SHEAR EFFECT ON THE ELASTIC-PLASTIC LARGE DISPLACEMENT STABILITY ANALYSIS OF PLANE STEEL FRAMES WITH MEMBERS RESTING ON ELASTIC FOUNDATION

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Abstract:

In this paper, a theoretical analysis is presented for estimating the in-plane large displacement elastic-plastic behavior of steel frames having members resting on elastic foundation subjected to either proportional or non- proportional increasing static loads and including shear deformation effect. The analysis adopts the beam- column approach and models the structure's members as beam-column elements. The formulation of the beam- column element is based on Eulerian approach allowing for the influence of the axial force on bending stiffness. Also, changing in member chord length due to axial deformation, flexural bowing and shear deformation effect are taken into account. The formation of tangent stiffness matrix for the member in local and global coordinates with geometric and material nonlinearly including shear effect have been presented. In a special procedure, the calculation of axial force and plastic moment capacity including shear effect has been explained. New interaction equations between the axial force, shear force and plastic moment capacity for box and I-steel sections are presented in this study. The present search has adopted the linear and nonlinear behaviors of soil and these behaviors have been presented by isolated springs at the nodes. The computational technique utilizes an incremental load approach with a Newton-Raphson iteration to satisfy joint equilibrium equations. In order to verify the efficiency of the present formulation, some case studies reported by previous researches are utilized. The investigation is extended to study the effect of shear deformation on the elastic plastic behavior of structures resting on elastic foundation. As a result of this investigation, several important conclusions are obtained, which assure the necessity of taking into account the shear effect in the analysis of large displacement elastic-plastic behavior of structures resting on elastic foundation.

Keywords: Shear effect, Stability, Large displacement, Elastic-plastic, Shear interaction, Soil effect.

تأثير القص في تحليل الإزاحات الكبيرة و الأستقرارية ألمرنة - أللدنة للهياكل ألحديدية ألمستوية و ألمرتبطة بأعضاء مسندة على أساس مرن

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ألخلاصة:

يتناول البحث دراسة التحليل النظري للسلوكية المرنة– اللدنة للهياكل الحديدية المستوية والتي تحتوي على أعضاء مستندة على أساس مرن ومعرضة لأحمال ساكنة متناسبة وغير متناسبة مع الأخذ بنظر الاعتبار الإزاحات الكبيرة الحاصلة فيها وتأثير القص. تبنت هذه الدراسة طريقة العمود العتبة. إن اشتقاق عنصر العمود العتبة قد تم بالاعتماد على طريقة اويلر مع الأخذ بنظر الاعتبار تأثير القوة المحورية على صلابة العزم ، كذلك اخذ بنظر الاعتبار تأثير التغيرات في طول الوتر نتيجة الانفعال المحوري وتقوس الانحناء وتأثير تشوه القص. بينت الدراسة اشتقاق مصفوفة الصلابة المماسية في نظام الإحداثيات ألموقعي والعام مع الأخذ بنظر الاعتبار تغير الشكل وتصرف المادة اللاخطي وتأثير القص بالإضافة إلى ذلك تم بيان طريقة خاصة لحساب القوة المحورية. تم استعمال علاقة لبيان التأثير المشترك بين القوة المحورية وسعة العزم اللدن مع الأخذ بنظر الاعتبار تأثير القص. تم اشتقاق معادلة جديدة لبيان التأثير المشترك بين القوة المحورية وقوة القص وسعة العزم اللدن للمقاطع الحديدية الصندوقية والمقاطع من نوع (I). تبنت الدراسة التصرف الخطي و اللاخطي للتربة وهذا التصرف قد مثل بنوابض في نهايات العنصر. لقد تم توضيح ألأسلوب الحسابي المستخدم لتتبع المنحني اللاخطي للمنشات. وقد استخدمت طريقة الحمل المتزايد مع تكرار نيوتن– رافسون لتحقيق معادلات التوازن في نهايات العنصر. ولغرض تقييم فعالية الدراسة المقدمة، تم مقارنة نتائج التحليل مع نتائج دراسات منشورة سابقا. تم توسيع هذه الدراسة لتتناول تأثير دراسة القص على السلوكية المرنة– اللدنة للمنشات الحاوية على أعضاء مستقرة على التربة. نتيجة لهذه الدراسة تم التوصل إلى عدة نتائج مهمة تؤكد ضرورة إدخال تأثير القص في التحليل المرن – اللدن للمنشات المستقرة على التربة.

Introduction:

In recent years, global attention and interest has grown in the nonlinear behavior of framed structures. The nonlinearity response of structures having members resting on elastic foundation such as (railway tracks, continuously supported pipe lines, piles and strip foundation) results from many effects some of these are the effect of large displacements and large strains (Geometrical nonlinearity), material nonlinearity and the effect of shear deformation. In structures with some of their members supported or driven into soil, the structural behavior of frameworks will be influenced significantly by the restrained caused by the foundation, and the amount of influence will be dependent on the flexural rigidity of the embedded members and the soil modulus (Bowels ١٩٩٦). Also, in these structures shear effect is important especially on the deflection, and its effect submit to many factors such as; span to depth ratio, the cross-section of the member, and soil modulus (Aydogan١٩٩٥) .

Substantial amount of literature on the nonlinear behavior of structures with members resting on soil with shear deformation effect has been undertaken in recent years. Al-Sarraf 1986 studied the shear effect on the elastic stability of frames. Modified stability functions for prismatic beam-columns having any solid cross-sectional shapes, laced or battened built-up sections were developed. Eisenberger and Yankelevsky 1987 presented an iterative procedure for the analysis of beams resting on nonlinear elastic foundation based on the exact solution for beams on a linear elastic foundation. The nonlinear characteristics of the foundation are approximated by the piecewise linear curve.

AL-Hachami 1997, presented large displacement elastic stability analysis of a beam-column resting or driven into elastic foundation. He adopted two approaches to representing the reaction of soil. The first approach represents the soil as isolated spring at nodes of abeam-column element and the second one, the elastic foundation is considered as uniformly distributed Winkler-type spring support. Onu 2000 produced a formulation leading to an explicit free-of-meshing stiffness matrix for the beam finite element on two parameter elastic foundation. Considering the shear deformation contribution, the formulation based on the exact solution of the governing differential equation. Jawad 2002, presented a theoretical analysis for estimating the in-plane large displacement elastic-plastic behavior of steel frames having prismatic members and resting on soil. Approximate expression presented to describe the interaction curves between the axial force and plastic moment capacity of wide rang of steel sections. The analysis has adopted the linear and nonlinear behaviors of soil and these behaviors have represented by isolated springs at nodes.

In the present study, a theoretical bases of the large displacement elastic-plastic analysis of plane steel structures having members resting on elastic foundation using a beam-column approach and including shear effect is presented. The formulation of the beam column element is based on Eulerian approach allowing the influence of the axial force on bending stiffness. Also changes in member chord length due to axial deformation and flexural bowing are taken into account. The formation of tangent stiffness matrix for the member in local and global coordinates with geometric and material nonlinearly has been presented. In a special procedure, the calculation of axial force has been explained. In order to verify the effect of shear in plastic moment capacity, new interaction equation between the axial force, shear force and plastic moment capacity of a box and I-steel sections is presented in this study. For soil representation, linear and nonlinear isolated spring behaviors are utilized and these behaviors have been represented by isolated springs at nodes. The algorithm of computer program (**NEPAPFSS)** is developed in this research. In order to display the effectiveness of this computer program, some examples reported by previous researches are utilized. As it is obvious from the literature review, there is no one using the beam-column approach (as far as author knowledge) to study the effect of shear on the elastic-plastic large displacements stability analysis of steel frames having members resting on elastic foundation.

Shear Deformation:

The basic theory of shear effect for a prismatic member and the methods that deals with shear deformation and the derivation of shear flexibility parameter for a solid member will be presented. In addition, the stability and bowing functions including shear effect will be presented.

Methods of Including Shear Deformation:

The effect of shear on curvature of element and consequently on the stability and bowing functions have a variable influence depending on type of member. This effect is analyzing by using two approaches as follows (Sideek 2005):

1- The first approach (total slope approach)

This approach is based on the assumption that the shear component of axial force is calculated from the total slope (bending and shear slope).

2- The second approach (bending slope approach)

This approach is based on the assumption that the shear component of axial force is calculated from the bending slope.

Shear Flexibility Parameter for Prismatic Member:

The shear effect on the elastic stability analysis depends on the type of structure, whether it is open or closed web structure. The open web structure exhibits relatively higher shear deformation than the solid one therefore it must be introduce a parameter which gives us a sense about how much the effect of shear will be on that structure. The derivation of shear flexibility parameter produced by Timoshenko and Gere 1961are presented as follows:

An element of length "L" of a structural member is considered as shown in **Fig.(1)**. This member is acted upon by axial force "Q", shearing force "V", and bending moment "M". The deformation is separated into two parts; (1) Bending and (2) Shear. Bending produces a change in slope in length "L" of ;

$$\phi L = \frac{ML}{EI} \qquad (1)$$

in addition, shear produces a change in slope of ;

$$\gamma = \frac{\bar{n}\, V}{GAv} \qquad (2)$$

Where,

EI : flexural rigidity of the member.
GA : shear rigidity of the member, and
$\bar{n}$: shear shape factor.
Av : effective shear cross-sectional area.

Thus, within the length (L), the ratio of change in slope caused by shear to that caused by moment is defined as “shear flexibility parameter (μ)” and it is equal to

$$\mu = \frac{\bar{n}\, Q_E}{GAv} \qquad (3)$$

Where ; Q_E : Euler load.

Typical values of the shear shape factor for solid member are giving as follows Timoshenko and Gere 1961.

Type of section	Shear shape factor ($\bar{n}$)
Rectangular cross-section	1.20
Circular cross-section	1.11
I – Section bent about minor axis	$1.2A / A_f = (1.4 - 2.8)$
I – Section bent about major axis	$A / A_w = (2 - 6)$

Where : A_f = area of two-flanges , A_w = area of the web, A = total cross-sectional area

Elastic Large Displacement Analysis with Shear Effect:

The following method of analysis is an extension of the elastic large displacement formulation presented in (Oran, and Kassimali 1976). The ideas previously reported are reviewed in the present study. The relationship between relative member deformations, θ_1,θ_2 and u and associated member end forces M_1,M_2 and Q of the elastic prismatic member shown in **Fig.(2)**,can be based on beam column theory as follows:

M_1= EI /L (C_1 θ_1+C_2 θ_2) (4)
M_2=EI /L ($C_2\theta_1$+$C_1\theta_2$) (5)
Q =EA /L (u –$C_{b.}$ L) (6)

In which C_1 and C_2 : Conventional elastic stability functions, where,
C_1: flexural stiffness factor.
C_2: flexural moment carry over-factor.
E: modulus of elasticity, I: moment of inertia, A: cross-sectional areal, L: initial member length.
u: axial displacement, and;

$$C_b= b_1 (\theta_1+\theta_2)^2 + b_2 (\theta_1-\theta_2)^2 \qquad (7)$$

is the length correction factor due to bowing action, with b_1,b_2 are bowing functions.

In the present study stability functions (C_1, C_2) and bowing functions (b_1, b_2) is replaced with modified functions including shear effect .Thus, equations (4) to (7) become:

M_1= EI /L ($\bar{C}_1$ θ_1+$\bar{C}_2$ θ_2) (8)
M_2=EI /L ($\bar{C}_2\theta_1$+$\bar{C}_1\theta_2$) (9)
Q =EA /L (u –$\bar{C}_{b.}$ L) (10)
$\bar{C}_b$= $\bar{b}_1$ $(\theta_1+\theta_2)^2$ + $\bar{b}_2$ $(\theta_1-\theta_2)^2$ (11)

Where $\bar{C}_1$, $\bar{C}_2$, $\bar{b}_1$, $\bar{b}_2$ are the modified stability and bowing functions including shear effect. Explicit expressions for stability and bowing functions including shear effect in terms of axial force parameter (q) and shear flexibility parameter are summarized in Appendix A.

$$q=\frac{Q}{QEuler}=\frac{QL^2}{\pi^2 EI} \qquad (12)$$

The incremental relationship between the member end forces and end displacements In global coordinates can be written as:

$$\{\Delta F\}= [T] \{\Delta V\} \qquad (13)$$

In which the [T] = the member tangent stiffness matrix in global coordinates and given by

$$\left[T\right]=\left[B\right]\left[t\right]\left[B\right]^T + \sum_{k=1}^{3} S_k \left[g^{(k)}\right] \qquad (14)$$

In which
S_k : the local forces which represents (M_1) and (M_2) and (QL) for k=1,2 and 3 respectively as shown in **Fig.(٢)**.
[$g^{(k)}$] : geometric matrices which have the expression given in the appendix A for k=1,2 and 3
[B] : the transformation matrix, which has the expression given in appendix A.
[t] : the member tangent stiffness matrix in local (Eulerian) coordinates which defined by

$$[t]=\frac{EI}{L}\begin{bmatrix} \bar{C}_1+\dfrac{G_1^{\,2}}{\pi^2 H} & \bar{C}_2+\dfrac{G_1 G_2}{\pi^2 H} & \dfrac{G_1}{H} \\ & \bar{C}_1+\dfrac{G_2^{\,2}}{\pi^2 H} & \dfrac{G_2}{H} \\ sym & & \dfrac{\pi^2}{H} \end{bmatrix} \qquad (15)$$

in which

$$G_1=\bar{C}'_1\theta_1+\bar{C}'_2\theta_2=-2\pi^2\left[\left(\bar{b}'_1+\bar{b}'_2\right)\theta_1+\left(\bar{b}'_1-\bar{b}'_2\right)\theta_2\right] \qquad (16)$$

$$G_1=\bar{C}'_2\,\theta_1+\bar{C}'_1\theta_2=-2\pi^2\left[\left(\bar{b}'_1-\bar{b}'_2\right)\theta_1+\left(\bar{b}'_1+\bar{b}'_2\right)\theta_2\right] \qquad (17)$$

and

$$H=\frac{\pi^2}{\lambda^2}+\bar{b}'_1(\theta_1+\theta_2)^2+\bar{b}'_2(\theta_1-\theta_2)^2 \qquad (18)$$

in which $\lambda=\dfrac{L}{\sqrt{I/A}}$ (19)

In addition, a prime superscript on $\bar{C}_i$ or $\bar{b}_i$ denotes a differentiation with respect to q. The stability and bowing functions, the geometric matrices and the transformation matrix are given in Appendix A.

Elastic- Plastic Analysis with Shear Effect:

An important step in the design of steel frame structures is the calculation of their ultimate load currying capacity. The accuracy of such calculation depends largely on numerical models used in the analysis procedure. As far as the effect of plasticity is concerned, a convenient concept in modeling the collapse mechanism of steel frames has been the introduction of zero length plastic hinges at a member ends (Oran, and Kassimali 1976). When steel frame undergoes a large plastic deformation, plastic hinges are expected to form at any section when moment of a member becomes equal to the plastic moment capacity of the member and the slope continuity at the end of the element, where the plastic hinge is located, will be destroyed. In other words, the particular end of the element can rotate independently of the joint and an additional degree of freedom is created. The material is assumed to be ideally elastic-plastic, and yielding is considered to be concentrated at member ends in the form of plastic hinges. A plastic hinge will form when the moment equals to the reduced plastic moment capacity (Mpc) of the member . The member is assumed to remain elastic between plastic hinges. The reversals of plastic hinge rotations are not taken into account.

The influence of axial force on the reduced plastic moment capacity of a plastic hinge (Mpc), is presented from the interaction between the bending moments and axial force based on exact interaction method that can be expressed for the box and I-sections, widely used in building frames. The exact interaction equations are

$$m = 1 - Xn^2 \qquad |n| \le R_w \frac{1 - R_f}{R_1} \tag{20}$$

$$m = Y(1 - |n|) - Z(1 - |n|)^2 \qquad |n| > R_w \frac{1 - R_f}{R_1} \tag{21}$$

Where

$$n = \frac{Q}{Q_y} \tag{22}$$

$$m = \frac{M_{pc}}{M_p} \tag{23}$$

Q = axial force.
Q_y = axial yield strength.
M_p = full plastic moment capacity of the cross-section.
M_{pc} = reduced plastic moment capacity of the cross-section due to effect of axial force.

$$R_w = \frac{2t_w}{b} \tag{24}$$

$$R_f = \frac{2t_f}{d} \tag{25}$$

$$R_1 = 1 - (1 - R_W)(1 - R_f) \tag{26}$$

$$R_2 = 1 - (1 - R_W)(1 - R_f)^2 \tag{27}$$

$$X = \frac{R^2{}_1}{R_w R_2} \tag{28}$$

$$Y = \frac{2R_1}{R_2} \tag{29}$$

$$Z = \frac{R^2{}_1}{R_2} \tag{30}$$

Another method is the approximate interaction method for the influence of axial force on the reduced moment capacity of a plastic hinge (Mpc), for W,S and M shapes can be expressed as [13]

$$M_{pc} = 1.18 M_p \left(1 - \frac{|Q|}{Q_y}\right) \qquad \text{when;} \quad \frac{|Q|}{Q_y} \rangle 0.15 \tag{31}$$

$$M_{pc} = M_p \qquad \text{when;} \quad \frac{|Q|}{Q_y} \le 0.15 \tag{32}$$

The effect of shear force on plastic moment capacity in general is much more complex than of axial force [14]. In addition to causing beam-column instability (which is included in stability functions), the presence of shear force tends to reduce the magnitude of the plastic moment capacity of the section. The reduction of the plastic moment capacity of the cross-section in the presence of shear force is presented from the interaction among the bending moment, shear and axial forces.

The special case of an I-section was considered by Baker and Heyman [14] to show the effect of together shear and axial force on plastic moment capacity. The full plastic section modulus of an I-section is given by

$$Z_p = b_f * t_f (D - t_f) + (\frac{D}{2} - t_f)^2 * t_w \qquad (33)$$

$$Z_p = Z_{pf} + Z_{pw} \qquad (34)$$

Where

Z_p: the full plastic section modulus of the I-section.

Z_{pf}, Z_{pw}: the plastic section modulus for the flanges and the web respectively.

t_f, t_w: thickness of the flanges and the web respectively.

D: depth of the I-section.

It is commonly assumed in elastic design that the shear stress is uniformly distributed over the web of an I-section and the flanges not contributing at all to the carrying of shear force [14]. Baker and Heyman [14] use these assumptions in plastic analysis and they gave the following equation for the reduced plastic section modulus for the web of an I-section due to the effect of shear and axial force ;

$$Zw = \left[\sqrt{1 - 3(\tau / F_y)^2} - \frac{(Q/Q_{yw})^2}{\sqrt{1 - 3(\tau / F_y)^2}} \right] Z_{pw} \qquad (35)$$

Where,

Z_w: Reduced plastic section modulus for the web due to effect of shear and axial force.

τ : Shear stress.

Q: Axial force.

Q_{yw}: Axial yield strength for the web ($Fy * A_w$).

A_w: Area of the web.

Suppose that the reduction ratio for the web due to the effect of shear and axial force is equal to N_c, thus, Eq. (35) can be written as:

$$Z_w = N_c * Z_{pw} \qquad (36)$$

To give the total reduced plastic section modulus, the flanges modulus (Zpf) can be reduced due to the effect of axial force alone. The reduction ratio can be obtained from the exact interaction equations [15] as follows:

$$\frac{Z_{pc}}{Z_p} = 1 - X * n^2 \qquad |n| \le R_w \frac{1 - R_f}{R_1} \qquad (٣٧)$$

$$\frac{Z_{pc}}{Z_p} = Y(1 - |n|) - Z(1 - |n|)^2 \qquad |n| > R_w \frac{1 - R_f}{R_1} \qquad (38)$$

Where:

Z_{pc}: reduced plastic section modulus of the I-section due to the effect of axial force.

$$M_c = 1 - X * n^2 \qquad |n| \le R_w \frac{1 - R_f}{R_1} \qquad (39)$$

Suppose that the reduction ratio due to effect of axial force is equal to Mc, thus, Eqs.(37 and 38) become

$$M_c = Y(1-|n|) - Z(1-|n|)^2 \qquad |n| > R_w \frac{1-R_f}{R_1} \tag{40}$$

Eqs. (37) and (38) can be written as:

$$Z_{pc} = M_c * Z_p \tag{41}$$

$$Z_{pc} = M_c\,(Z_{pf} + Z_{pw}) \tag{42}$$

$$Z_{pc} = M_c * Z_{pf} + M_c * Z_{pw} \tag{43}$$

The second term of equation (43) can be replaced by Eq. (36) to obtain a general interaction equation for the plastic section modulus for I and box sections including the effect of shear and axial force (Zpcs), so equation (43) becomes:

$$Z_{pcs} = N_c * Z_{pw} + M_c * Z_{pf} \tag{44}$$

Where:

Z_{pcs}: is the reduced plastic section modulus for box or I- section due to the effect of shear and axial force.

In terms of plastic moment capacity, equation (44) becomes:

$$M_{pcs} = N_c * M_{pw} + M_c * M_{pf} \tag{45}$$

M_{pcs}: The reduced plastic moment capacity for box or I-section due to the effect of shear and axial force.

M_{pw}, M_{pf}: are the full plastic moment capacity for the web and the flanges respectively.

The member stiffness relationships previously described for elastic large displacement analysis are based on the assumption that each member is rigidly connected to joints at both ends. When plastic (or real) hinges are presented, the relative member end rotations at the released ends are obtained from relative member force deformation relationships (Eq.8 and Eq.9) Thus, for a member with plastic hinge at end 1 ,

$$\theta_1 = \frac{LM_{PCS1}}{EI\bar{C}_1} \quad \frac{\bar{C}_2}{\bar{C}_1}\theta_2 \tag{46}$$

for member with plastic hinge at end 2

$$\theta_2 = \frac{LM_{PCS2}}{EI\bar{C}_1} \quad \frac{\bar{C}_2}{\bar{C}_1}\theta_1 \tag{47}$$

and for a member with plastic hinges at both ends

$$\theta_1 = \frac{L}{EI}\frac{\left(\bar{C}_1 M_{PCS1} - \bar{C}_2 M_{PCS2}\right)}{\left(\bar{C}^2{}_1 - \bar{C}_1{}^2\right)} \tag{48}$$

$$\theta_2 = \frac{L}{EI}\frac{\left(\bar{C}_1 M_{PCS2} + \bar{C}_2 M_{PCS1}\right)}{\left(\bar{C}^2{}_1 - \bar{C}_1{}^2\right)} \tag{49}$$

In Eqs.(46 to 49), the M_{pcs1} and M_{pcs2} are the reduced plastic moment capacity at ends 1 and 2 respectively. It may be seen that these equations (Eqs.(46 to 49) which are developed in this study are written in general expression for the case of real (Mpcs = 0) or/and plastic hinges. In the case of a member with a plastic hinge at one end and a real hinge at the other , the relative member end rotations ,θ_1 and θ_2 can be expressed in terms of M_{pcs1} or M_{pcs2} and C_1 and C_2 by using Eqs.(48 and 49) with putting Mpcsi = 0 for real hinge at end i .For example , for a member with a real hinge at end 1 and a plastic hinge at end 2

$$\theta_1 = -\frac{LM_{pcs}}{EI}\frac{\bar{C}_2}{(\bar{C}_1^2 - \bar{C}_2^2)} \tag{50}$$

$$\theta_2 = \frac{LM_{pcs}}{EI}\frac{\bar{C}_2}{(\bar{C}_1^2 - \bar{C}_2^2)} \tag{51}$$

It may be noted that , for prismatic member with rigidly connected to joints at both ends the plastic moment capacities are equal for both ends.

The tangent stiffness matrix in local coordinates , [t] , can be developed for a member with hinges by eliminating the released coordinate(s) from Eqs.(8 to 10) and differentiating the resulting relations ,term by term ,with respect to the remaining coordinates. If incremental changes in plastic moment capacities are neglected, i.e. , plastic hinges are treated as real hinges, the nonzero elements of the 3x3 matrix, [t], are:

For a member with a hinge at end 1

$$t_{22} = Z_{22} - \frac{Z^2{}_{12}}{Z_{11}} \tag{52}$$

$$t_{23} = t_{32} = Z_{23} - \frac{Z_{12}Z_{13}}{Z_{11}} \tag{53}$$

$$t_{33} = Z_{33} - \frac{Z^2{}_{13}}{Z_{11}} \tag{54}$$

For a member with a hinge at end 2

$$t_{11} = Z_{11} - \frac{Z^2{}_{12}}{Z_{22}} \tag{55}$$

$$t_{13} = t_{31} = Z_{13} - \frac{Z_{12}Z_{23}}{Z_{23}} \tag{56}$$

$$t_{33} = Z_{33} - \frac{Z^2{}_{23}}{Z_{22}} \tag{57}$$

and, for a member with hinges at both ends

$$t_{33} = AEL \tag{58}$$

In Eqs. (52 to 57), [Z] = the local tangent stiffness (denoted by [t] in Eq. (15) for a member with no releases). The local tangent stiffness matrices as given by Eq. (52 to 57) can be used for members with real as well as plastic hinges. As indicated previously, incremental changes in the plastic moment capacities are not included in Eq.(52 to 57) (i.e.ΔM_{pcsi} = 0). In addition Eq. (58) (for a member with releases at both at ends) assumes that the effect of incremental changes in the flexural bowing term on member axial force is negligible (i.e. ΔC_b= 0). It should be noted that these simplifications are limited to the tangent stiffness matrices and do not extend to the system equilibrium equations. As the order of [t] is 3x3 even the presence of member releases, the transformations out lined in Eq.(14) can be used to obtain the member tangent stiffness matrix [T] in the global system.

Soil-Structure Interaction

In this study, the soil represents as isolated linear and nonlinear Winkler-type springs at nodes of beam column element. The coefficient stiffness of spring is added to tangent stiffness matrix of structure in global coordinates. The modulus of subgrade reaction gives the relationship between the soil pressure and the resulting deflection. The behavior of the soil under compressive loading is nonlinear as verified by the results of plate load test in the field. The isolated nonlinear Winkler-type spring at nodes of a beam- column element can approximately be modeled using the two-constant hyperbolic equation as shown in **Fig.(3)**, which takes the following form

$$p = \frac{\delta}{a + b\delta} \tag{59}$$

$$K_n = \frac{a}{(a + b\delta)^2} \tag{60}$$

Where

P : the lateral load on beam-column element that is concentrated at the node.

δ : the lateral displacement of the node.

K_n : the normal subgrade reaction of soil.

"a" and "b" are the physical parameters required for the hyperbolic equation, which can be obtained from the load-settlement curve of the plate-load test.

In the present study, the negative stress of soil is not allowed in the analysis. So, the stiffness of tension force spring was not added to the total tangent stiffness matrix of structure.

Computational technique:

Most numerical schemes to solve nonlinear systems of equations are numerical-iterative processes that use a series of linear solutions to approximate the nonlinear solution. In the present study, the nonlinear response of the structures is generated by incremental load approach with the conventional Newton-Raphson type of iteration performed at each load level to satisfy the joint equilibrium. The computational technique is shown graphically in **Fig.(4)** for a single degree of freedom system. To terminate the iteration process, a convergence criterion based on the displacement is used here through comparing the incremental change in displacement vector {Δx}, where the convergence is assumed to occur when the inequality

$$\left[\frac{\sum_i (\Delta x_i)^2}{\sum_i (x_i)^2}\right]^{1/2} \le e \qquad (61)$$

is satisfied simultaneously and independently for each group, where "e" represents prescribed tolerance which chosen here to be equal to 0.001.

An important computational difficulty arises in determining member end forces M_1,M_2 and Q, from the relative member deformations θ_1,θ_2 and u this is due to the fact that the expression for member axial force Q, as given in Eq.(10) involves bowing functions b_1 and b_2 , which, in turn, are functions of the axial force parameter q. In the presence of hinges, the problem is further complicated by the fact that rotations at the released ends (Eqs.46 to 51) are also functions of Q. the problem can be solved, however, by the following iterative procedure.

Noting that q , is only unknown in Eq.(10) let:

$$K(q) = \frac{\pi^2}{\lambda^2} q + \bar{C}b - \frac{u}{L} = 0 \qquad (62)$$

Let q_i be an approximate of this equation. By using a first-order Taylor series expansion,Eq.(62) can be rewritten as

$$K(q_i + \Delta q_i) = K(q_i) + K'(q_i)\Delta\Delta_i = 0 \qquad (63)$$

in which a prime superscript denotes a differentiation with respect to q , and

$$K'(q) = \frac{\pi^2}{\lambda^2} + \bar{C}'b \qquad (64)$$

A new value is thus obtained for q:

$$q_{i+1} = q_i + \Delta q_i = q_i - \frac{K(q_i)}{K'(q_i)} \qquad (65)$$

Moreover, the iteration continues until $\Delta q < e$.

In Eq. (64) $\bar{C}'_b$ is given by

$$\bar{C}'_b = \bar{b}'_1(\theta_1 + \theta_2)^2 + \bar{b}'_2(\theta_1 - \theta_2)^2 + 2\bar{b}_1(\theta_1 + \theta_2)(\theta'_1 + \theta'_2) + 2\bar{b}_2(\theta_1 - \theta_2)(\theta'_1 - \theta'_2) \qquad (66)$$

In which, θ'_i terms = zero at member ends when rigidly connections to the joints. The rotations at such rigid ends are defined by the global joint displacements and remain constant during this iteration process. For a member with hinges, θ'_i terms at the released ends are given by differentiation of Eqs.(46 to 51) with respect to q. Thus, for a member with plastic hinge at end 1

$$\theta'_1 = \frac{L}{EI\,\bar{C}_1^2}\left(M'_{pcs\,1}\bar{C}_1 - M_{pcs\,1}\bar{C}'_1\right) + \frac{\theta_2}{\bar{C}_1^2}\left(\bar{C}'_1\bar{C}_2 - C_1 C'_2\right) \qquad (67)$$

For member with a plastic hinge at end 2

$$\theta'_2 = \frac{L}{EI\,\bar{C}_1^2}\left(M'_{pcs\,2}\,\bar{C}_1 - M_{pcs\,2}\,\bar{C}'_1\right) + \frac{\theta_2}{\bar{C}_1^2}\left(\bar{C}'_1\,\bar{C}_2 - C_1 C'_2\right) \tag{68}$$

And for member with plastic hinges at both ends

$$\theta'_1 = \theta'_2 = \frac{L}{EI(\bar{C}_1 + \bar{C}_2)}\left[M'_{pcs} - M_{pcs}\frac{(\bar{C}'_1 + \bar{C}'_2)}{(\bar{C}_1 + \bar{C}_2)}\right] \tag{69}$$

In these equations the values of M'_{pcs} can be determined by differentiating of reduced moment capacity with respect to the axial force parameter (q) for bending, axial and shear interaction (Eq. (45)).

$$M'_{pcs} = M_{pw} * N'_c + M_{pf} * M'_c \tag{70}$$

Where

$$N'_c = \frac{-2n_w * n'_w}{\sqrt{1 - 3(\tau / f_y)^2}} \tag{71}$$

$$n_w = \frac{Q}{Q_{yw}} \tag{72}$$

$$n'_w = \frac{\pi^2 EI}{L^2 Q_{yw}} \tag{73}$$

$$M'_c = -2X * |n| * n' \qquad |n| \le R_w \frac{1 - R_f}{R_1} \tag{74}$$

$$M'_c = -Y * n' - 2 * Z * n' * (1 - |n|) \qquad |n| > R_w \frac{1 - R_f}{R_1} \tag{75}$$

$$n' = \frac{\pi^2 EI}{L^2 Q_y} \tag{76}$$

In the case of real hinges $M_{pcs} = 0$ and $M'_{pcs} = 0$

<u>Numerical Examples:</u>

A computer program (**NEPAPFSS**) is developed in the present study to carry out the shear effect in large displacement elastic-plastic stability analysis of plane steel frames with members resting on elastic foundation .The computer program is coded in (Quick-Basic) language to be used on computers. The computer program consist of a main routine and many subroutines, each of these subroutines has been designed to deal with a part of the analysis. In order to verify the reliability of this computer program and the effectiveness of the present formulation, some examples reported by previous researches are utilized.

Example No.1 represents a fixed-fixed beam resting on elastic foundation under uniform load. The geometry, load and material condition of this example is shown in **Fig.(5)**.The aim of this example is to illustrate the effect of the nonlinearity behavior of soil , shear deformation effect in the analysis of the structure .Chen 1998 solved this problem assuming that the subgrade reaction of soil is linear. Jawad 2002 re-solved this example assuming linear and nonlinear subgrade reaction. The same assumptions of above researchers for soil representation are used to re-analyze this example. A good agreement is achieved between the results of the present study and those obtained by Chen 1998. The problem is presented in present work with six and eight elements considering the effect of shear deformation. **Table (1)** gives the values of the mid-span deflection of the beam for previous works and the present study. The maximum percentage of shear effect for the mid-span deflection is about (234.7٪) and (442.3٪) for linear and nonlinear spring behavior respectively. This shows the great role of shear deformation in increasing lateral deflection for linear and nonlinear soil analysis as shown in **Fig.(7)**. **Fig. (6)** shows the deflected shape of this example for linear and nonlinear spring behavior with and without shear effect.

Example No.2 represents a fixed-fixed beam under concentrated load at mid span. The geometry ,load and material condition of this example is shown in **Fig.(٨)**. This case study is considered to check the effectiveness of the new interaction equation (Eq.45) utilized in the present study which including axial, bending and shear interaction effects and to show the effect of considering shear interaction in the analysis. Two types of steel sections are used W 12×31 and W14×426 (light and heavy sections respectively). Zhou and Chen 1985 gave the interaction curves for these sections based on the exact interaction equations Eqs.(20 and 21)for the interaction between axial force and bending moment. In present study, the interaction curves for the above sections is plotted based on equation (45) for the reduced plastic moment capacity due to effect of together shear and axial force. From **Fig. (8)** it can be seen that the ratio of (M_{pc}/M_p) is reduced about (11٪) from unity for light section and about (9٪) for heavy section when shear interaction is considered in the analysis. The full plastic moment capacity for heavy section is larger than the light section. Thus; the reduction in the ratio (M_{pc}/M_p) due to shear effect for light section is larger than the heavy one.

Example No.3 represents a one-bay four-story frame resting on elastic foundation shown in **Fig.(9)**. The frame has been chosen for studying the effect of shear interaction in the analysis of frame structure when a base of the frame changes from a fixed base to elastic foundation by using steel embedded pile. In this study, this example is solved without including shear interaction by using approximate interaction method and including shear interaction to show the effect of shear on the behavior of structure. Pile length is six meter. Ten elements have been used for modeling the pile. The frame is analyzed for the value of the lateral load parameter (r = 0.5). The value of subgrade reaction of soil varies with the depth and the pile length is divided into two zones. The value of the subgrade reaction of the lower zone soil is larger than the upper one by 34% .The value of tangential modulus of subgrade reaction (Ks) is equal to zero. The horizontal displacement of the top right joint of the frame versus the applied loading results obtained from the present analysis for all cases of this example as shown in **Figures (10) and (11). Table (2)** gives the results of present analysis. From the results obtained from all cases of this example, it can be seen that the ultimate load capacity decreases by a ratio (1.05-7.65)٪ and the horizontal displacement of joint A increases about (3.18-6.1) ٪ when including shear effect in the analysis. The sequence of plastic hinges will be changed when the effect of shear interaction is taken into account. In addition, it may be seen that using the elastic foundation supported instead of fixed support tend to reduce the ultimate load capacity about (0.1- 6.67)% .

Conclusions:

Based on the results obtained in the present study, several conclusions may be obtained. These may be summarized as follows:

1. The large displacement elastic-plastic stability analysis of plane steel structures having Members resting on soil and including shear effect can be accurately predicted using the beam column approach in the analysis.
2. A comparison between the results of linear and nonlinear behavior of soil obtained from elastic analysis shows that shear effect reached to (234.7٪) for linear soil behavior and about (442.3٪) for nonlinear soil behavior.
3. The ultimate load capacity decreases with a ratio (1.05-7.65)٪ and the displacement increases with a ratio (3.17-6.1)٪ when shear interaction is taken into account; in addition, the sequence of plastic hinges will change.
4. It has been found that using the elastic foundation supported instead of fixed support tends to reduce the ultimate load capacity with a ratio (0.1-6.67) ٪.
5. Shear effect is governed by the value of shear flexibility parameter. In addition, the shear effect is more pronounced for members with small span to depth ratio.
6. From the above important conclusions, the effect of shear should be taken into account in the large displacement elastic-plastic analysis of structures having members resting on soil to get the real behavior of structures.

References:

1. Bowels, J. E., “Foundation Analysis and Design.” Mc Graw-Hill, International Editions, Fifth Edition.1996.
2. Aydogan .M., “Stiffness- Matrix Formulation of Beam with Shear Effect on Elastic Foundation” Journal of Structural Engineering, ASCE, Vol.121, No.9, 1995, pp.1265-1270.
3. Al-Sarraf, S.Z., “Shear Effect on the Elastic Stability of Frames”, The Structural Engineer, Vol. 64B, No. 2, June 1986, pp. 43-46.
4. Eisenberger, M. and Yankelevesky, D.Z., “Exact Stiffness Matrix for Beams on Elastic Foundation “International Journal of Computers and Structures, Vol.21, 1987, pp.1355-1359.
5. Al-Hachami, E.K., “Large Displacement Analysis of Structures with Applications of Piles and Submarine Pipelines.” Ph.D. Thesis, University of Technology, Baghdad, 1997.
6. Onu, G., “Shear Effect in Beam Finite Element on Two Parameter Elastic Foundation” Journal of Structural Engineering, ASCE, Vol.126, No.9, 2000, pp.1104-1107.
7. Jawad, F.A., “Elastic-Plastic Large Displacement Stability Analysis of Plane Steel Frames with Members Resting on Soil” M.Sc. Thesis, University of Kufa, Najaf, 2002.
8. Sideek, K.F., “Exact Solution of the Shear Effect on the Analysis of Laced and Battened Beam-Column “Engineering Journal of the University of Qatar ,Vol.18,2005,p.p.105-120.
9. Timoshenko, S.P. and Gere, J.M., “Theory of Elastic Stability”. Mc Grow-Hill Book Company, 2nd Edition, 1961.
10. Oran, C., and Kassimali, A., “Large Deformations of Framed Structures under Static and Dynamic Loads.” International Journal of Computers and Structures, Vol.6, No.6, 1976, PP. 539-547.

11. Duan, L. and Chen, W. F., “A Yield Surface Equation for doubly Symmetrical Sections.” Engineering Structures, Vol.12, April 1990, pp.114-119.
12. Baker, J. and Heyman, J., “Plastic Design of Frames”. Cambridge at the University press, 1969.
13. Zhou, S. P. and Chen, W. F., “Design Criteria for Box Columns under Biaxial Loading.” Journal of Structural Engineering, ASCE, Vol.111, No.12, 1985 PP.2643-2658.
14. Chen, C. N., “Solution of Beam on Elastic Foundation by DQEM.” Journal of Engineering Mechanics, ASCE, Vol.124, No.12, 1998, PP.1381-1384.

Table (1) The results of example No.1.

TYPE OF SUPPORT	KN_1 TON/IN^2	KN_2 TON/IN^2	PRESENT STUDY (WITHOUT SHEAR INTERACTION)		PRESENT STUDY (WITH SHEAR INTERACTION)	
			Pu (tons)	Sequence of plastic hinges	Pu (tons)	Sequence of plastic hinges
Fixed base	----------	-----------	14.5	Fig .(12-A)	14.306	Fig .(12-A)
Elastic Support (Linear)	0.133	0.133	14.442	Fig .(12-C)	14.29	Fig .(12-B)
Elastic Support (Nonlinear)	a_1=7.5 b_1=0.029	a_2=7.5 b_2=0.029	14.429	Fig .(12-D)	14.028	Fig .(12-E)
Elastic Support (Linear)	0.099	0.133	14.386	Fig .(12-H)	14.161	Fig .(12-I)
Elastic Support (Linear)	0.133	0.179	14.458	Fig .(12-F)	13.352	Fig .(12-G)

Table (2) The results of example No.3

Type 0f spring	Reference of the analysis	Number of elements	Mid span deflection (m)		% effect of shear deformation $(\frac{\delta 2-\delta 1}{\delta 1})*100$
			Without shear effect (δ1)	With shear effect (δ2)	
Linear spring behavior	Chen 1998	8	0.1812	----	----
	Jawad 2002	7	0.1803	----	----
	Present study	6	0.1861	0.6229	234.7
		8	0.1865	0.6241	234.64
Non-linear spring behavior	Al-Rubai	10	0.208	----	----
	Jawad 2002	10	0.210	----	----
	Present study	6	0.214	1.1605	442.3
		8	0.215	1.1584	438.8

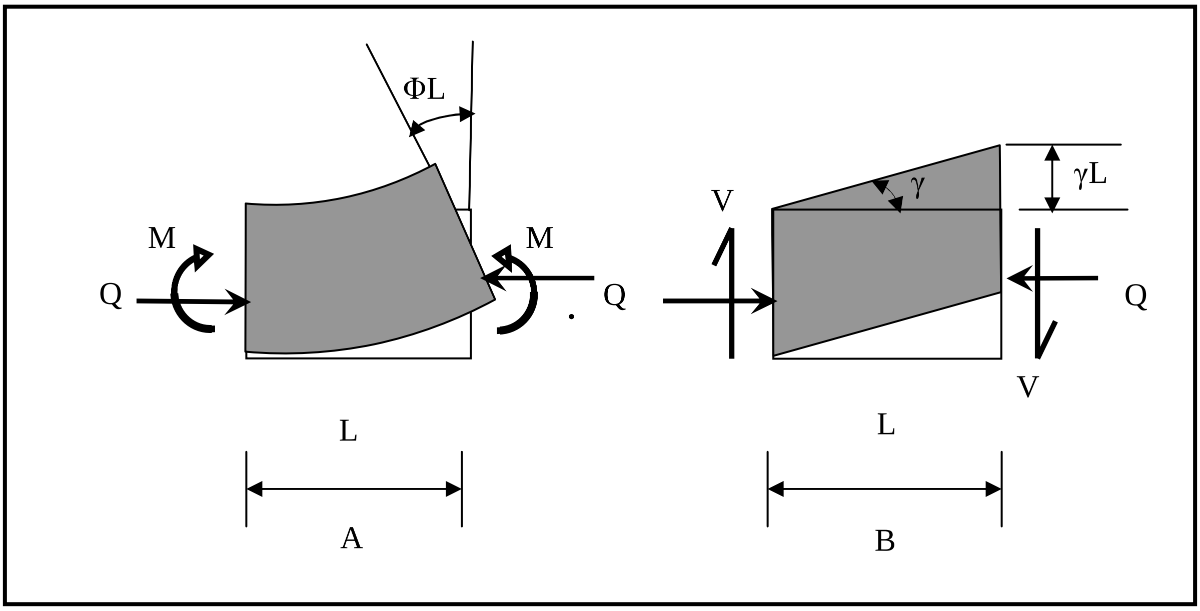


Fig. (1) Element of length (L) of Solid Structural Member Under (A) Bending (B) Shear

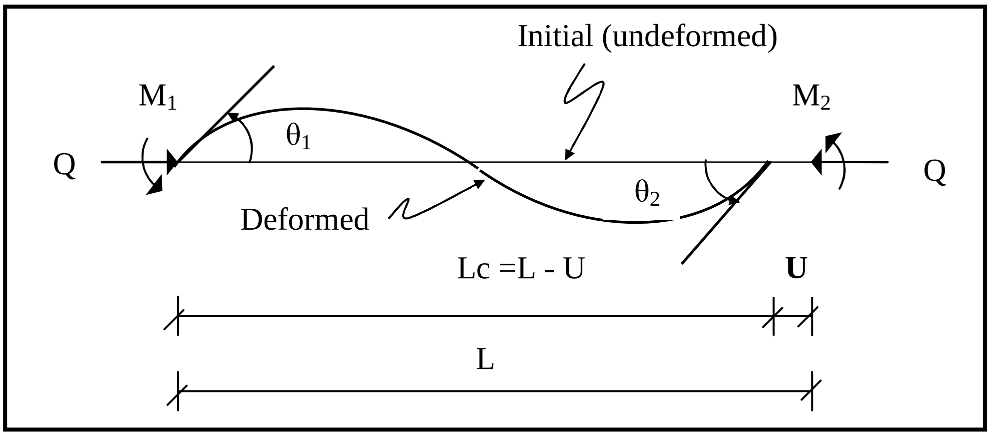


Fig. (2) Member end force and relative deformation for a beam-column element

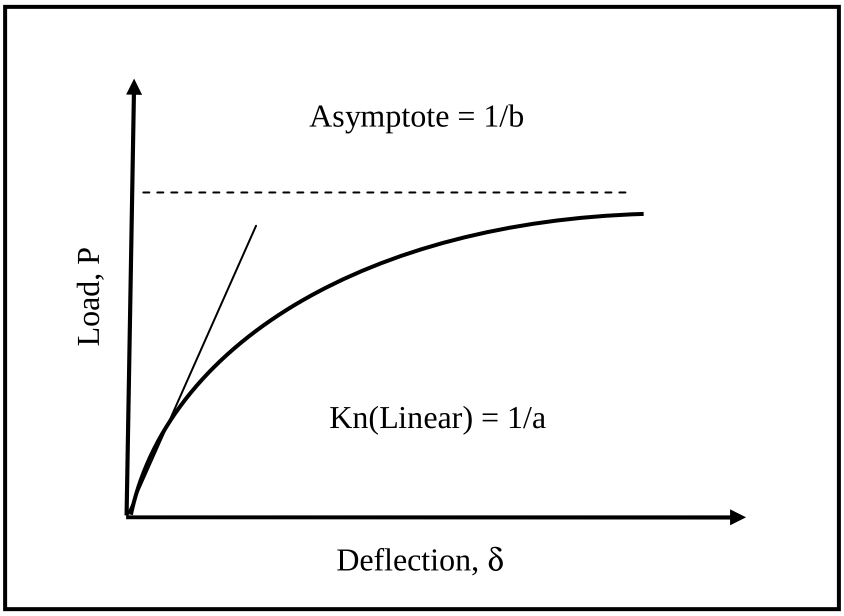


Fig. (3) Typical Two Constant Hyperbolic Equation to Model the Nonlinear Soil Behavior

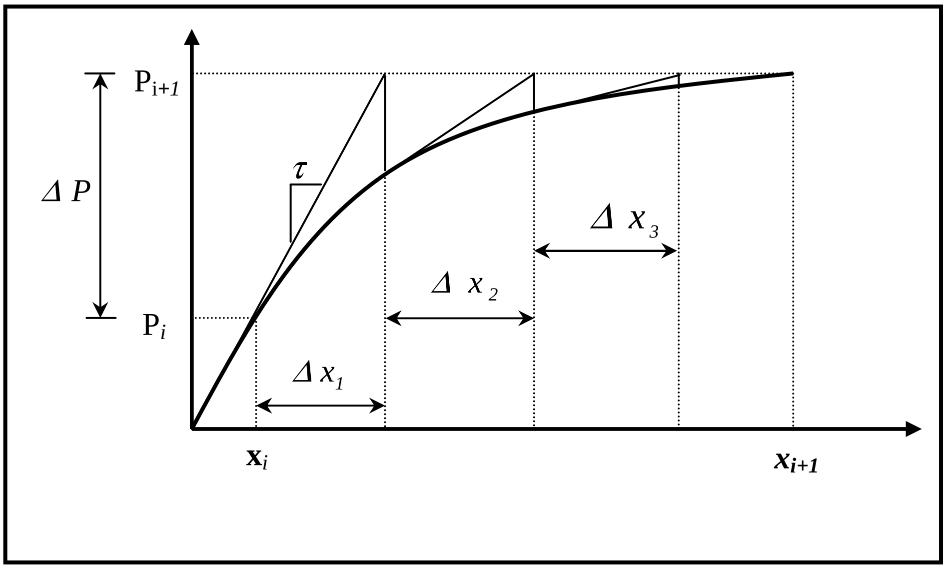
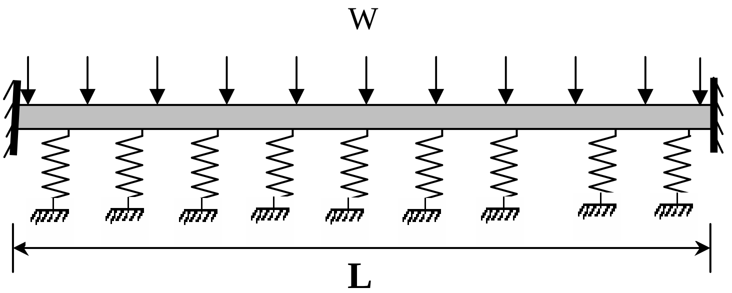


Fig. (4) Computational Technique.



W= 1 kN /m, I = 1 m^4
L = 3 m, A = 2.29 m^2
a = 1 m^2/kN, b = 10 m /kN

Fig. (٥) The Geometry, Soil Properties and Loading Conditions of Beam of Example No.1

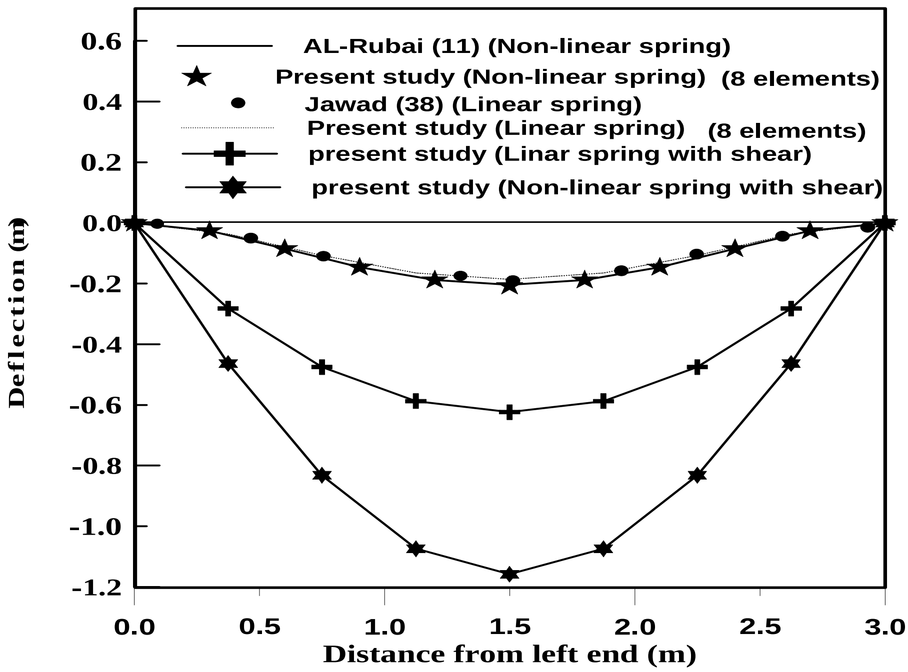


Fig.(6) Deflected Shape with and Without Shear Effect of Example No.1

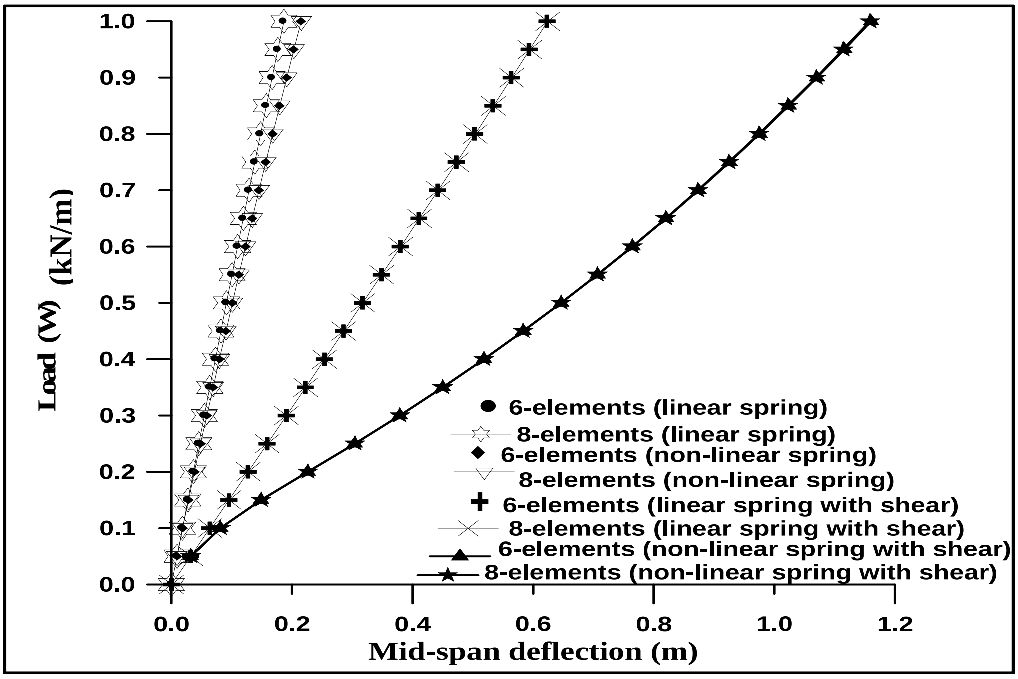


Fig.(٧) Load-Deflection Curves of Example No.1

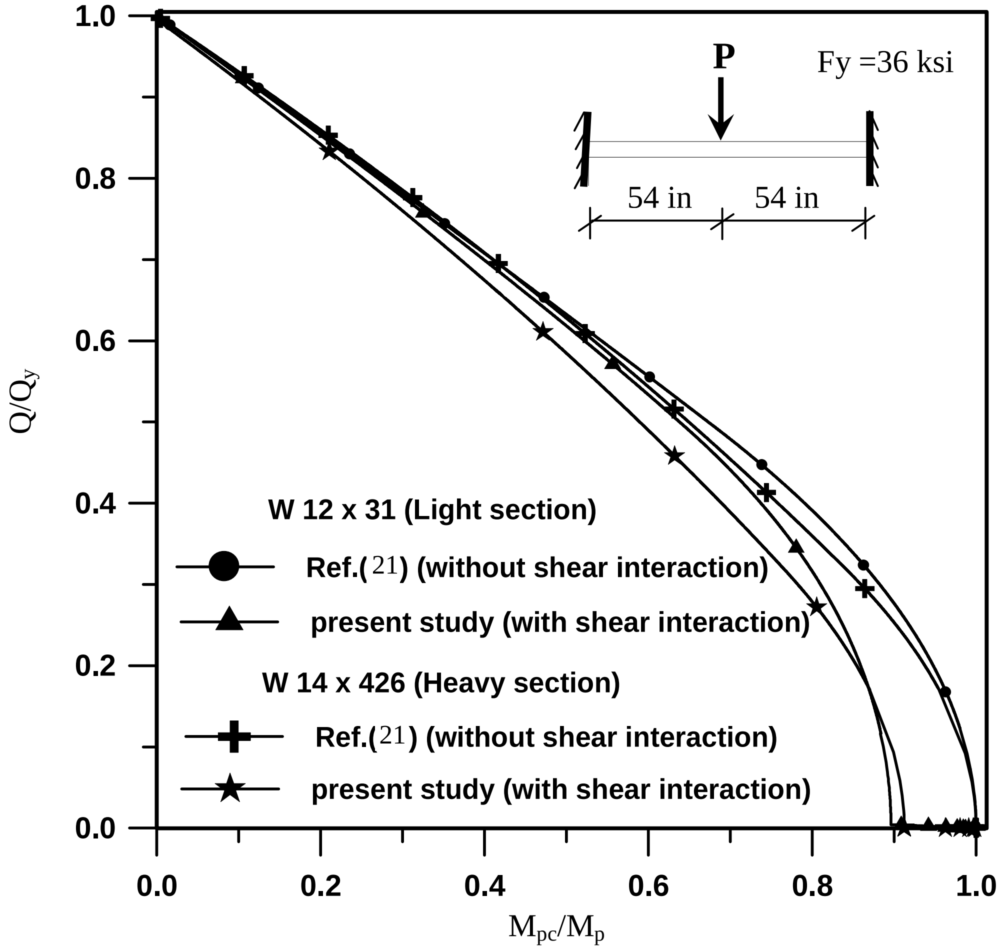


Fig.(8) Interaction Diagrams with and Without Shear Effect of Example No.2

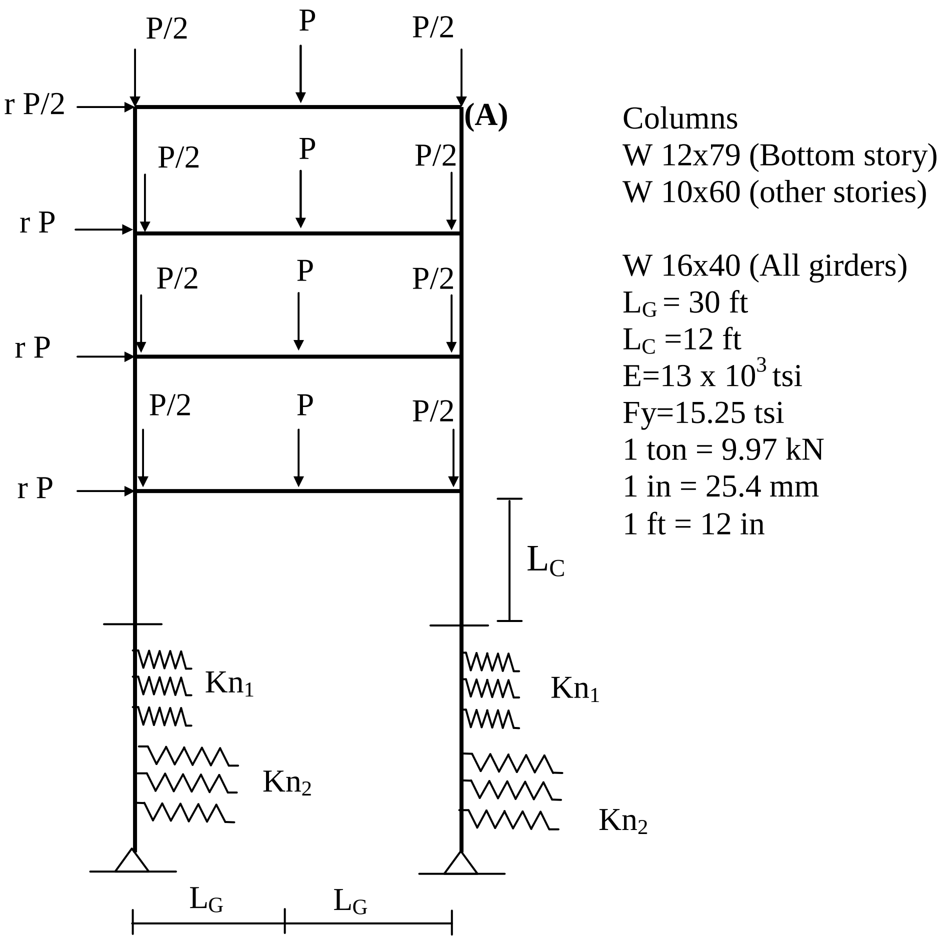


Fig.(9) The Geometry, Material and Loading Conditions of Example No.3

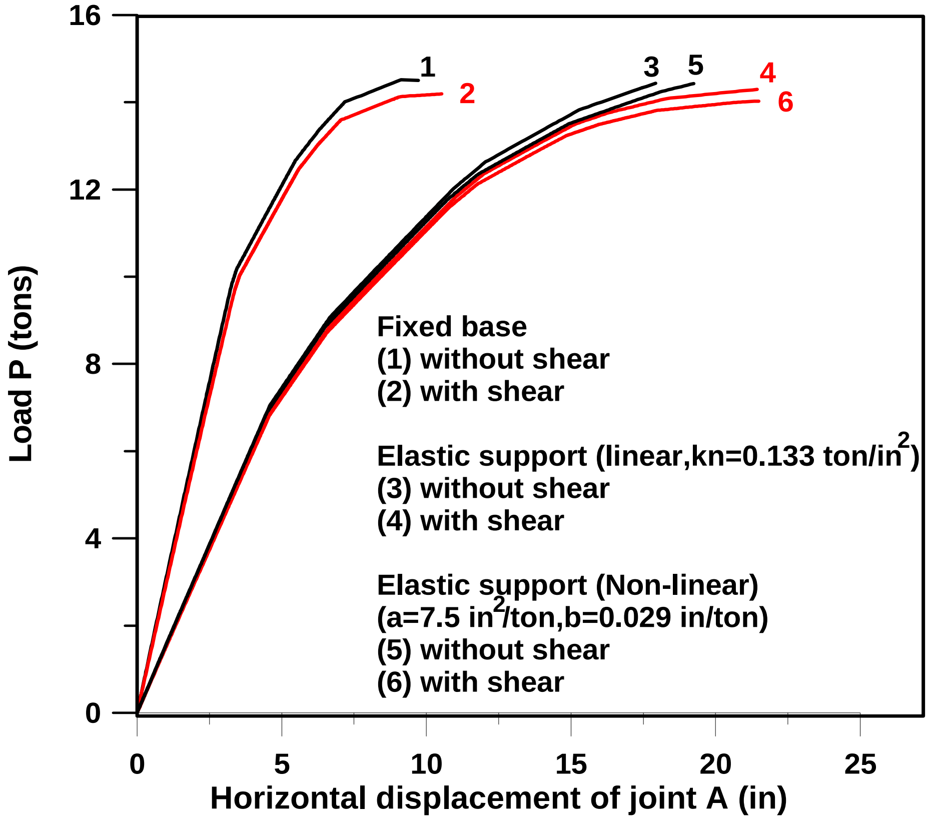


Fig.(10) Load-Displacement Curves for Fixed Base and Constant Normal Subgrade Reaction with and Without Shear Effect of Example No.3

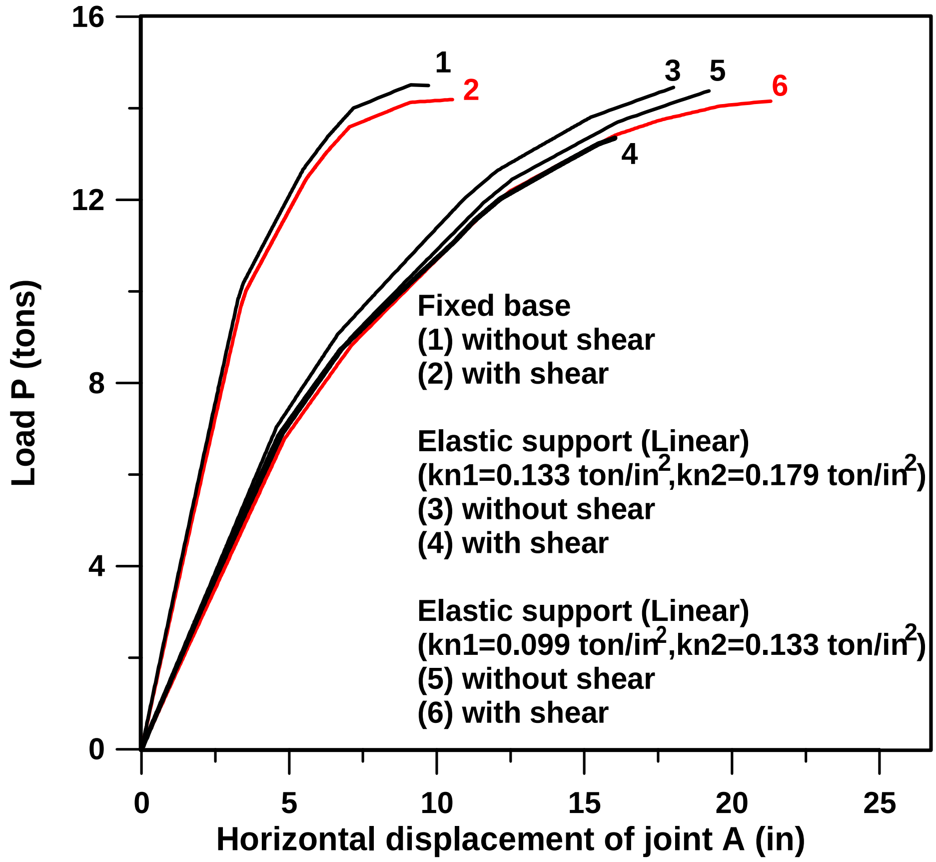


Fig. (11) Load -Displacement Curves for Fixed Base and Variable Normal Subgrade Reaction with and Without Shear Effect of Example No.3

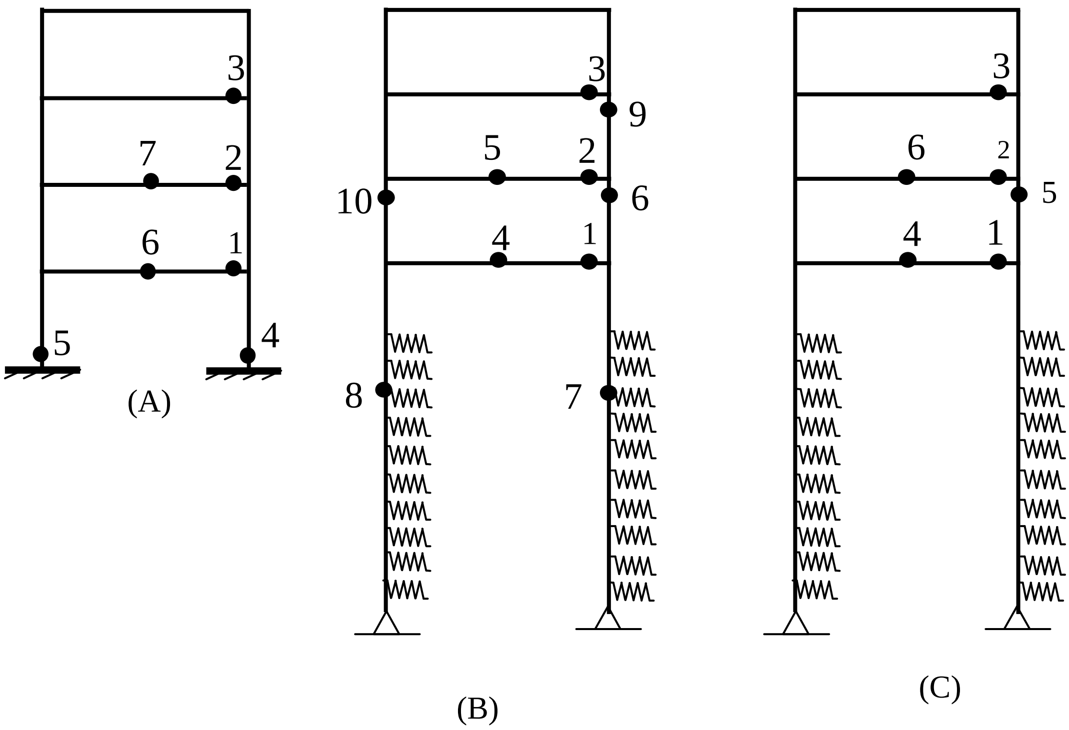


Fig.(12) Sequence of Plastic Hinges at Collapse Load for All Cases of Example No.3

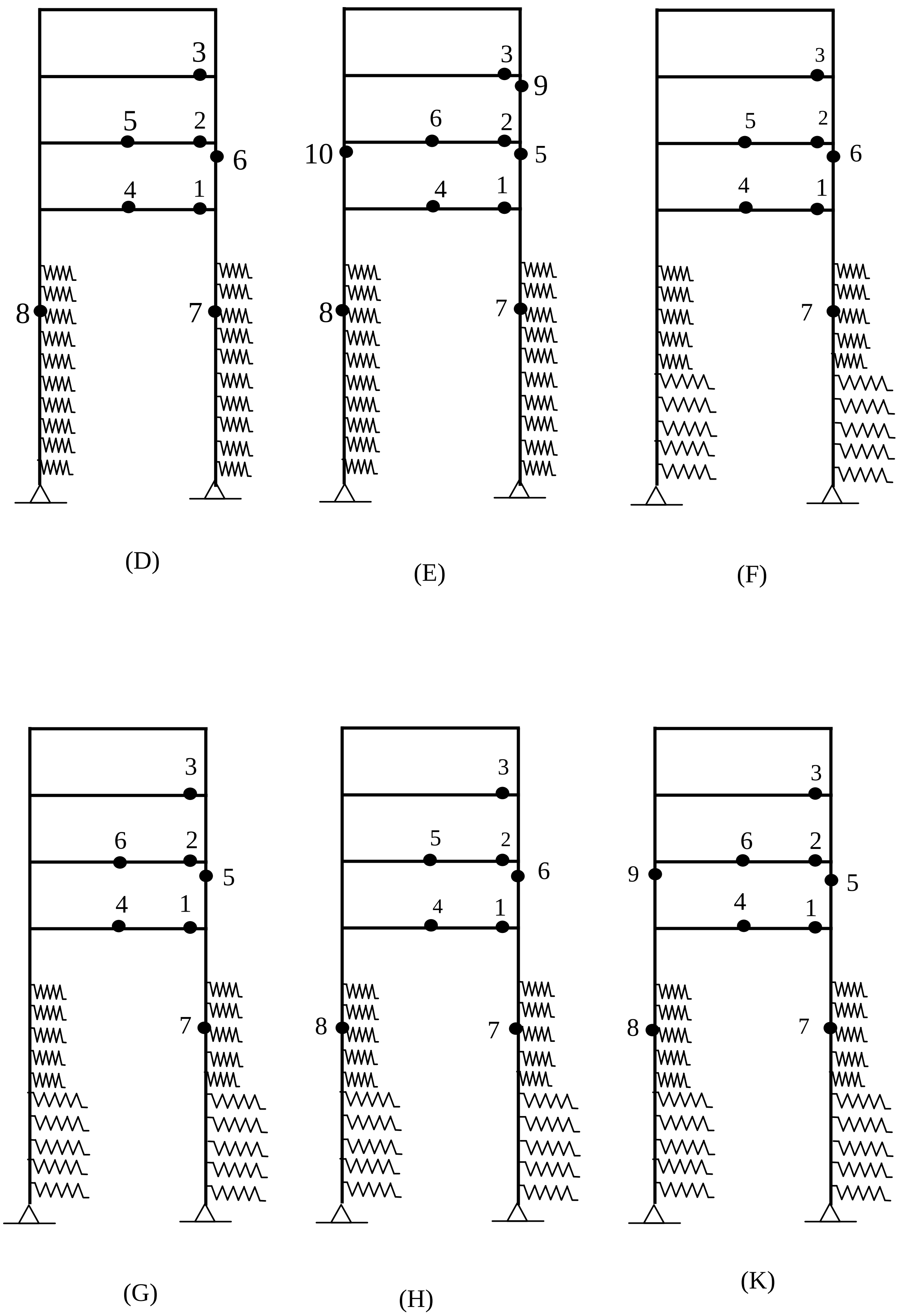


Fig. (١٢) Continue

Appendix A:

1.Stability functions including shear effect

1.1 Compressive axial force (i.e. q>0) [3]

$$\bar{C}1 = \frac{\bar{\alpha}(1-2(1-\mu q)\bar{\alpha}\cot 2\bar{\alpha})}{\tan\bar{\alpha}-\bar{\alpha}(1-\mu q)} \tag{A-1}$$

$$\bar{C}_2 = \frac{(2(1-\mu q)\bar{\alpha}-\sin 2\bar{\alpha})(\bar{\alpha}(1-2\mu q)\bar{\alpha}\cot 2\bar{\alpha}))}{(\tan\bar{\alpha}-\bar{\alpha})(1-\mu q)(\sin 2\bar{\alpha}-2(1-\mu q)\bar{\alpha}\cos 2\bar{\alpha})} \tag{A-2}$$

in which

$$\bar{\alpha} = \frac{\pi}{2}\sqrt{\bar{q}} = \frac{\pi}{2}\sqrt{\frac{q}{1-\mu q}} \tag{A-3}$$

μ : shear flexibility parameter.

q : axial force parameter.

1.2 Tensile axial force (i.e. q < 0)

$$\bar{C}_1 = \frac{\bar{\alpha}(1-2(1+\mu q)\bar{\alpha}\coth\bar{\alpha})}{\tan\bar{\alpha}-\bar{\alpha}(1+\mu q)} \tag{A-4}$$

$$\bar{C}_2 = \frac{(2(1+\mu q)\bar{\alpha}-\sinh 2\bar{\alpha})(\bar{\alpha}(1-2(1+\mu q)\bar{\alpha}\coth 2\bar{\alpha})}{(\tanh\bar{\alpha}-\bar{\alpha}(1+\mu q))(\sinh 2\bar{\alpha}-2(1+\mu q)\bar{\alpha}\cosh 2\bar{\alpha})} \tag{A-5}$$

Where $\bar{\alpha} = \frac{\pi}{2}\sqrt{\bar{q}} = \frac{\pi}{2}\sqrt{\frac{q}{1+\mu q}}$ (A-6)

1.3 Zero axial force (i.e. q = 0) [19]

$$\bar{C}_1 = \frac{4\pi^2+12\mu}{\pi^2+12\mu} \tag{A-7}$$

$$\bar{C}_2 = \frac{2\pi^2-12\mu}{\pi^2+12\mu} \tag{A-8}$$

2.Bowing functions including shear effect

2.1 Compressive axial force (i.e. q > 0)[19]

$$\bar{b}_1 = \frac{(\bar{C}1+\bar{C}2)^2}{4\pi^4 q^2}\left[\frac{2\bar{\alpha}(2\bar{\alpha}+\sin 2\bar{\alpha}\cos 2\bar{\alpha})}{\sin^2 2\bar{\alpha}} - 2 + \frac{(\bar{\alpha}\cos\bar{\alpha}-\bar{\alpha})(2\bar{\alpha}-\sin 2\bar{\alpha})}{\sin^2 2\bar{\alpha}}\right] \tag{A-9}$$

$$\bar{b}_2 = \frac{(\bar{C}1-\bar{C}2)^2}{8\pi^4 q^2}\left[\frac{(2\bar{\alpha}-2\bar{\alpha}\cos 2\bar{\alpha})(2\bar{\alpha}-\sin 2\bar{\alpha})}{\sin^2 2\bar{\alpha}}\right] \quad \text{(A-10)}$$

2.2 Tensile axial force (i.e. q<0)

$$\bar{b}_1 = \frac{(\bar{C}1+\bar{C}2)^2}{4\pi^4 q^2}\left[\frac{2\bar{\alpha}(2\bar{\alpha}+\sinh 2\bar{\alpha}\cosh 2\bar{\alpha})}{\sinh^2 2\bar{\alpha}} - 2 + \frac{(\bar{\alpha}\cosh 2\bar{\alpha}-\bar{\alpha})(2\bar{\alpha}-\sinh 2\bar{\alpha})}{\sinh^2 2\bar{\alpha}}\right] \quad \text{(A-11)}$$

$$\bar{b}_2 = \frac{(\bar{C}1-\bar{C}2)^2}{8\pi^4 q2}\left[\frac{(2\bar{\alpha}-2\bar{\alpha}\cosh 2\bar{\alpha})(2\bar{\alpha}-\sinh 2\bar{\alpha})}{\sinh^2 2\bar{\alpha}}\right] \quad \text{(A-12)}$$

3. The Geometric matrices $[g^{(k)}]$ and the transformation matrix [B] [15,16]

$$\left[g^{(1,2)}\right] = \frac{1}{(1+\delta)^2}\begin{bmatrix} -2mn & m^2-n^2 & 0 & 2mn & -(m^2-n^2) & 0 \\ m^2-n^2 & 2mn & 0 & -(m^2-n^2) & -2mn & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2mn & -(m^2-n^2) & 0 & -2mn & m^2-n^2 & 0 \\ -(m^2-n^2) & -2mn & 0 & m^2-n^2 & 2mn & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A-13)}$$

$$\left[g^{(3)}\right] = \frac{1}{(1+\delta)}\begin{bmatrix} -n^2 & mn & 0 & n^2 & -mn & 0 \\ mn & -m^2 & 0 & -mn & m^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ n^2 & -mn & 0 & mn & -m^2 & 0 \\ -mn & m^2 & 0 & mn & -m^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(A-14)}$$

$$[B] = \begin{bmatrix} \frac{-n}{1+\delta} & \frac{-n}{1+\delta} & m \\ \frac{m}{1+\delta} & \frac{m}{1+\delta} & n \\ 1 & 0 & 0 \\ \frac{n}{1+\delta} & \frac{n}{1+\delta} & -m \\ \frac{-m}{1+\delta} & \frac{-m}{1+\delta} & -n \\ 0 & 1 & 0 \end{bmatrix} \quad \text{(A-15)}$$

in which

$m = \text{Cos}\alpha$, $n = \text{Sin}\alpha$

$\delta = - u/L = - (L - L_c)/L$

α = the angle between the local and the global coordinates in the deformed configuration of the member.

L_c = the member chord length of deformed configuration.

EFFECT OF COMPRESSIVE STRENGTH AND REINFORCEMENT RATIO ON STRENGTHENED BEAM WITH EXTERNAL STEEL PLATE

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Abstract:

The present study is an experimental comparison between the effect of increasing the compressive strength of the section and increasing the reinforcement ratio on the results of strengthening reinforced concrete beams with external steel plates of constant dimensions.
The experimental program consists of testing ten reinforced concrete beams. Five of them are without external steel plates to be the original specimens while the other five ones are provided with steel plates of same dimensions glued at the bottom face of the beams.
Three values of compressive strength (f'c) were used in this study which were (22, 45 and 71MPa) and also three ratios of internal reinforcement (ρ) which were (0.01411, 0.02116 and 0.03445) to investigate their effects on the strengthened beams behavior.
The results showed that the cracking load and the ultimate load can be increased up to (150% and 137%) respectively. Also, by increasing the section compressive strength all the properties of the strengthened beam can be improved while by increasing the reinforcement ratio the deflection and cracking can be reduced to improve the elastic behavior of the beam.

Keywords: Strengthened beam, external plate, deflection ductility, restraining.

تاثير قوى الشد القصوى ونسبة التسليح على العتبة الناتئة المدعمة بصفيحة من الحديد الخارجية

هشام عبد الطيف نعمان

الجامعة المستنصرية كلية الهندسة

الخلاصة:

الدراسة الحالية عبارة عن مقارنة عملية بين تأثير زيادة مقاومة الانضغاط للمقطع وزيادة نسبة التسليح على نتائج تقوية العتبات الخرسانية المسلحة باستخدام صفائح فولاذية ثابتة الأبعاد. يتكون البرنامج العملي من فحص عشر عتبات خرسانية. خمس