THERMAL DESIGN OF TUBE BANKS IN CROSS FLOW BASED ON MINIMUM THERMODYNAMIC LOSSES

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Abstract

Tube banks are widely used in crossflow heat exchangers. Usually, the methods for its design are the NTU or LMTD methods, while in this research the Entropy Generation Method is used. By assuming constant tube wall temperature, a general dimensionless expression for the entropy generation rate is obtained by considering a control volume around a tube bank and applying the conservation equations for mass and energy with the entropy balance. A comparison of the design is accomplished for a tube banks of different stream velocity, lengths and diameters. The heat transferred rate, ambient and tube wall temperatures are 20kW, 300K, and 365K, respectively. From the comparison of the design with the entropy generation rates, the optimal design is obtained. A single objective function is used which is the dimensionless entropy generation rate N_s subjected to the constraints of diameters and pitch ratio. This method of optimization can be applied for any constraints on the system which is the Lagrange optimization method. The effects of tube diameter, tube length, dimensionless pitch ratios, front cross-sectional area of the tube bank, and heat load are examined with respect to its role in influencing optimum design conditions and the overall performance of the tube banks. It is demonstrated that the performance is better for higher air velocities and larger dimensionless pitch ratios. Compact tube banks perform better performance for smaller tube diameters.
Key words: entropy , generation ,tube bank , crossflow ,performance

التصميم الحراري لحزمة أنابيب في جريان متقاطع بالاعتماد على اقل الخسائر الثرموديناميكية

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الخلاصة

إن حزم الأنابيب واسعة الاستخدام في المبادلات الحرارية ذات الجريان المتقاطع . كما إن الطرق الشائعة لتصميها هي طريقة NTU وطريقة LMTD اما في هذا البحث نستخدم طريقة جديدة وهي طريقة تولد الأنتروبي Entropy Generation Method . تم افتراض إن درجة حرارة جدران الأنابيب ثابتة حيث تم الحصول على تعبير رياضي عام لمعدل تولد الأنتروبي بشكل لابعدي dimensionless من خلال تطبيق معادلات حفظ الكتلة والطاقة مع موازنة للطاقة والأنتروبي على حجم التحكم C.V المأخوذ حول حزمة الأنابيب. في هذا البحث تم مقارنة تولد الأنتروبي لحزمة أنابيب ذات أقطار و أطوال وسرع جريان خارجي مختلفة يكون معدل انتقال الحرارة فيها 20kW ودرجة حرارة جو 300K ودرجة حرارة جدران الأنابيب 365K ومن مقارنة معدلات تولد الأنتروبي تم

اختيار التصميم الأمثل. تمت الأمثلية لدالة هدف واحدة وهي معدل تولد الأنتروبي اللابعدية N_s وليقيود علىالأقطاروسب الخطوة اللابعدية حيث إن هذه الطريقة ممكن ان تطبق أية قيودconstraints تضاف للنظام وهي طريقة لاكرانج للأمثلية. كذلك تم دراسة تأثير كل من قطر الأنبوب, مساحة مقطع حزمة الأنابيب بالإضافة إلى نسب الخطوة اللابعدية Pitch Ratio على التصميم والأداء الأمثل لمنظومة حزمة الأنابيب. تم التوصل إلى إن الأداء الأمثل يكون عند سرعة جريان خارجي عالية ونسب خطوة لابعدية عالية. إن حزمة الأنابيب المتراصة يكون الأداء الأمثل لكلا النظامين لأقطار أنابيب صغيرة.

Nomenclature

A	surface area of a single tube, m^2
A_t	total heat transfer area, m^2
D	tube diameter, m
E	specific energy, W
f	friction factor
g, l	equality and inequality constraints
h_{avg}	average heat transfer coefficient of tubes, $W/m^2.K$
i	number of imposed constraints
k	thermal conductivity, $W/m.K$
LF	Lagrangian function
L	length of tube, m
N	total number of tubes, $N_T N_L$
n	number of design variables
N_L	number of rows in streamwise direction
N_s	dimensionless entropy generation rate, $\dot{S}_{gen}/(Q^2 U_{max} / k_f v T_a^2)$
N_T	number of rows in spanwise direction
Nu_D	Nusselt number based on tube diameter
P	pressure, Pa
Q	heat transfer rate over the boundaries of control volume, W
Re_D	Reynolds number, DU_{max}/ν
$\dot{S}_{gen}$	total entropy generation rate, W/K
S_L	tube streamwise pitch, mm
S_T	tube spanwise pitch, mm
T	absolute temperature, K
U	air velocity, m/s
Pr	Prandtl number,
v	specific volume of fluid, m^3/kg
x_i	design variables

Greek Symbols

γ aspect ratio, L / D

ν kinematic viscosity of fluid, m^2 / s

ρ fluid density, kg / m^3

Subscripts

a ambient

f fluid

in inlet of control volume

out exit of control volume

T thermal

W wall

Superscripts

* optimum

Introduction

Tube banks are usually arranged in an in-line or staggered manner, where one fluid moves across the tubes, and the other fluid at a different temperature passes through the tubes. This research is interested to determine an optimal design of the tube banks in crossflow using entropy generation minimization method. The crossflow correlations for the heat transfer and pressure drop are employed to calculate entropy generation rate. A careful review of existing literature reveals that most of the studies are related to the optimization of plate heat exchangers and only few studies are related to tube heat exchangers. Bejan(1982) extended that concept and presented an optimum design method for balanced and imbalanced counterflow heat exchangers. He proposed the use of a N_s as a basic parameter in describing heat exchanger performance. This method was applied to a shell and tube regenerative heat exchanger to obtain the minimum heat transfer area when the amount of units was fixed. Aceves-Saborio et al.(1989) extended that approach to include a term to account for the exergy of the heat exchanger material. Ordonez and Bejan(2000), Bejan(2001), and Bejan(2002) demonstrated that the optimal geometry of a counterflow heat exchanger can be determined based on thermodynamic optimization subject to volume constraint. Entropy generation rate is generally used in a dimensionless form. Peters and Timmerhaus(1991) presented an approach for the optimum design of heat exchangers. They used the method of steepest descent for the minimization of annual total cost. They observed that this approach is more efficient and effective to solve the design problem of heat exchangers. Optimization of plate-fin and tube-fin crossflow heat exchangers was presented by Shah et al.(1978) and Van den Bulck(1991). They employed optimal distribution of the *UA* value across the volume of crossflow heat exchangers and optimized different design variables like fin thickness, fin height, and fin pitch. In two different studies, Stanescu et al.(1996) and Matos et al.(2001) demostrated that the geometric arrangement of tubes/cylinders in cross-flow forced convection can be optimized for maximum heat transfer subject to overall volume constraint. They used FEM to show the optimal spacings between rows of tubes. Vargas, et al.(2001) documented the process of determining the internal geometric configuration of a tube bank by optimizing the global performance of the installation that uses the crossflow heat exchanger.

Problem Formulation

The irreversibility of this system is also due to heat transfer across the nonzero temperature difference T_w - T_a and due to the total pressure drop across the tube bank. First law of thermodynamics for the control volume can be written as

$$Q = \dot{m}(h_{out} - h_{in}) \tag{1}$$

From the second law thermodynamics

$$\dot{S}_{gen} = \dot{m}(s_{out} - s_{in}) - \frac{Q}{T_w} \tag{2}$$

Gibbs equation $\left[dh = Tds + (1/\rho)dP\right]$ can be written as:

$$h_{out} - h_{in} = T_a(s_{out} - s_{in}) + \frac{1}{\rho}(P_{out} - P_{in}) \tag{3}$$

Combining Eqs. (1) and (3), we get:

$$Q = \dot{m}T_a(s_{out} - s_{in}) - \frac{\dot{m}}{\rho}\Delta P \tag{4}$$

From Eqs. (2) and (4), we get:

$$\dot{S}_{gen} = \left(\frac{Q^2}{T_a T_w}\right)R_{tube} + \frac{\dot{m}}{\rho T_a}\Delta P \tag{5}$$

where R_{tube} is the tube wall thermal resistance, $\dot{m}$ is the mass flow rate through the tubes and ΔP is the pressure drop across the tube bank and can be written as

$$R_{tube} = \frac{\Delta T}{Q} = \frac{1}{h_{avg}A} \tag{6}$$

$$\dot{m} = \rho U \; N_T S_T L \tag{7}$$

$$\Delta P = N_L f\left(\frac{1}{2}\rho U^2\right) \tag{8}$$

Khan(2004) has developed following analytical correlation for dimensionless heat transfer coefficient for the tube banks:

$$Nu_D = \frac{h_{avg}D}{k_f} = C_1 \mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3} \tag{9}$$

Where and C_1 is a constant which depends upon the longitudinal and transverse pitch ratios, arrangement of the tubes, and thermal boundary conditions. For isothermal boundary condition, it is given by:

$$C_1 = \left[0.25 + \exp(-0.55 S_L)\right] S_T^{\;0.285} S_L^{\;0.212} \tag{10}$$

Khan et al.(2005) digitized thier experimental data and fitted into single correlations for the friction and correction factors for each arrangement. These correlations can be used for any pitch ratio $1.05 \le S_L$ or $S_T \le 3$ and Reynolds number in the laminar flow range. They are

$$f = K_1\left[\frac{0.233 + 45.78}{(S_T - 1)^{1.1}\,\mathrm{Re}_D}\right] \tag{11}$$

Where K_1 is a correction factor depending upon the flow geometry and arrangement of the tubes. It is given by:

$$K_1 = 1.009\left(\frac{S_T - 1}{S_L - 1}\right)^{1.09/\mathrm{Re}_D^{0.0553}} \qquad (12)$$

Using Eqs. (6) - (9), the entropy generation rate can be simplified to

$$\dot{S}_{gen} = \frac{Q^2 / T_a T_w}{C_1 N \pi L k_f \,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}} + \frac{N f \rho U^3 (S_T - 1) L}{2T_a} \qquad (13)$$

For external flow, Bejan (1996) used the term $Q^2 U / k_f v T_a^2$ to nondimensionalize entropy generation rate in Eq. (14). So the dimensionless entropy generation rate can be written as

$$N_s = \frac{T_a / T_w}{C_1 N \pi \gamma \,\mathrm{Re}_D^{3/2}\,\mathrm{Pr}^{1/3}} + \frac{1}{2} f N \gamma B \,\mathrm{Re}_D^2 (S_T - 1) \qquad (14)$$

Where $B = \rho v^3 k_f T_a / Q^2$

Optimization Procedure

If $f(\mathbf{x})$ represent the dimensionless entropy generation rate that is to be minimized subject to equality constraints

$$g_i(x_1, x_2, ..., x_n) = 0 \qquad (15)$$

and inequality constraints

$$l_k((x_1, x_2, ..., x_n) \geq 0\,, \qquad (16)$$

then the complete mathematical formulation of the optimization problem may be written in the following form:

$$\min imize \qquad f(x) = N_s(x) \qquad (17)$$

Subject to the equality constraints

$$g_i(x) = 0, \qquad i = 1,2,....,m \qquad (18)$$

and inequality constraints

$$l_i(x) \geq 0, \qquad i = m+1, m+2,....., n \qquad (19)$$

In this research, the design variables x are:

$$x = (x_1, x_2, x_3,....., x_n)^T = [D, H, W, L, U, Q] \qquad (20)$$

Inequality constraints are:

$$D \geq 10\ mm \qquad (21)$$

$$1.25 \leq \frac{S_L}{D} \leq 3 \qquad (22)$$

$$1.25 \leq \frac{S_T}{D} \leq 3 \qquad (23)$$

$$\gamma \geq 20 \qquad (24)$$

The objective function can be defined by using Lagragian function as follows:

$$\mathrm{LF}(\mathrm{x}, \lambda, \chi) = N_s(x) + \sum_{i=1}^{m} \lambda_i g_i(x) - \sum_{i=m+1}^{n} \chi_i l_i(x) \qquad (25)$$

where λ_i and χ_i are the Lagrange multipliers. The λ_i can be positive or negative but the χ_i must be greater than or equal zero. In addition to Kuhn-Tucker conditions, the other necessary condition for x^* to be a local minimum of the problem, under consideration, is that the Hessian matrix of L should be positive semidefinite, i.e.

$$v^T \nabla^2 [(x^*, \lambda^*, \chi^*)] v \geq 0 \qquad (26)$$

For a local minimum to be a global minimum, all the eigen-values of the Hessian matrix should be greater than or equal zero. A system of non-linear equations is obtained, which can be solved using numerical methods such as a multivariable Newton-Raphson method. In this study, the same approach is used to optimize the overall performance of a tube bank in such a manner that all relevant design conditions combine to produce the best possible tube bank for the given constraints. The optimized results are then compared. A simple procedure was programmed in MATLAB, which solves the system of *N* non-linear equations using the multivariable Newton-Raphson method.

Results and Discussion

The problem is solved for different pitch ratios and the overall performance is compared for both NTU and LMTD methods. **Figure 2** shows the effect of tube diameter on the heat transfer from the system of tube banks based on the three different NTU, LMTD, and $\dot{S}_{gen}$ methods. It is show that the linear relation between the tube diameter and the heat transfer rate based on the LMTD method, while in the $\dot{S}_{gen}$ and NTU methods , the relation was not linear. This behavior due to the different mathematical formula between each of three methods. LMTD neglect the pressure drop effect, while the $\dot{S}_{gen}$ take the pressure as the first parameter in its mathematical relation. NTU and $\dot{S}_{gen}$ method gives the same amount of heat transfer at D=2.05 mm which gives the more accuracy for the NTU method. Also the NTU method gives a good convergence for tube diameter less than 2.05 mm, but diverge for the diameter larger than 2.05mm.

Figure 3 shows the real interpretation for effect of tube bank length on the heat transfer rate. The LMTD and NTU method gives higher heat transfer rate than $\dot{S}_{gen}$ method. This behavior due to the pressure drop effect. The recommendation for this case is to use entropy generation method to study the tube banks performance when the length is variable.

Figure 4 shows the real interpretation for the effect of tube banks length on the heat transfer rate. There is a note from the above figure which is at the velocity 12 m/s, the NTU, LMTD, and $\dot{S}_{gen}$ methods gives the same estimation of the heat transfer rate. Also at velocities less than 12 m/s, NTU and $\dot{S}_{gen}$ converge in estimation of heat transfer rate, while at velocities larger than 12 m/s, the $\dot{S}_{gen}$ diverge from the NTU and LMTD method because of the pressure losses. The recommendation for this case is to use any method to estimate the heat transfer rate when the air velocity is 12 m/s.

Figure 5 shows the effect of the tube pitch on the heat transfer rate. It is noted that the amount of heat transfer decrease when the pitch is increase. Also when the $S_T \approx 2mm$. The LMTD, $\dot{S}_{gen}$, and NTU methods gives the same estimation of the heat rate. Heat transfer rate increase after the pitch of 2mm value because of the increasing of the heat transfer area. The recommendation for this case is to use any method for heat transfer rate estimation when pitch is larger than 2mm.

Conclusions

1- This research shows that, for the given volume of the tube bank and heat duty, the dimensionless entropy generation rate depends on ambient and wall temperatures, total number of tubes, longitudinal and transverse pitch ratios, Reynolds and Prandtl numbers, and aspect ratio. After fixing ambient and wall temperatures, all these parameters depend on tube diameter and the approach velocity for given longitudinal and transverse pitches.
2- An entropy generation minimization method is applied as a unique measure to study the thermodynamic losses caused by heat transfer and pressure drop for a fluid in crossflow with tube banks.
3- A general dimensionless expression for the entropy generation rate is obtained by considering a control volume around a tube bank and applying the conservation equations for mass and energy with the entropy balance.
4- Any method can be used for heat transfer rate estimation when pitch is larger than 2mm. Also entropy generation method can be used to study the tube banks performance when the length is variable.

5- N_s method is better than NTU and LMTD methods to find the optimum thermal design for heat sink.

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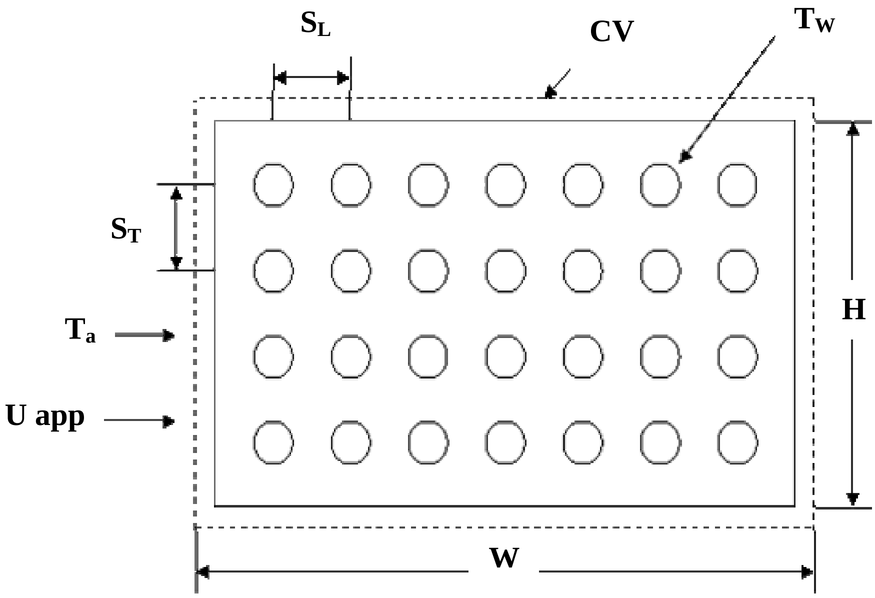

Fig. 1: Front View Control Volume for Calculating $\dot{S}_{gen}$ for the Tube Banks of Length L.

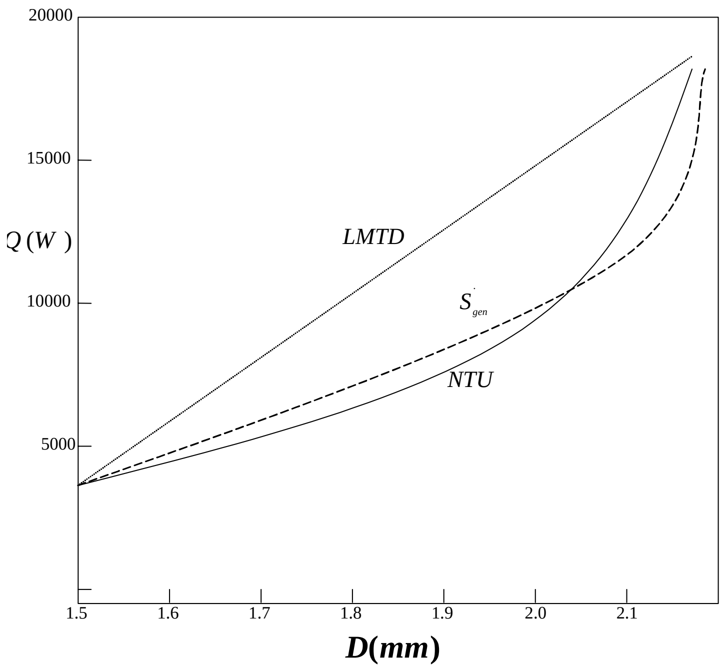

Fig. 2: Comparison of optimum heat loss between LMTD, $\dot{S}_{gen}$, and NTU methods based on tube diameter.

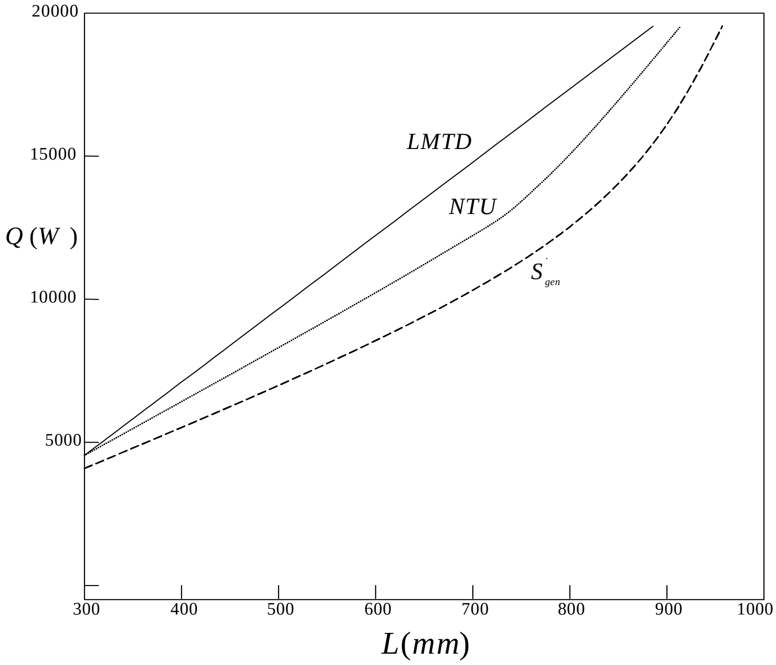

Fig. 3: Comparison of optimum heat loss between LMTD, $\dot{S}_{gen}$, and NTU methods based on tube length.

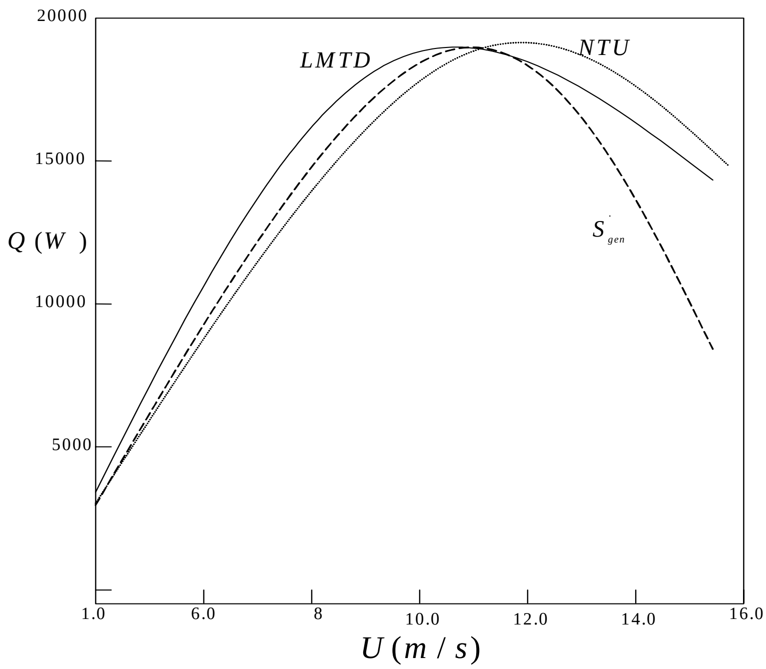

Fig. 4: Comparison of optimum heat loss between LMTD, $\dot{S}_{gen}$, and NTU methods based on air velocity.

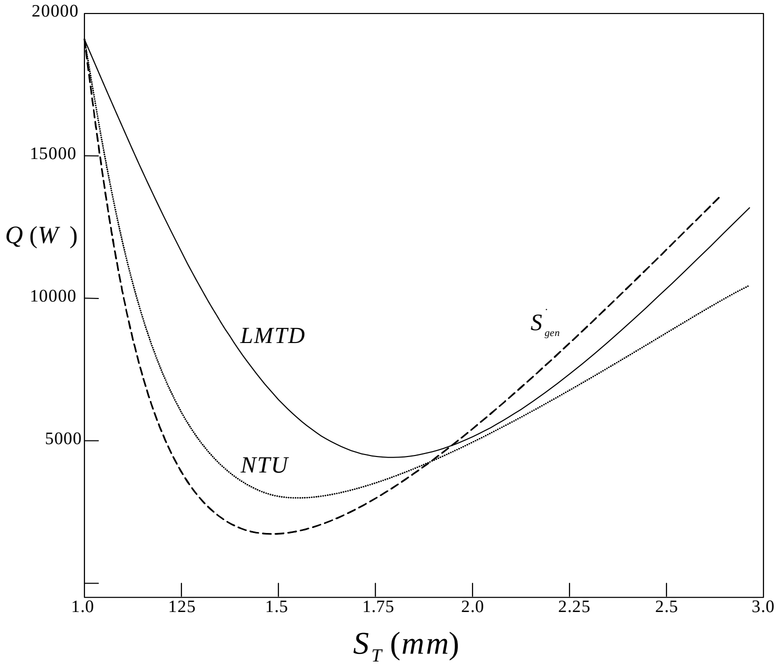

Fig. 5: Comparison of optimum heat loss between LMTD, $\dot{S}_{gen}$, and NTU methods based on tube on tube pitch.