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Analytical Modeling of Stresses in the Wall 0f the Human Heart

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Abstract

The mechanical function of the heart is governed by the contractile properties of the cells, the mechanical stiffness of the muscle and connective tissue, and pressure and volume loading conditions on the organ. Although ventricular pressures and volumes are available for assessing the global pumping performance of the heart, the distribution of stress and strain that characterize regional ventricular function and change in cell biology must be known. The mechanics of the equatorial region of the left, ventricle was modeled by a thick-walled cylinder. The tangential (circumferential) stress, radial stress and longitudinal stress in the wall of the heart have been calculated. There are also significant torsional shear in the wall during both systole and diastole. In addition there should exist shear stress in the wall of the heart due to action of pressure on the curved surface.

***Keyword*:** stresses, human heart, mechanical function

1. Introduction

The heart is a powerful muscle that pumps blood throughout the body by means of coordinated contraction. This powerful muscle is slightly larger than a clenched fist. The primary function of the heart is fundamentally mechanical to pump oxygen-rich blood through all parts of the body. A human heart beats at an average of 100,000 times per day. During that time, it pumps more than 17000 liter of blood throughout the entire body, compensating for acute changes in demand for blood flow and adapting to sustained changes in the applied loads.

The mammalian heart consists of four pumping champers, the right and left atria and right and left ventricles [1, 2].

2. The walls of the heart

The ventricular walls are the most important structures with regard to the pumping action of the heart. Generally the wall of the heart consists of three distinct layers.

a) An inner layer called the endocardium. This layer lines the inside of each cardiac chamber. It is a thin serous membrane.

b) A middle layer called myocardium. It is the functional tissue that endows the heart with its ability to pump, blood. The myocardium consists primarily of myocytes that are arranged into local parallel muscle fibers, which in turn are embedded in an extra cellular matrix consisting largely of types I and III collagen. The orientation of the muscle fibers change with position in the wall.

c) The outer most layer or epicardium is also a thin serous layer consisting largely of a two dimensional plexus of collagen and some elastic fibers [3].

The pericardium is a sac-like membranous structure that surrounds the heart. It holds the heart in a fixed geometric position and isolates the heart from other structures in the thorax, to prevent adhesions and the spread of infection [4, 5].

3. Geometry

The basics of the model of the left ventricular mechanics have been described in details in references [6, 7]. Briefly, the left ventricle is represented by a thick-walled three dimensional pressure vessel whose thickness and curvature vary regionally and temporally. In the normal heart, the ventricular walls are thick at the equator and the base of the left ventricle and thin at the left ventricle apex and right ventricular free wall. Nonetheless, the ratio of wall thickness to radius is always that the heart can only be treated accurately by the most sophisticated theories for thick-walled cylinder [3, 8, 9]. Because of similarity of geometry the equatorial region of the ventricle is simulated most accurately.

4. Stress in left ventricular wall

The left ventricular can probably be described as a prolated spheroid, noting that the equatorial region can be well approximated as a thick walled cylinder [3]. Moreover, a cylindrical geometry admits reasonable simple descriptions of rather complex motion radial inflation, axial extension, torsion and transmular shear; all of which have been measured in the beating and arrested heart [3].

Figure (1), designates the inside radius of the annul by (a) and the outside radius (b), the internal pressure by (p_i) and the external pressure by (p_o). Then it can be shown that tangential (σ_t) and radial (σ_r) stresses exist and whose magnitude is [21]:

$$\sigma_t = \frac{p_i a^2 - p_o b^2}{b^2 - a^2} + \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2} \quad \ldots(1)$$

$$\sigma_t = \frac{p_i a^2 - p_o b^2}{b^2 - a^2} - \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2) r^2} \quad \ldots(2)$$

In the above equations, positive stresses indicate tension and negative stresses indicate compressive stresses.

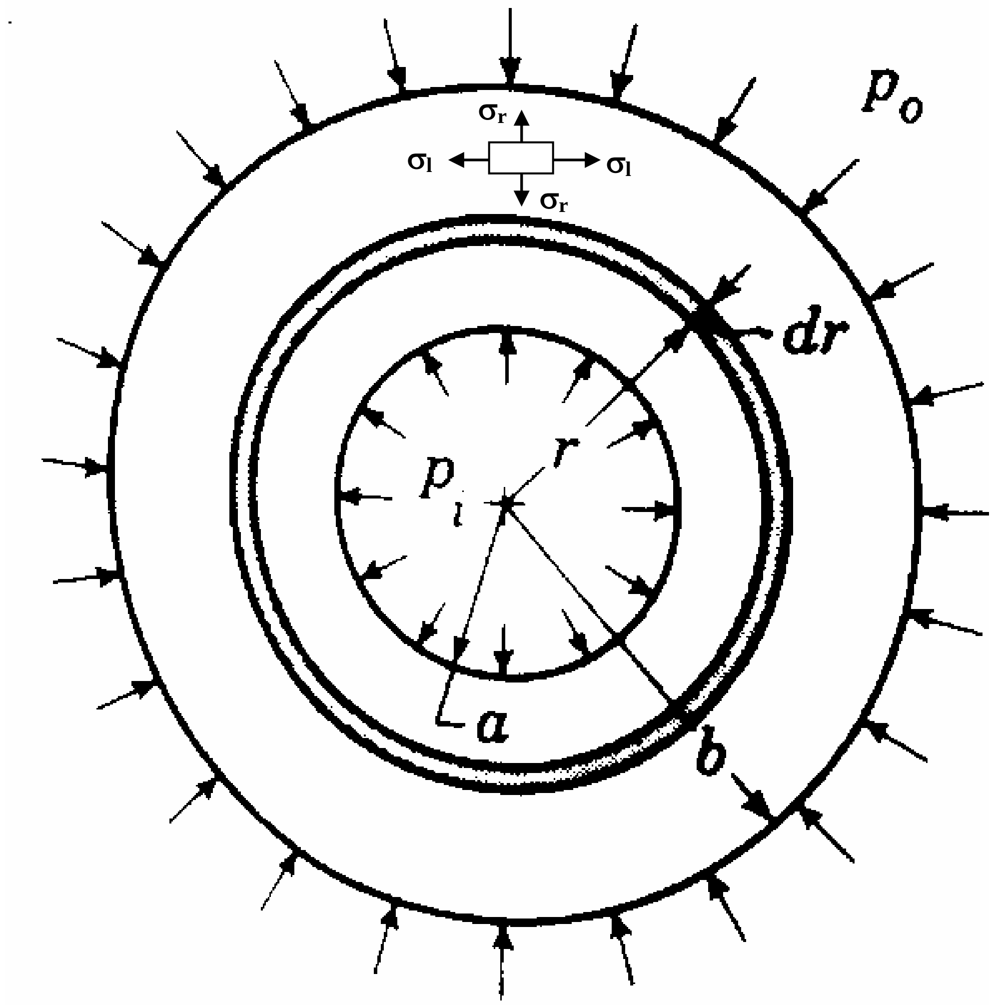

Fig. 1. A Cylinder Subjected To Internal And External Pressure (21).

In normal heart, p_0 is small and can be neglected [3], so that:

$$\sigma_t = \frac{p_i\, a^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \qquad \ldots(3)$$

$$\sigma_r = \frac{p_i\, a^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \qquad \ldots(4)$$

The distribution of stresses across the wall thickness is non-uniformly as shown in fig (2). The tangential stress σ_t is tensile while the radial stress is compressive diminishing from p_i at the inside surface to zero at the outside surface.

From the distribution diagrams, the maximum stresses occur at the inner surface where r=a and their magnitudes are [21]:

$$\sigma_t = p_i\,\frac{b^2 + a^2}{b^2 - a^2} \qquad \ldots(5)$$

$$\sigma_r = -p_i \qquad \ldots(6)$$

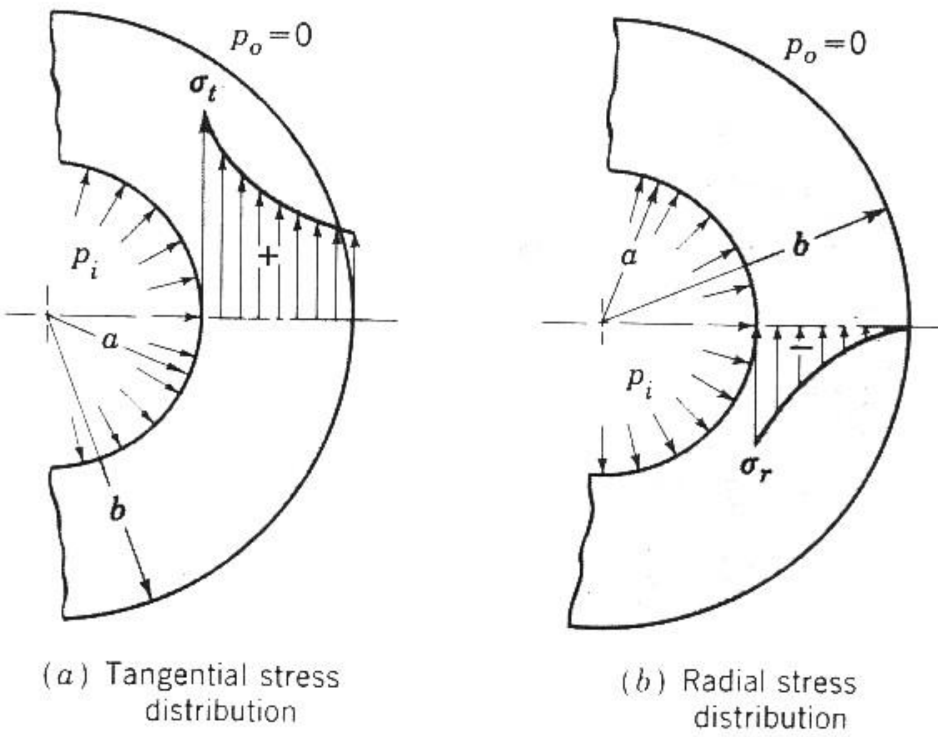

Fig. .2. Distribution of Stresses in a Thick-Walled Cylinder Subjected to Internal Pressure Only [21].

Since the internal pressure is taken by the heart, longitudinal stresses will exist of the magnitude will exist [21],

$$\sigma_l = \frac{p_i r_i^2}{r_0^2 - r_i^2} \qquad \ldots(7)$$

The above mentioned relations are classic relations that are derived in introductory mechanics of material courses. The results are good for cylinder subjected to a uniform inflated pressure p_i. Not restricted to particular class of material, they are examples of universal results which are particularly important in biomechanics [3].

There are also significant shear stresses in the wall during both systole and diastole (fig 3). Plane (torsional) shears are negative during diastole, and are consistent with a small left-handed torsion of the left ventricle during filling, and positive as the twist of the ventricles reverses during ejection[10]. The distribution of this stress along any radius varies linearly with the radial distance from the axis of the heart. In addition because myocardial fibers are wrapped toroidally around the central layers of the wall in the real left ventricle, [11, 12, 13], shear stress should also exist in the wall of the heart throughout the cardiac cycle, due to the action of a pressure on the curved surface.

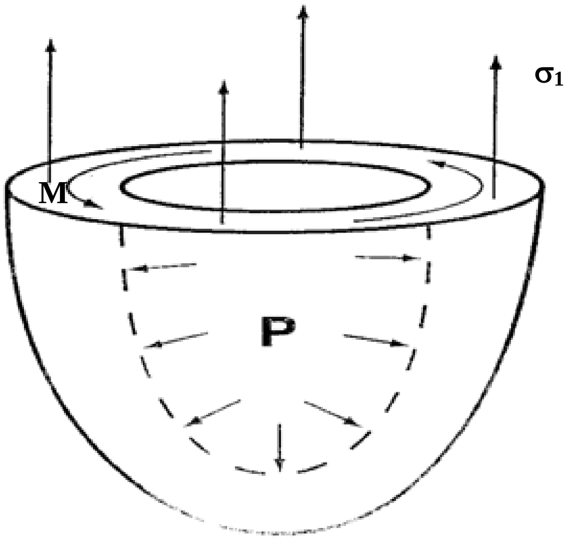

Fig. 3. Free Body Diagram of the Left Ventricle, Showing of the Longitudinal Stress and the Torsional Moment (τ is the shear stress, M is the torsional moment).

5. Discussion of the results

1. The elastic circular hydraulic system of the heart causes simple harmonic motion repeated about 75 times per minute. Due to this motion stress is induced in the wall of the heart with its maximum at the inside surface of the wall. Since the stresses vary in magnitude and direction, in each cycle, the inside surface of the heart will be sensitive to any natural defect in the wall and that may lead to fracture in the inside surface of the wall.
2. In calculating the stresses, simplified assumptions such as homogeneous wall has been used [3]. Finding by Novak etal [14], Kang etal [15] and Hunten etal [16] suggest that the myocardium is not homogeneous, and that the endocardium and epicardium also exhibit distinct behavior [17, 18]. Viewed in this manner our results must thus be considered to reflect homogenized or mean properties which can be sufficient in some analyses, but certainly not in all.
3. Minor [19] noted "each heart beat or cardiac cycle is an intricate sequence of electrical and mechanical events". Although a full understanding of cardiac function will thus require an electromechanical analysis [10, 19, 20]. Here it has been focused on the mechanical alone (due to limited time).
4. To avoid any assumption that can more or less affect the result of analytical modeling, experimental test should be carried out. Unfortunately no experiment in laboratory is comprehensive because of the loss of the in vivo environment which includes hormonal influences and control by the nervous system on the function and properties of the cells, tissues or organ. Again in vivo experiments or test, intact organs are not without concern. From the mechanics and biology points of views the intact organ is typically so complicated that the interpretation of data often requires many simplified assumptions, which in turn may musk salient features of the behavior of manifestation of many contributors that are often difficult to delineate. Therefore, we must consider many different classes of test with the hope that together they will reveal most of the feature of interest [3].

6. Conclusion

The left ventricle is represented by thick walled three dimensional pressure vessels. Due to the complex motions of the ventricle, radial inflation, axial extension, torsion and transmular shear. Stresses are induced in the wall of the ventricles.

1. Tangential and radial stresses exist. The distribution of these stresses across the wall thickness is non-uniformly.
2. Since the internal pressure is taken by the heart longitudinal stress will exist.
3. There are also significant shear stresses in the wall during both systole and diastole. They are negative during diastole when filling and positive as the twist of the ventricles reverses during ejection.
4. In addition, because myocardial fibers are rapped toroidally around the central layers of the wall in the real left ventricle, shear stress should also exist in the wall of the heart throughout the cardiac cycle, due to action of a pressure on the curved surface.

Since all these mentioned stresses vary in magnitude or and direction in each cycle, the inside surface of the heart will be sensitive to any natural defect in the wall and this may leading to fracture inside the wall.

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نموذج تحليل الإجهادات داخل قلب الإنسان

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الخلاصة

الحركة الميكانيكية للقلب، مسيطرة بالخاصيات التقلصية للخلايا، الصلادة الميكانيكية للعضلات والنسيج الرابط وضغط وحجم الحمل على العضو بالرغم من ان ضغط وحجم البطين متوفرة لتقييم اداء الدفع الكلي للقلب، لكن توزيع الجهد والتوتر (strain, stress) التي توصف وظيفة البطين موضوعيا والتغير في حيوية الخلية يجب ان تعرف.

ان ميكنيكية المنطقة القلبية للبطين تمثل بالاسطوانة السميكة الجدران وعلى هذه الفرضية تم حساب الجهد المحيطي والجهد لشعاعي وجهد الطول في جدار القلب. خلال تقلص وانقباض القلب يتكون جهد لي Torsion stress في الجدار اضافة وجود جهد قص Shear stress في جدار لقلب لعمل الضغط على الاسطح المقوس في البطين.