aperture = number of element × interelement spacing) according to Eq. (23). It can be also shown that when interference relative bandwidth is (0.3), the amount of drop in the output SINR is (3.504dB) for four elements and (3.608) for nine elements.

Figure (8) shows the same reasons mentioned above that the output INR is higher for the case of nine element due to increase in the array aperture.

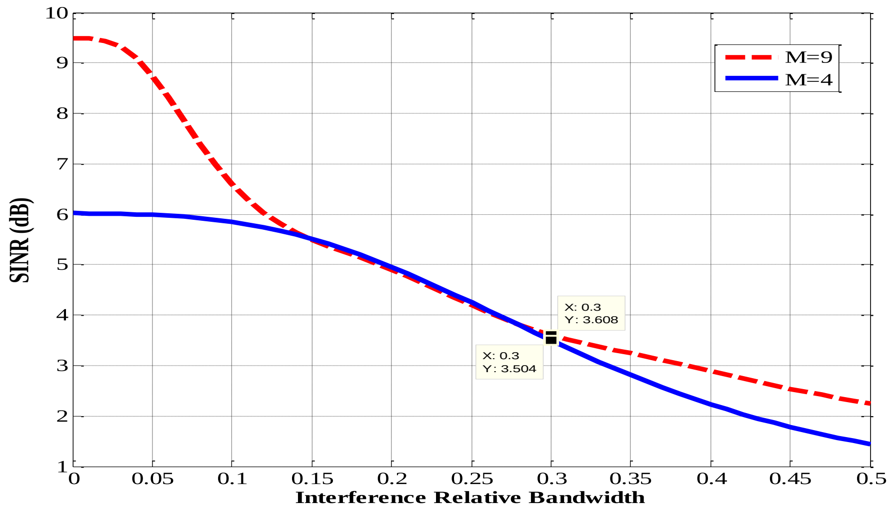

Fig. 7. SINR plot versus interference relative bandwidth for different number of antennas.

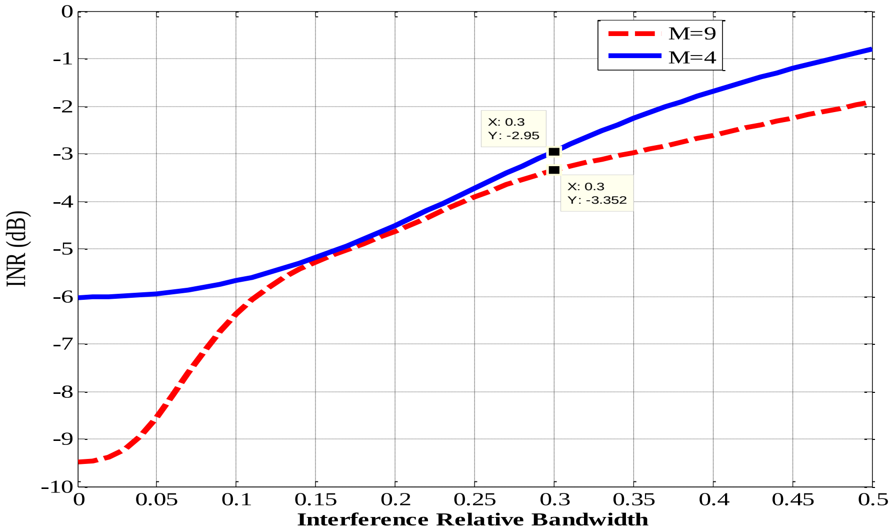

Fig. 8. INR plot versus interference relative bandwidth for different number of antennas.

5. Conclusion

It is found from the presented result in this paper there is a significant mutual effect between array parameters and receiving signal bandwidth. Theses mutual effects can be defend as follows

Generally the performance of adaptive array system with narrowband processor is degraded with the increase of received signal bandwidth. The degree of degradation is depending on the array parameters as well as on the value of received signal bandwidth. The increase in the number of array element causes an increase in the array aperture which makes the array antenna sensitive to the bandwidth of received signal, wide aperture cause more degradation in the output SINR. The received signal angle of arrival plays a role on the amount of bandwidth performance that the narrowband signal can be peer. It is found that the received signal is at the broadside angle the system is more sensitive for the received signal band width. The increase in the interelement spacing cause an in increase in the aperture of antenna which makes the effects of the received signal bandwidth more significant.

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آثار التفاعل المتبادل بين معاملات مصفوفة الهوائيات واستقبال إشارات النطاق الترددي

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الخلاصة

ان وجود مصفوفة توازن معقدة واحدة في كل عنصر من عناصرالمصفوفة المتكيفية تكفي فقط لمعالجة الإشارات ذات الحزمة الترددية الضيقة. أن قابلية النظام لتخميد التداخل ستسوء بزيادة الحزمة الترددية لمصدر التداخل. تم فحص اداء مصفوفة الهوائيات المتكيفة للحزمة الضيقة مع خوارزمية (LMCV). تم دراسة التأثيرات المتباينة بين عناصرمعاملات مصفوفة الهوائيات وتأثيرات عرض الحزم للإشارات المستلمة. تم استخدام معاملات قياس الجودة (SINR) و (INR) في تقيم هذه التاثيرات على عمل المنظومة. لقد توصل البحث الى أن مقدار التدهور في (SINR) يزداد بصورة ملحوظة بزيادة عدد الهوائيات والمساحة الفاصلة بين الهوائيات وكذلك عندما تكون زاوية الوصل للإشارة المستلمة قريبة من end fire.