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Experimental Estimation of Critical Buckling Velocities for Conservative Pipes Conveying Fluid

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Abstract

Conservative pipes conveying fluid such as pinned-pinned (p-p), clamped–pinned (c-p) pipes and clamped-clamped (c-c) lose their stability by buckling at certain critical fluid velocities. In order to experimentally evaluate these velocities, high flow-rate pumps that demand complicated fluid circuits must be used.

This paper studies a new experimental approach based on estimating the critical velocities from the measurement of several fundamental natural frequencies .In this approach low flow-rate pumps and simple fluid circuit can be used.

Experiments were carried out on two pipe models at three different boundary conditions. The results showed that the present approach is more accurate for estimating the critical velocities of p-p and c-p pipes. However, for c-c pipes it was not so unless a higher flow rate is used.

Keywords: *Conservative pipes, critical buckling velocity***.**

1. Introduction

It has been demonstrated that the natural frequencies for conservative pipes conveying fluid such as pinned-pinned, clamped-pinned and clamped-clamped pipes decrease with the increasing of the fluid velocities [1] .At certain critical values of the fluid velocities the fundamental natural frequency can be dropped to zero .At this condition pipes can lose their stability by static divergence or buckling [2] and [3].

Evaluating the critical velocities of buckling is an important task in pipes design .They separate the range of velocities at which the pipe becomes safe and stable from the unstable range.

Many early researchers studied the stability for conservative pipe conveying fluid like Bishop and Fawzy [4], Gregory and Paidoussis [5] and recently, Kuiper and Metrlkine [6] and Wang and Ni [7] .Their investigations were focused on evaluating the critical fluid velocities for buckling instability and stability characteristics for pinned-pinned, clamped–pinned and clamped-clamped pipes conveying fluid. These studies were based on the solution of the equation of motion by either approximate analytical or numerical methods.

Long [8] carried out many experiments on pinned–pinned pipes to show the effect of the fluid velocity on the natural frequencies .Benjamin [9] made a visual investigation for the dynamics of articulated pipes (two degree of freedom pipes) with different supports .Many experiments measured the critical fluid velocities of buckling and recorded the flutter phenomena photographically. Gregory and Paidoussis [10] investigated experimentally the main dynamical features of the cantilever pipes such as the effects of fluid speed and mass ratio on flutter instability and the natural frequencies.

Critical velocities of buckling instability are normally high and it is difficult to measure them experimentally unless high flow rate pumps are used. As a result a complicated fluid circuit will result and the problem of turbulence may arise [11].

Owing to these difficulties, a new approach is presented in the present work for evaluating the critical velocities of buckling from a set of experimental data .Such data was obtained from measuring the fundamental natural frequencies at relatively low fluid velocities.

2. The Present Approach

The relations between the fundamental natural frequencies and the fluid velocities for conservative pipes conveying fluid were derived by Wang and his co-workers [12] in 2010.

They gave the following expressions for the three conservative pipes conveying fluid:-

$$w = p^{2}\sqrt{\frac{EI}{m_p + m_f}}.\sqrt{1-\frac{m_f V^2}{p^2 EI}} \quad \text{for(p-p)} \quad \ldots(1)$$

$$w = 3.927^2\sqrt{\frac{EI}{m_p + m_f}}.\sqrt{1-\frac{0.747 m_f V^2}{3.927^2 EI}} \quad \text{for(c-p)} \quad \ldots(2)$$

$$w = 4.73^2\sqrt{\frac{EI}{m_p + m_f}}.\sqrt{1-\frac{0.55 m_f V^2}{4.73^2 EI}} \quad \text{for(c-c)} \quad \ldots(3)$$

Now, eqs.(1-3) may be written in the following dimensionless forms :-

$$\Omega = p^2\sqrt{1-\frac{U^2}{p^2}} \quad \ldots(4)$$

$$\Omega = 4.73^2\sqrt{1-\frac{U^2}{6.378^2}} \quad \ldots(5)$$

$$\Omega = 3.927^2\sqrt{1-\frac{U^2}{4.543^2}} \quad \ldots(6)$$

Where :-

$$U = VL.\sqrt{m_f / EI} \quad \ldots(7)$$

and

$$\Omega = \omega L^2.\sqrt{(m_f + m_p)/EI} \quad \ldots(8)$$

Squaring and arranging eqs.(4-6) gives:-

$$\frac{\Omega^2}{p^4} = 1-\frac{U^2}{p^2} \quad \ldots(9)$$

$$\frac{\Omega^2}{3.927^4} = 1-\frac{U^2}{4.543^2} \quad \ldots(10)$$

$$\frac{\Omega^2}{4.73^4} = 1-\frac{U^2}{6.378^2} \quad \ldots(11)$$

When Ω=0, Eqs .(9-11) give the theoretical critical velocities of buckling U_c for the three types of the conservative pipes which are:-

$U_c = \pi, 4.543, 6.378$

Furthermore, when U=0 ,eqs .(9-11) give the dimensionless fundamental frequencies Ω_b of the corresponding beams which are:-

$\Omega_b = \pi^2, (3.927)^2, (4.73)^2$

In this way Eqs .(9-11) can be put in the following general form:-

$$\frac{\Omega^2}{\Omega_b^2} = 1-\frac{U^2}{U_c^2} \quad \ldots(12)$$

Or :-

$$\Omega^2 = \Omega_b^2 - (\Omega_b/U_c)^2 U^2 \quad \ldots(13)$$

Noting that eq.(13) represents a straight line in Ω^2-U^2 plain.

According to eq.(13), when a set of fundamental frequencies at given fluid velocities are plotted Ω^2 against U^2 ,the result is a straight line .From this line the value of the critical velocity can be calculated. This is the basic concept of the present approach.

In summary, the critical velocities can be estimated from the measured data of the fundamental natural frequencies, for a given model by performing the following steps:-

1- Measuring the fundamental natural frequencies at a range of fluid velocities.
2- Plotting the data in Ω^2against U^2.
3- Fitting the data to a straight line and determining its equation.
4- Comparing the equation of the fitted line with eq.(13) to find the critical velocity *Uc*

This approach enables using any available fluid velocities, however, when the flow rate is relatively low (less that 30 l/min) the variation in the fundamental frequencies become insensitive for most models.

3. Apparatus and Test Models

Table 1 shows the specifications of the two test models used in this work.

Table 1,
Specifications of the Test Pipe Models [13].

Model No.	Material	D_o m	t m	L (m)	ρ kg/m^3	E GN/m^2
1	Aluminum	0.01	0.001	1.25	2650	62
2	PVC	0.011	0.0011	1.25	1000 (at 30 C^o)	7.12 (at 30 C^o)

The test rig consists of two main parts; the foundation and the two end supports.

The foundation was constructed from brown-walnut wood and reinforced by 2 mm thickness steel plates. See Fig .(2)

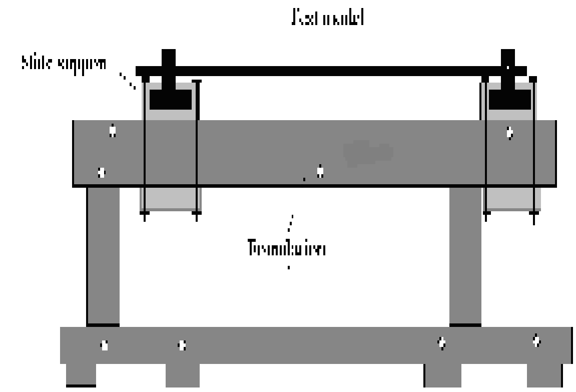

Fig.1. Schematic Layout for the Wooden Test Rig.

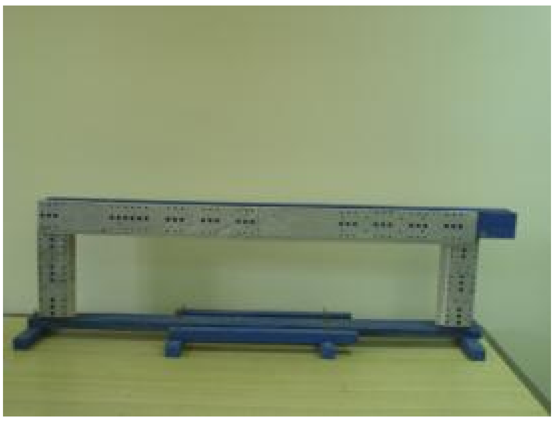

Fig.2. A photograph for the Test Rig.

Wood was chosen for the foundation since it has the following advantages over the metallic materials:-

1- It is simple to process.
2- It has a higher structural damping making it a good vibration-absorber for undesired vibrations that can be transmitted from the test-model supports.
3- The ratio E/ρ of most woods is about 25 kN.m/g as compared with 26 kN.m/g for the most common metallic material [14] .Thus their dynamical behavior are nearly similar.

The two end supports were designed to fulfill the various requirements for pinned and clamped conditions. They consisted of two main parts; the cast iron base and the steel holders.

For pin support, a ball-bearing assembly was used to insure zero moment and displacement see fig.(3).

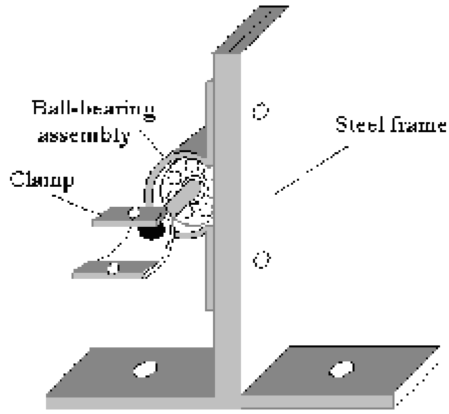

Fig. 3. Configuration of Pinned Support.

For clamp support, steel clamps were provided to insure zero slope and displacement necessary for clamped end condition.

The various boundary conditions for the three conservative pipes can be performed by using the proper supports selection.

Water was used as a flowing fluid .To measure the water velocity a flow meter was fitted at the inlet of the test model. The water circuit is shown in Fig.(4). The main components of this circuit are the collecting tank (150 litters) ,centrifugal pump (60 l/min,20 m), control valve (gate type) and the test pipe model.

In the case of PVC pipe models a digital thermometer was used to record the operating temperature since their mechanical properties are temperature dependant .The sensor of the thermometer was fitted on to the pipe outer surface.

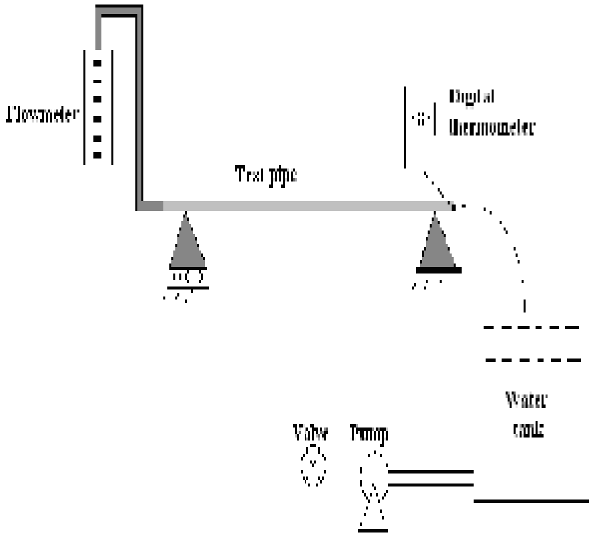

Fig. 4.Schematic Diagram for Test Model and Water Circuit.

The block diagram of the various components of the electric vibration circuit used in this work are shown in fig.(5).

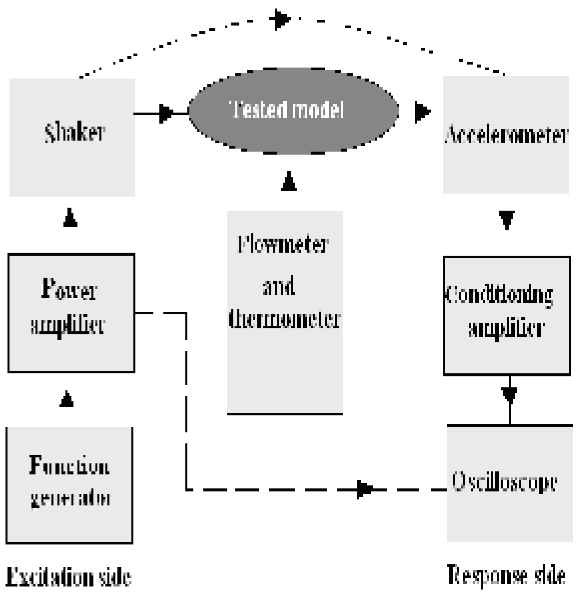

Fig.5.Block diagram for vibration circuit (the dash lines are for calibration mode).

Prior to testing, the densities and modulus of elasticity of the test models were checked and the accuracy of the instruments were tested.

To check the modulus of elasticity for the test pipe models simple experiments were carried out. In these experiments the lowest three natural frequencies of the pipe models without fluid and under clamped –clamped conditions were measured .Then the modulus of elasticity is calculated from the following formula:

$$E=\frac{m_p L^4 w^2}{I B_n^2} \qquad \text{for n=1, 2, 3} \qquad \ldots (14)$$

Where:

B_n =1.5π, 2.5π, 3.5π

The densities of the model materials were a checked by weighting pipe samples of length of 2cm by using a three-digit electronic balancer. The results showed good agreement between the measured and the calculated values.

The accuracy of the flow meter was tested in a calibration process .In this test the water was allowed to full a reservoir of five liters capacity through the meter. The average time required for this process was recorded and the flow rate was calculated .The calculated flow rate was compared with the meter reading. The results gave good agreement.

Calibrating the variation of the vibration instruments focus was made on checking the overall accuracy due to measuring the frequency which is the main interest in this work. The accuracy of the amplitude was not taken into account .This is true since the natural frequencies can be measured by observing the resonance frequencies rather than their amplitudes. For this purpose the instruments were connected as shown in the dash line paths of Fig.(5). In this arrangement the accelerometer was directly mounted on the vibrating part of the shaker and an electrical feedback signal was taken from the shaker input .The detected frequency (f_i) by the accelerometer was fed to channel A of the oscilloscope while the feedback frequency (f_d) was fed to channels B. The shaker was allowed to vibrate at specified sets of frequencies generated by the Function generator (f_s) . These frequencies were 100, 500 and 1000 Hz. The two output frequencies displayed on the oscilloscope were observed and recorded .Then ,the overall accuracy for the exciter and the response sides for the three selected frequencies were calculated from the following relations :

% over all accuracy of the exciter side

$$\eta_{ex} = \frac{fs - f_i}{fs} x100$$

% over all accuracy of the response side

$$\eta_{rs} = \frac{f_i - f_d}{f_i} x100$$

The calculated accuracies are shown in Table (2) . This table shows that the accuracies are high and hence the errors are small.

Table2,
Overall Accuracies of Exciter and Response Side Circuits at Some Selected Frequencies.

f_s	f_i	f_d	η_{ex}	η_{rs}
100	99.8	99.2	99.8%	99.2%
500	492	490	98.4%	98%
1000	990	985	99%	98.5%

4. Experimental Procedures

The fundamental natural frequencies of the two models were measured at different water velocities and boundary conditions .In all cases the fundamental natural frequencies were firstly measured at zero flow rate and then the water flow rates are increased to 10 ,20 ,30 ,40 ,50 and 60 l/min .This was achieved by adjusting the controlling valve .It is important to mention that, since in some flow rates the flow is turbulent, the water was allowed for flowing about ten minutes prior any measuring..

In measuring the natural frequencies the test models were subjected to forced vibration under the effect of the harmonic force generated by the shaker to insure only force transmission to the models and avoiding the effect of the moment the connection between the models and the shaker shaft was made as a pin-joint connector by using a ball-bearing assembly.

It is important to mention here that, it is necessary to minimize the amplitude of the excitation force due to the following reasons:

1- Insure a linear vibration since the amplitude remains small and within the elastic limit.
2- Reduce the effect of the modulation which may be taken between the vibrational wave of the test model and that of the foundation structure.
3- Minimize the effect of the impact which may be taken between the rotated parts such as the pinned supports and connectors.
4- Minimize the effect of structural and Coulomb friction damping in the models.

For this purpose the amplitude of the shaker was kept small within 2 V. This was done by adjusting the gain of the power amplifier .To compensate this reduction, the sensitivity of the accelerometer was raised to a maximum value by adjusting the gain of the conditioning amplifier to about 30mV.
With the above arrangements refined vibrational waves which are necessary for accurate measuring were obtained on the oscilloscope.

The response of the pipe models were picked by the accelerometer and displayed via the oscilloscope.

In measuring the fundamental natural frequencies the excitation frequency of the shaker was gradually increased from zero observing the response until a sharp amplitude (resonance) was reached .At this instant the frequency was recorded .

Fig.(6) shows photograph of natural frequency tests for the PVC model at p-p boundary conditions .

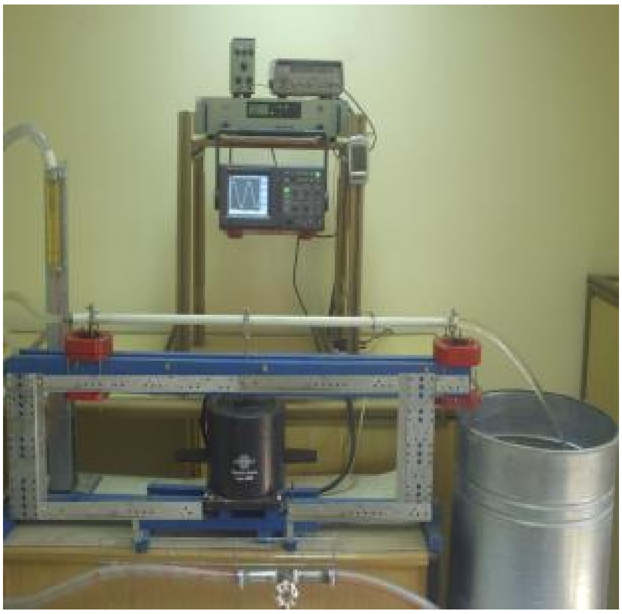

Fig.6. A photograph for p-p, PVC pipe.

5. Results and Discussions

For the illustration purpose the theoretical critical flow rates of buckling of the two models at different boundary conditions were calculated and presented in table 3 .Table 3 shows the required high flow rates for the buckling of the two models.

The measured natural frequencies and the associated fluid flow rates for the two models at different boundary conditions are given in table (4) and (5).

Table 3,
Critical Flow Rates (Qc) for the Test Models at Different Boundary Conditions.

Model No.	Material	Q_c (p-p) l/min	Q_c (c-p) l/min	Q_c (c-c) l/min
1	Aluminum	238	357	476
2	PVC	101	151.5	202

Table 4,
Measured Fundamental Natural Frequencies (Hz) for Aluminum Pipe at Different Boundary Conditions .

Q(l/min)	0	10	20	30	40	50	60
p-p	13.2	13.0	12.9	12.6	12.4	11.9	11.8
c-p	17.75	17.85	17.6	17.6	17.5	17.4	16.7
p-p	25.7	25.7	25.77	25.6	25.57	25.2	25

Table 5,
Measured Fundamental Natural Frequencies (Hz) for PVC pipe at Different Boundary Conditions .

Q(l/min)	0	10	20	30	40	50	60
p-p	7.2	6.6	6	5.9	5.4	5	2.5
c-p	8.1	8.2	8	7.75	7.3	6.7	6
c-c	12	11.8	11.5	11.2	11	10.4	9.9

The dimensionless frequencies Ω and velocities U of the measured data then calculated from eqs.(7) and (8) .

In figs (7-12) the Ω^2 and U^2 for the two models at the three considered boundary conditions are plotted and fitted to straight lines

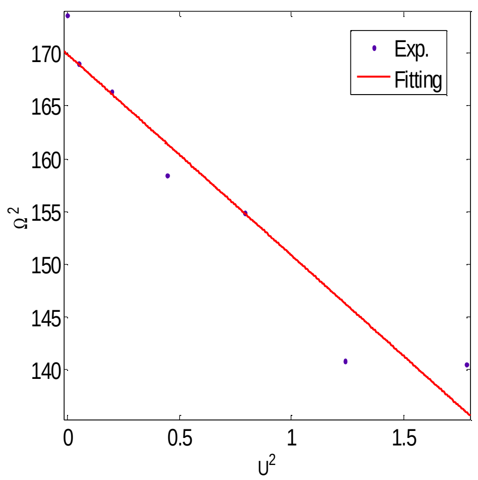

Fig.7.Experimental Plot of Ω^2 against U^2 for p-p Aluminum Pipe.

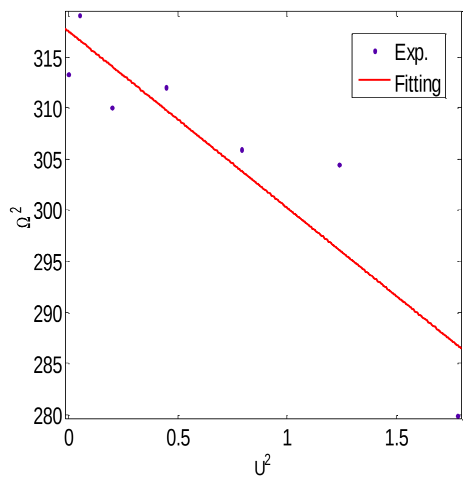

Fig. 8. Experimental plot of Ω^2 against U^2 for c-p aluminum pipe.

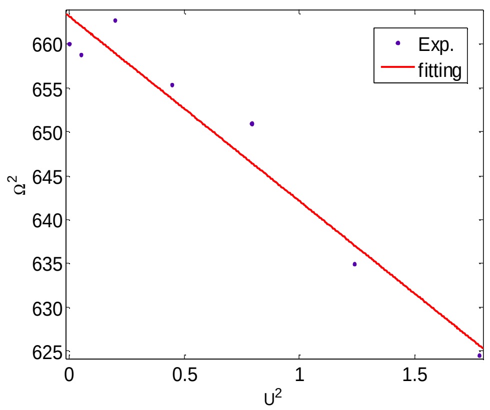

Fig. 9. Experimental Plot of Ω^2 Against U^2 for c-c Aluminum Pipe.

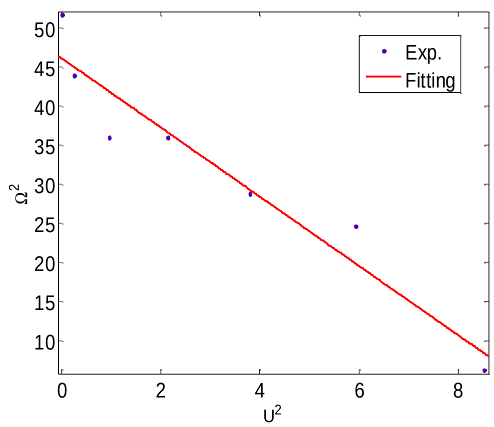

Fig. 11. Experimental plot of Ω^2 against U^2 for c-p PVC Pipe.

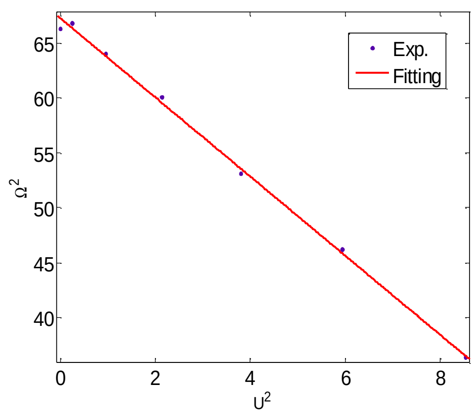

Fig.10. Experimental Plot of Ω^2 Against U^2 for p-p PVC Pipe.

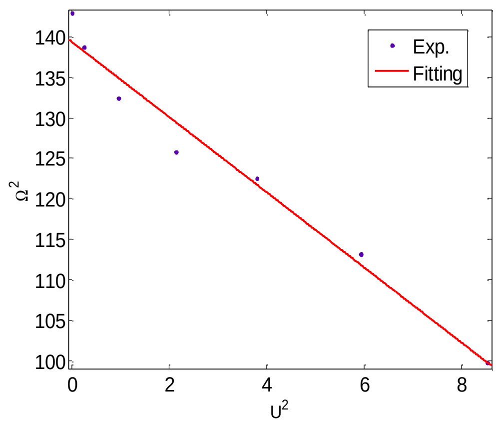

Fig. 12. Experimental Plot of Ω^2 Against U^2 for c-c PVC Pipe.

Then, the equations of the fitted lines were evaluated and are given in Table 6.

Table 6,
Equations of the Fitted Lines for Figs. (7) to (12).

Figure no.	Equation of fitted line	Boundary conditions
7	Ω^2 =169.9 -19.03 U^2	p-p
8	Ω^2 =317.5 -17.27 U^2	c-p
9	Ω^2 =663.2 -21.05 U^2	c-c
10	Ω^2 =46.05 -4.406 U^2	p-p
11	Ω^2 =67.29 -3.604	c-p
12	Ω^2 =139.3 -4.63 U^2	c-c

Comparing the equations given in Table 6 with eq.(13) gives the estimated dimensionless critical velocities .Finally, using eq.(7), the critical velocities for the two models were calculated.

For example, to find the critical velocity of p-p aluminum model -
The equation of the fitted line from Table 6 is:

Ω^2 =169.9-19.03U^2

According to eq.(13) :-

Ω_b^2=169.9 and $(\Omega_b/U_c)^2$= 19.03

Solving these equations give :

U_c =٢.٩٨٨٠

Now by using eq.(7) and substituting the aluminum pipe specifications shown in Table 1,one get the following critical velocity :-

V_c = 42.5032 m/s

The results of the estimated and theoretical critical velocities of the two models and the associated errors are given in the Tables 7 and 8.

Table 7,
Estimated and Theoretical Critical Velocities for Aluminum Pipe .

Boundary conditions	Critical velocity(m/s) Estimated	Theoretical	Error%
p-p	42.5032	44.6884	-4.9
c-p	60.9916	64.0114	-4.7
c-c	79.8437	89.3767	-10.1

Table 8,
Estimated and Theoretical Critical Velocities for PVC Pipe .

Boundary conditions	Critical velocity Estimated	Theoretical	Error%
p-p	18.2238	17.7091	2.9
c-p	24.3573	25.3664	-3.95
c-c	30.9194	35.4182	-12.4

Tables 7 and 8 indicate that there is generally some errors between the theoretical and the estimated values of critical velocities these errors can be attributed to many reasons such as :

1. It is impossible to produce perfect boundary conditions in practice.
2. Many sources of error such as instruments, reading, etc.
3. The effect of damping.

Also, Tables 7 and 8 show that for pinned-pinned and clamped-pinned cases, the estimated critical velocities are very closed to the theoretical values since the errors are small. However for clamped-clamped case the errors are relatively high for the two models .This can be attributed to the fact that the critical velocities for clamped-clamped pipe are higher than those for other boundary conditions . This will decrease the slope of the line given in eq.(13) .The small slope means that the variation in the fluid velocity lead to a small variation in the natural frequencies. This small variation is difficult to be detected experimentally unless a wider range of velocities are considered .Thus, for accurate estimation the flow rate must be increased for clamped-clamped case .

6. Conclusions

The present approach provides an alternative simple experimental method for evaluating the critical velocities of buckling instead of the classical methods which require high flow rate pumps and sophisticated fluid circuits.

The validity of present approach was checked by comparing its results with the theoretical ones. From the experimental results of two pipe models at different boundary conditions it can be concluded that, the present approach is more accurate for pinned-pinned and clamped–pinned pipes since the resulting errors are small .However for clamped-clamped pipes the accuracy is less unless higher flow rates are used.

Notations and Nomenclatures

p-p	Pinned-pinned
c-p	Clamped – pinned
c-c	Clamped- clamped
A_f,A_p	Fluid and pipe cross sectional area ,respectively.(m^2)
D_o,D_i	Outer and inner pipe diameter ,respectively .(m)
E	Modulus of elasticity.(N/m^2)
I	Moment of inertia (m^4)
L	Pipe length.(m)
m_f, m_p	Fluid and pipe mass per unit length, respectively .(kg/m)
Qc	Critical flow rate (l/min)
U	Dimensionless fluid velocity.
U_c	Dimensionless critical buckling velocity
V	Fluid velocity.(m/s)
V_c	Critical velocity of buckling (m/s)
Ω	Dimensionless frequency = $\omega L^2[(m_f + m_p) /E I]^{1/2}$
ω	Circular frequency.(rad/sec)
Ω_b	Dimensionless natural frequency of beams
ρ	Fluid density (kg/m^3)

7. References

[1] Paidoussis M..P.," Fluid–Structure Interactions: Slender Structures and Axial Flow", 1. New York: Academic Press; 1998.

[2] Weaver D.S., and Unny T.E., "On the Dynamic Stability of Fluid Conveying Pipe"ASME,,48-50,1973

[3] Rao C. and Simhab H.. "Critical Velocity of Fluid-Conveying Pipes Resting on Two-Parameter Foundation" J.Soun.Vib.Vol.302, 387-397, 2007.

[4] Bishop R.E. and Fawzy I., "Free and Forced Oscillation of Vertical Tube Containing a flowing Fluid" Philosophical Transactions of Royal Society of ,1970
[5] Gregory R..W.and Paidoussis M..P. ,"Unstable Oscillation of Tubular Cantilevers Conveying Fluid. I. Theory ". Proc Royal Soc London A Vol.293, 512–27, 1966.
[6] Kuiper G.L. and Metrlkine A..V., 'On stability of a Clamped-Pinned Pipe Conveying Fluid', Heron, Vol 49 No.3, 211-230,2004.
[7] Wang L. and Ni Q., " A Note on The Stability and Chaotic Motions of a Restrained Pipe Conveying Fluid"J.Soun.Vib.Vol.296, 1079–1083,2006
[8] Long R.H., "Experimental and Theoritical Study of Transverse vibration of a Tube Containing Flowing Fluid,"J.appl.mech.Vol.22,65-68, 1955.
[9] Benjamin T.B., "Dynamics of System of Articulated Pipes Conveying Fluid-I Experimental “Proc. Roy. Soc.(London),Ser.A 261,488-498,1961.
[10] Blevins D.R., "Flow-Induced Vibration", 1977.
[11] Gregory R.W.and Paidoussis M..P., ,"Unstable Oscillation of Tubular Cantilevers Conveying Fluid. II. Experiments". Proc. Roy. Soc. (London) Ser. A 293,528–42, 1966.
[12] Huang Y., Liu Y., Li B., Li Y. and Yue Z., "Natural Frequency Analysis of Fluid Conveying Pipeline with Different Boundary Conditions," Nuclear Engineering and Design Vol. 240,461–467, 2010.
[13] ASHRAE Systems and Equipment Handbook (SI),41.3-41.4, 2000
[14] Lawrence H. Van Vlack, "Elements of Materials Science and Engineering",1989.

تخمين تجريبي لسرع الانبعاج ألحرجه للأنابيب الناقلة للموائع ذات الطاقة المحفوظة

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الخلاصة

الأنابيب الناقلة للمائع ذات الطاقة المحفوظة تفقد استقرارها بالانبعاج عند سرع حرجه ولغرض حساب هذه السرعة تجريبيا يجب استخدام مضخات عاليه التصريف مما يسبب تعقيدات لمنظومة السائل.

في هذا البحث تم تقديم طريقه تجريبية جديدة تعتمد على تخمين السرع ألحرجه من قياس عده ترددات طبيعيه أساسيه وبهذه ألطريقه يمكن استخدام سرع قليله نسبيا ومنظومة سائل بسيطة .

تم إجراء تجارب على نموذجين من الأنابيب وتحت ظروف إسناد منوعه فبينت النتائج أن هذه ألطريقه ملائمة لحساب السرع ألحرجه للأنابيب ذات الإسناد بسيط- بسيط و بسيط- محكم بينما تكون اقل من ذلك للأنابيب المحكمة من جهتين إلا إذا تم زيادة مقدار الجريان المستخدم.