ADVANCED MODEL FOR THE EFFECTIVE MOMENT OF INERTIA TAKING INTO ACCOUNT SHEAR DEFORMATIONS EFFECT

Dr.Haider K. Ammash
in Civil Engineering,
University of Al-Qadisiya

Mr. Muthana H. Muhaisin
in Civil Engineering,
University of Al-Qadisiya

Abstract

The study aims to present anew form of the effective moment of inertia by enhancement Branson's model to take the effect of several factors such load type(concentrated, uniformly distribution, and two points) loads, shear deformations affect are also considered. These deformations depend on the span to depth ratio. The results of the presented model were compared with (experimental results, Branson's model results, and results of other models). The results of the present model give best agreement with experimental results than Branson's and the other models; the results showed that the effective moment of inertia reduced by about 27% for span to depth ratio of (20 to 5) due to shear deformation effects.

Keywords: **beams, reinforced concrete, deflection, effective moment of inertia, shear deformation**.

نموذج متقدم لعزم القصور الذاتي الفعال يأخذ بنظر الاعتبار تأثير التشوهات القصية

د.حيدر كاظم عماش
قسم الهندسة المدنية
جامعة القادسية

مثنى حسين محيسن
قسم الهندسة المدنية
جامعة القادسية

الخلاصة

تهدف الدراسة إلى تقديم صيغة جديدة لعزم القصور الذاتي الفعال بتحسين نموذج برانسون لأخذ بنظر الاعتبار عدة عوامل مثل نوع الحمل (مركز،موزع بانتظام، مركز في نقطتين)وتشوهات القص بالاعتماد على نسبة الفضاء إلى السمك ونسبة حديد التسليح (حديد التسليح في منطقة الشد الى نسبة حديد التسليح في منطقة الضغط). نتائج النموذج المقدم قورنت مع(نتائج عملية، نتائج نموذج برانسون و نتائج نماذج أخرى). النموذج المقدم يعطي نتائج أقرب للنتائج العملية من نموذج برانسون والنماذج أخرى، كذلك بينت النتائج إن عزم القصور الذاتي الفعال يقل بنسبة ٢٧% لنسبة فضاء\ السمك من (20الى٥) نتيجة تشوهات القص.

Notations

b_1=top flange width.
b_2=bottom flange width.
b_w=web width.
d = Effective depth of tension region.
d' = Effective depth of compression region
f_r=modulus of rupture
H=total depth of beam.
I_e : Effective moment of inertia.
I_{cr} : Cracked moment of inertia.
I_g : Gross moment of inertia.
M_a: Applied external moment.

M_{cr}: Cracking moment
n=modular ratio.
ρ = Ratio of steel area at tension region.
ρ' = Ratio of steel area at compression region

Introduction

Deflections of reinforced concrete flexural members were the focus of several research activities for many years. It is prime importance in the determination of the deflection of beams is calculation of the moment of inertia (I) of the beam, since its value changes along the span length from (Ig) for uncracked sections to (I_{cr}) for cracked sections.

Branson developed a well known expression for the effective moment of inertia (I_e) over the entire length of the simply supported beam in the following form:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr} \tag{1}$$

The ACI Building Code adopted Branson's equation and it first appeared in the 1971 edition of the publication and remains the recommended way of calculating the effective moment of inertia for the purpose of calculating the deflection of a reinforced concrete member.

Since its adoption by the ACI Code in 1971, Branson's model has been continually opposed. The reasons vary, but center around the accuracy of the model. Design engineers argue that the cumbersome calculation of I_{cr}, especially for flanged sections, is complex and time consuming (Grossman 1981). They also argue that the effort required is not justified by the final product. Grossman (1981) states that the estimated deflection obtained by using Branson's model is, at best, within ±20 % of experimental deflections obtained in a controlled lab setting. Another argument against Branson's model is that its empirical nature can produce gross errors when applied to beams that are heavily or lightly reinforced and/or subjected to non-uniform loads. Researchers have shown, that in some instances, Branson's model can produce values that are 100% in error (Fikry and Thomas 1998). The arguments and concerns prompted various researchers to study the validity of Branson's equation. The subsequent research produced numerous simplifications and enhancements to the Branson model.

Modifications to the I_e method

1 Method (1)

In 1991 scholars from King Saud University in Riyadh, Saudi Arabia published findings from research they conducted to determine if non-uniform load configurations are accurately accounted for by Branson's effective moment of inertia model (Al-Zaid, Al-Shaikh, and Abu-Hussein 1991). The research compared theoretical moment of inertia values to experimental moment of inertia values obtained from subjecting reinforced concrete members of rectangular cross-section to a uniform load, a mid-span concentrated load, a third-point load, and a mid-span concentrated load combined with a uniform load. The service load moment applied to the member was the same for each load configuration. It was observed that the experimental moment of inertia values for a member subjected to a mid-span concentrated load was 12% greater than that experienced by a member subjected to a third-point load and 20% greater than the experimental moment of inertia exhibited by a member subjected to a uniform load.

The experimental values proved that Branson's model can not be accurate for all loading cases. Equation (1) returns a value comparable to the experimental value for the uniform loading case, which means that if the member is loaded with a concentrated load at mid-span the stiffness of the member would be significantly underestimated. The researchers addressed the discrepancy by suggesting that Branson's model be generalized by modifying it to the form of Equation (2).

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^m \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^m\right] \times I_{cr} \tag{2}$$

where:
m : experimentally determined exponent.

In their report the researchers showed that by generalizing Equation (1) and in-turn solving for m (Equation 3) for each load case that the discrepancy could be eliminated.

$$m = log\left(\frac{I_{exp} - I_{cr}}{I_g - I_{cr}}\right) \Big/ log\left(\frac{M_{cr}}{M_a}\right) \tag{3}$$

where:
I_{exp} : Experimental moment of inertia.

2 Method (2)

The researchers argued that the discrepancy revealed in Branson's model was caused by the different lengths over which a beam cracks due to a specific load condition (Al-Zaid, et. al. 1991). Therefore, the authors suggested a model (Equation 4), similar in form to Branson's model, that incorporated the ratio of cracked length to overall length which inherently accounted for the variation in the effective moment of inertia caused by different cracked lengths (Equation 4).

$$I_e = \left(\frac{L_{cr}}{L}\right)^{m*} \times I_{cr} + \left[1 - \left(\frac{L_{cr}}{L}\right)^{m*}\right] \times I_g \qquad (4)$$

where:

m^* : experimentally determined exponent.

L_{cr} : cracked length of the member.

L = length of member

The proposed model is bounded by $I_e = I_g$ when $L_{cr} = 0$, and $I_e = I_{cr}$ when the cracked length covers nearly the entire length of the member. The exponent m' is calculated using Equation (3). In theory, the exponent m^* is solely a function of the reinforcement ratio. This theory was later expanded on by the same researchers (Al-Shaikh and Al-Zaid 1993).

$$m^* = log\left(\frac{I_g - I_{exp}}{I_g - I_{cr}}\right) \Big/ log\left(\frac{L_{cr}}{L}\right) \qquad (5)$$

where:

I_{exp} : experimental moment of inertia.

The researchers exhibited that the "modified" form of Branson's model and the proposed model incorporating cracked length both produce effective moment of inertia values relatively close to experimental moment of inertia values when the proper exponent is employed.

As a continuation of the aforementioned study, two of the authors later executed an experimental program to study the effect that reinforcement ratio (***ρ***) plays on a reinforced concrete member's effective moment of inertia (Al-Shaikh and Al-Zaid 1993). The experimental program was conducted by applying a mid-span concentrated load to reinforced concrete beams, of rectangular cross-section, containing varying amounts of reinforcement. The test specimen labels and reinforcement quantities were:

Reinforcement Label	**Reinforcement Ratio**
Lightly	0.8
Normally	1.4
Heavily	2.0

The study revealed that Branson's model underestimated the effective moment of inertia of all test specimens. The underestimation of Ie was approximately 30% in the case of a heavily reinforced member and 12 % for a lightly reinforced specimen. Beyond the previously observed behavior of a reinforced concrete member subjected to a mid-span concentrated load (Al-Shaikh and Al-Zaid 1993), it is obvious that reinforcement ratio affects the accuracy of Branson's model especially when the member is heavily reinforced. Therefore, by curve fitting, the authors derived an expression (Equation 6) to calculate the exponent m for use in Equation (3) which was introduced in the aforementioned study by the same authors.

$$\mathrm{m} = 3.0 - 0.8 \times \rho \tag{6}$$

where:

m = experimentally determined exponent ρ = reinforcement ratio.

The authors also applied the more general Equation (4), introduced in their earlier research, and to the values obtained from this experiment. The experimental values were used to develop Equation (7) to determine the exponent m' for Equation (4)

$$m^{*} = \beta \times \frac{M_{cr}}{M_{a}} \tag{7}$$

where: m^{*} = experimentally determined exponent and $\beta = 0.8\,\rho$

where: ρ = reinforcement ratio The use of Equation (4) may be better suited when considering the affects of reinforcement ratio on the effective moment of inertia, because the discrepancy created by load configuration is already taken into account by the cracked length term of the equation, which leaves the exponent m' dependent only on the reinforcement ratio.

3 Method (3)

In 1998, a new model was proposed by Fikry and Thomas were derived an effective moment of inertia model from basic concrete flexural response theory. Their focus was developing an effective moment of inertia model that eliminated the laborious I_{cr} calculation associated with Branson's model and more accurately accounted for variations in reinforcement ratio as well as load configuration. The derivation of the new model was based on an approximation for I_{cr}, which the authors called I_{cre}.

The authors began their derivation with a cracked, singly reinforced, rectangular cross-section concrete member. They then derived I_{cr} as a function of two variables (η and ρ) and represented it in the form of Equation (8),

$$I_{cre} = (\alpha + \beta\eta\rho) \times \left(\frac{bd^{3}}{12} \right) \tag{8}$$

where, I_{cre} = approximate moment of inertia, α = constant (given in literature), β = constant (given in literature), η = modular ratio, ρ = reinforcement ratio, b = width of member, and d = effective depth of reinforcement, this derivation achieved their first goal (eliminating the I_{cr} calculation) and the approximation was within 6% of the cracked moment of inertia of all test specimens. The cracked moment of inertia approximation was then expanded to flanged cross-sections and doubly reinforced, rectangular and flanged cross-sections.

4 Proposed Model

The deflections caused not only by changes of curvature but also by changes of shear deformations those are not always negligible, especially in the case of beams with span/depth ratio($L/H< 10$) and in the case of amore pronounced of shear forces.

The proposed model for the effective moment of inertia takes into account several effects such as (1. Type of loading, 2.(Span/depth) ratio, 3. Reinforcement ratio, 4. Ratio of (compression/tension) reinforcement, 5. (Effective depth/web width) ratio). The proposed model takes the following form:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^{(\xi+\gamma)} \times I_g + \left[(\xi+\gamma) - \left(\frac{M_{cr}}{M_a}\right)^{(\xi+\gamma)}\right] \times I_{cr} \times (\xi+\beta) \tag{9}$$

where

$$\begin{aligned} \alpha &= 5.26 - 0.525\left(\frac{d}{b_w}\right) \\ \beta &= \frac{\rho'}{\rho} \\ \eta &= \frac{I_g}{I_{cr}} \\ \gamma &= f_l - \frac{(\rho' + \alpha\rho)n}{\eta} \\ \xi &= \frac{H}{L} \\ I_g &\le I_e \le I_{cr}(\xi+\beta) \end{aligned} \tag{10}$$

The cross and cracked moment of inertia for beam with deferent cross sections as shown in figure below can be calculated as

$$\begin{aligned} I_g = {} & \frac{b_w h^3}{12} + b_w h\left(\frac{h}{2} - y_g\right)^2 + \frac{(b_1 - b_w)h_{f_1}{}^3}{12} + \left(c - \frac{h_{f_1}}{2}\right)^2 + (b_2 - b_w)h_{f_2}\left(y_g - \frac{h_{f_2}}{2}\right)^2 \\ & + \frac{(b_2 - b_w)h_{f_2}{}^3}{12} + (b_2 - b_w)h_{f_2}\left(y_2 - \frac{h_{f_2}}{2}\right)^2 \end{aligned} \tag{11}$$

$$I_{cr} = \frac{b_w c^3}{3} + \frac{(b_1 - b_w) {h_{f_1}}^3}{12} + (b_1 - b_w) h_{f_1} (c - \frac{h_{f_1}}{2})^2 + (n-1) A_{s_1} (c - d_1)^2$$

$$+ n \times A_{s_2} (d - c)^3$$

$$M_{cr} = \frac{f_r I_g}{y_g} \quad , y_2 = H - y_g \quad , c = \left[\frac{-a_1 + \sqrt{(a_1^2 + 4a_0 a_1)}}{2a_0} \right] \qquad (12)$$

$$\boldsymbol{a_0 = \frac{b_w}{2}}$$

$$\boldsymbol{a_1 = (n-1) A_{s_1} + n A_{s_2} + (b_1 - b_w) h_{f_1}}$$

$$\boldsymbol{a_2 = (n-1) A_{s_1} d_1 + n A_{s_2} d + (b_2 - b_w) h_{f_2}}$$

where,

f_l= factor depend on loading type such as:

1. Distributed load =1.25
2. Two point load =1.0
3. Concentrated load =0.75

Properties and Abilities of the Program

The computer program (EMIRCM)(Effective Moment of Inertia for Reinforced Concrete Members) is designed to deal with reinforced concrete members with many types of cross section with inclusion of transverse shear deformation effect and subjected to many types of loads. The computer program is coded in FORTRAN 90 language executed by PC Pentium IV 2800 MHz full cache Intel processor compatible computer with 2.0 GB RAM. The properties and abilities of this program may be summarized as follows:

1- Many types of loading such as (distributed loads, two point loads, and concentrated load).

2- Many types of cross section of members.

3- Using three different types of methods.

Numerical Examples

In order to verify the reliability of the adopted proposed method, some case studies reported by other researchers are utilized and compared with experimental and Branson' model, and Al-Zaid et al. model.

1 Comparison with experimental investigations of reinforced concrete members under concentrated loading at mid span

a- Reinforced concrete simply supported beam under concentrated loading (with *L*/*H*=12.5)

The accuracy of the results of the present analysis of real panels is checked through comparing with the experimental and numerical results studied by Al-Zaid et al. [1991] on simply supported reinforced concrete members and with Branson's model. The dimensions and material properties of the beams, as shown in **Figure (2).**

Figure (3) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental results obtained by Al-Zaid, et al. [1991] with difference not more than (0.5%) with the experimental investigation while the difference between the Branson's model with the experimental results more than (17%) and so the difference between Al-Zaid et al. model with the experimental results more than (20%) at ultimate load stage. The load-deflection results are listed in **Table (1).**

b- Reinforced concrete simply supported beam under concentrated loading (with *L/H*=٦ . ٥٣)

one in a series of beams tested by (Bresler and Scordelis) was also examined the beam is simply supported and subjected to a concentrated load at mid span, The dimensions and material properties of the beams as shown in **Figure (4)**.

In the present study, this beam is analyzed using the proposed method with factor for typing of loading (f_l=0.75).

Figure (5) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental obtained by Bresler and Scordelis with difference about than (12%) with the experimental investigation while the difference between the Branson's model with the experimental results more than (42%) and so the difference between Al-Zaid et al. model with the experimental results more than (47%) at ultimate load stage. The load-deflection results are listed in **Table (2).**

c- Reinforced concrete simply supported beam under concentrated loading (with *L/H*=٦ . ٥5)

one in a series of beams analyzed by **kreshna** was also examined the beam is simply supported and subjected to a concentrated load at mid span, The dimensions and material properties of the beams as shown in **Figure (6).**

Figure (7) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental obtained by **kreshna** with difference not more than (12%) with the experimental investigation. This test shows the effect of area of steel at compression region on the behavior of reinforced concrete members. The load-deflection results are listed in **Table (3).**

d- Reinforced concrete simply supported beam under concentrated loading (with *L*/*H*=7.5)

Nurnbergerova et al. tested several reinforced concrete beams with I-cross section. The dimensions and materials properties of the beams as shown in **Figure (8)**

Figure (9) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental obtained by Nurnbergerova et al. with difference not more than (4%) with the experimental investigation while the difference between the Branson's model with the experimental results more than (42%) and so the difference between Al-Zaid et al. model with the experimental results more than (44%) at ultimate load stage. The load-deflection results are listed in **Table(4).**

2 Comparison with experimental investigations of reinforced concrete members under Distributed loading

a- Reinforced concrete simply supported beam under distributed loading (with *L*/*H*=12.5)

The accuracy of the results of the present analysis of real panels is checked through comparing with the experimental and numerical results studied by **Al-Zaid et al.** [**1991**] on simply supported reinforced concrete members and with Branson's model. The dimensions and material properties of the beams, as shown in **Figure (10).**

Figure (11) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental obtained by Al-Zaid, et al. with difference not more than (8%) with the experimental investigation while the difference between the Branson's model with the experimental results more than (15%) and so the difference between Al-Zaid et al. model with the experimental results more than (6%) at ultimate load stage. The load-deflection results are listed in **Table (5).**

3 Comparison with experimental investigations of reinforced concrete members under two point loading

a- Reinforced concrete simply supported beam under two point loading (with *L*/*H*=12.5)

The accuracy of the results of the present analysis of real panels is checked through comparing with the experimental and numerical results studied by **Al-Zaid et al.** [**1991**] on simply supported reinforced concrete members and with Branson's model. The dimensions and material properties of the beams, as shown in **Figure (12).**

Figure (13) shows a comparison with the experimental and the numerical results for the deflection at mid span. The results obtained from the present study give good agreement with the experimental obtained by Al-Zaid, et al. with difference not more than (3%) with the experimental investigation while the difference between the Branson's model with the experimental results more than (3%) and so the difference between Al-Zaid et al. model with the experimental results more than (7%) at ultimate load stage. The load-deflection results are listed in **Table(6).**

4 Parametric Study

a- Effect of tension reinforcement steel ratio of flexural on the effective moment of inertia

A simply supported rectangular cross section beam subjected to concentrated loading at mid span was analyzed with a range of (ρ) from (0.5-3.0%).

Figure (14) shows the effective moment of inertia ratio-applied moment ratio curve for the reinforced concrete member with a range of steel ratio (0.5-3.0%). The following properties of the beam are (H=200 m, b=200 mm, E_c=29.634×10^6 kN/m^2, L=2500 mm,f_y=153 **MPa**,$f_{c'}$=**38.12 MPa**).

b- Effect of slenderness ratio on the effective moment of inertia

A two simply supported reinforced concrete beams subjected to concentrated loading at mid span were analyzed with a range of slenderness ratio (L/H) (5-20).

Figure (15) and **(16)** show the effective moment of inertia ratio-applied moment ratio curve for the reinforced concrete member with a range of slenderness ratio (5-20). The following properties of the rectangular cross section beam are (H=200 m, b=200 mm, E_c=29.634×10^6 kN/m^2, L=2500 mm,f_y=413 **MPa**,$f_{c'}$=**38.12 MPa**) and the properties of beam with I-section were mentioned at **Figure(7)**. From these **figures** can be noticed that the effective moment of inertia is reduced by about 27% for the range of slenderness ratio (20-5) which can be attributed to shear deformations effect which increase with decreasing of slenderness ratio where this effect was neglected by the other models.

d- Effect of compression tension reinforcement steel ratio on the effective moment of inertia

A simply supported rectangular cross section beam subjected to concentrated loading at mid span was analyzed with a range of ($A_{s'}/A_s$) from (0.0-0.5%).

Figure (17) and **(18)** shows the effective moment of inertia ratio-applied moment ratio curve for the reinforced concrete member with a range of ($A_{s'}/A_s$) from (0.0-0.5%). The following properties of the beam are (H=200 m, b=200 mm, E_c=29.634×10^6 kN/m^2, L=2500 mm, f_y=413 **MPa**, $f_{c'}$=**38.12 MPa**). From these **figures** can be noticed that the effective moment of inertia is increased by about 21% for the range of (($A_{s'}/A_s$)) (0.0-0.5) which can be attributed to compression reinforcement. The increase compression reinforcement lead to reduce the deformations of reinforced concrete members. Where this effect was neglected by the other models.

Conclusions

The research was presenting a new form of the effective moment of inertia by enhancement Branson's model taking into account the effect of several factors such as type of loading, shear deformations, reinforcement ratio. the results of the presented model were compared with (experimental results, Branson's model results, and results of other models). The results of the present model give best agreement with experimental results than Branson's and the other models. The following conclusions are drawn with regard to the results obtained for the present study such as:

1. The results showed that the effective moment of inertia reduced by about 27% for span to depth ratio of (20 to 5) due to shear deformation effects.
2. The present model gives good agreement with the experimental results for all types of cross section.

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Table (1): Load-deflection results of reinforced concrete simply supported beam under concentrated load at mid span

Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.00	0.00	0.00	0.00	0.00
9.88	1.05	1.44	1.00	1.03
11.82	1.50	2.17	1.35	1.39
13.76	1.95	2.915	1.71	1.80
15.70	2.50	3.64	2.10	2.24
17.65	3.05	4.34	2.51	2.73
19.59	3.63	5.02	2.94	3.24
21.53	4.23	5.67	3.38	3.79
23.47	4.77	6.31	3.82	4.37
25.59	5.40	6.98	4.32	5.04
27.71	6.13	7.65	4.83	5.74
31.77	7.31	8.90	5.84	7.17
35.30	8.47	9.97	6.73	8.51

Table (2): Load-deflection results of reinforced concrete simply supported beam under concentrated load at mid span

Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.0	0.00	0.00	0.00	0.00
50.6	0.60	0.49	0.49	0.51
101.2	1.40	1.59	1.24	1.41
149.5	2.25	2.55	2.04	2.48
200.1	3.20	3.48	2.90	3.78
250.7	4.20	4.40	3.76	5.23
299.0	5.50	5.26	4.60	6.73
349.6	7.20	6.17	5.49	8.41
400.2	9.20	7.06	6.37	10.20
450.8	12.40	7.96	7.26	12.07
460.0	14.20	8.13	7.42	12.41

Table (3): Load-deflection results of reinforced concrete simply supported beam under concentrated load at mid span

Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.00	0.00	0.00	0.00	0.00
44.83	0.52	0.81	0.75	0.65
89.65	1.3	2.07	1.78	1.69
134.48	2.25	3.17	2.82	2.90
179.31	3.65	4.27	3.90	4.32
224.14	5.2	5.37	4.98	5.85
260.00	6.35	6.21	5.82	7.12

Table (4): Load-deflection results of reinforced concrete simply supported beam under concentrated load at mid span

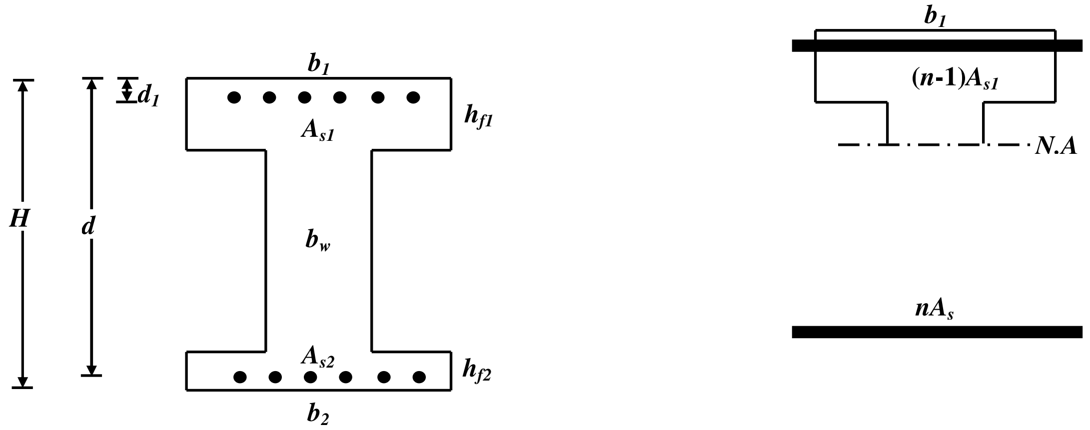
Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.0	0.00	0.00	0	0
25.0	0.40	0.39	0.35	0.38
50.0	0.85	1.04	0.85	0.96
75.0	1.40	1.60	1.37	1.66
100.0	2.10	2.16	1.91	2.45
125.0	3.00	2.70	2.44	3.29
150.0	4.10	3.25	2.98	4.21
175.0	5.10	3.80	3.53	5.18
200.0	6.10	4.35	4.07	6.19
225.0	7.25	4.89	4.61	7.24
250.0	8.50	5.43	5.15	8.33
275.0	9.50	5.16	4.87	7.79
300.0	10.70	6.52	6.23	10.63
325.0	11.7	7.07	6.77	11.82
350.0	12.70	7.61	7.32	13.04
375.0	13.85	8.15	7.86	14.29
400.0	15.00	8.69	8.40	15.56

Table (5): Load-deflection results of reinforced concrete simply supported beam under distributed load

Total Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.00	0.00	0.00	0.00	0.00
17.95	1.21	1.23	1.40	1.15
20.30	1.67	1.61	1.92	1.49
22.36	2.10	1.97	2.40	1.81
24.41	2.50	2.34	2.90	2.17
26.18	2.90	2.67	3.32	2.50
28.53	3.43	3.13	3.88	2.97
31.77	4.16	3.77	4.63	3.68
35.60	5.10	4.54	5.50	4.60
39.42	6.00	5.318	6.32	5.60
43.24	6.95	6.093	7.13	6.67
47.36	7.90	6.92	7.97	7.90
50.90	8.85	7.63	8.68	9.01
54.72	9.83	8.39	9.43	10.28
58.84	10.84	9.21	10.22	11.70

Table (6): Load-deflection results of reinforced concrete simply supported beam under two point loads

Total Load (kN)	Deflection (mm)			
	Experimental results	Branson method	Al-Zaid,et al. method	Present Study
0.00	0.00	0.00	0.00	0.00
11.73	0.91	0.95	0.89	0.88
14.21	1.48	1.62	1.39	1.28
15.79	1.87	2.10	1.74	1.55
18.04	2.45	2.81	2.27	2.00
19.85	2.91	3.38	2.73	2.39
21.65	3.43	3.96	3.20	2.80
25.48	4.51	5.14	4.21	3.79
29.54	5.55	6.32	5.31	4.96
33.38	6.63	7.39	6.35	6.18
37.21	7.68	8.42	7.37	7.50
41.27	8.89	9.49	8.46	9.00
45.11	10.18	10.48	9.47	10.50



(a) uncracked cross section

(b) Transformed cracked section

Figure (1): Details of uncracked and cracked cross section

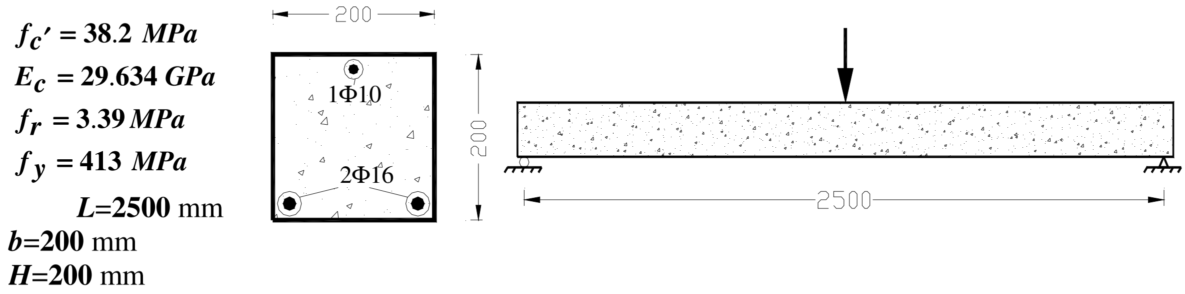


Figure (2): Details of a reinforced concrete simply supported beam under concentrated load at mid span with (L/H=12.5)(Al-Zaid, et al.)

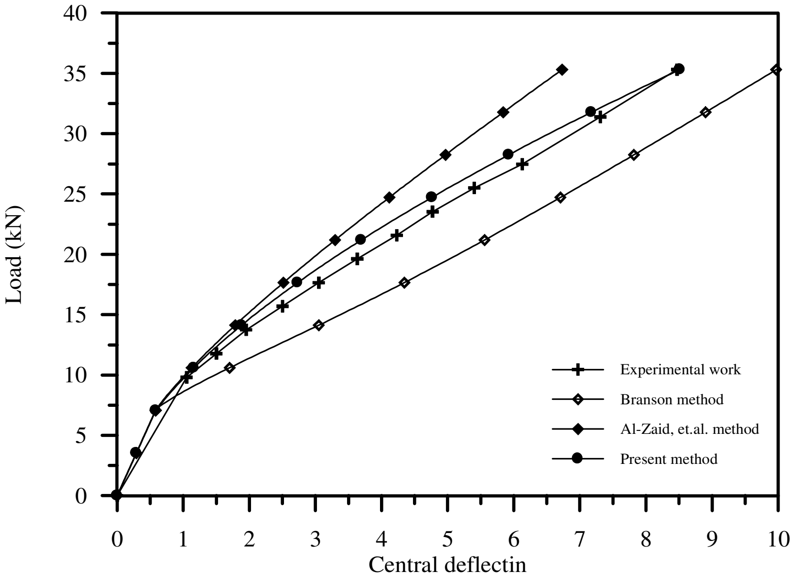


Figure (3): Load-deflection curve of reinforced concrete simply supported beam under concentrated load at mid span (Al-Zaid, et al.)

$f_c' = 24.1\ MPa$

$E_c = 23.037\ GPa$

$f_r = 3.04\ MPa$

$f_y = 345\ MPa$

L=**3660** mm

b=**305** mm

H=**560** mm

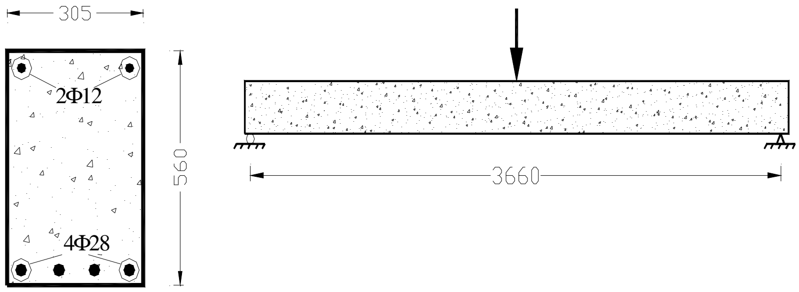


Figure (4): Details of a reinforced concrete simply supported beam under concentrated load at mid span with (*L/H*=6.53)(Bresler and Scordelis)

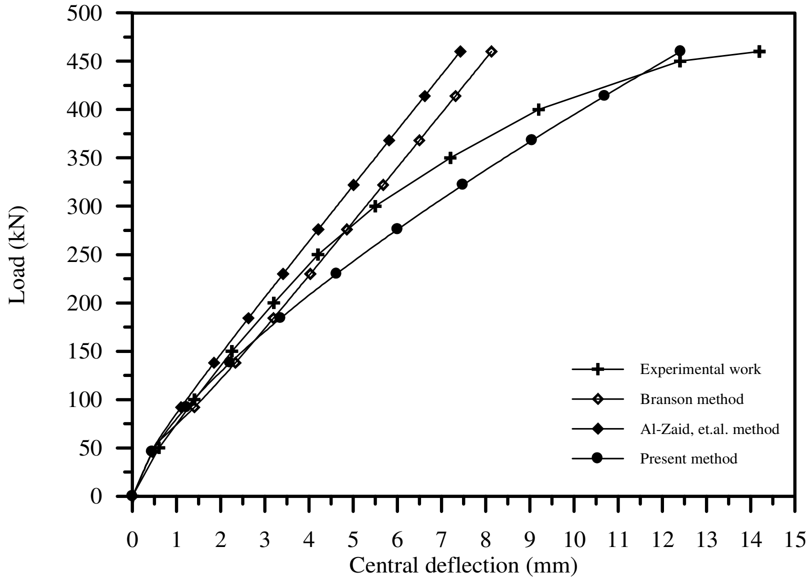


Figure (5): Load-deflection curve of reinforced concrete simply supported beam under concentrated load at mid span (Bresler and Scordelis)

$f_c' = 24.1\ MPa$

$E_c = 23.037\ GPa$

$f_r = 3.00\ MPa$

$f_y = 345\ MPa$

L=**3657** mm

b=**225** mm

H=**558** mm

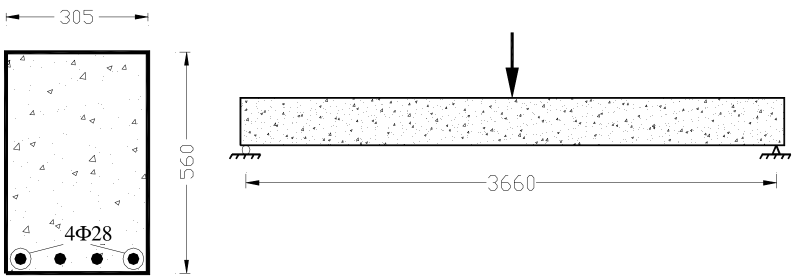


Figure (6): Details of a reinforced concrete simply supported beam under concentrated load at mid span with (*L/H*=6.55)(kreshna)

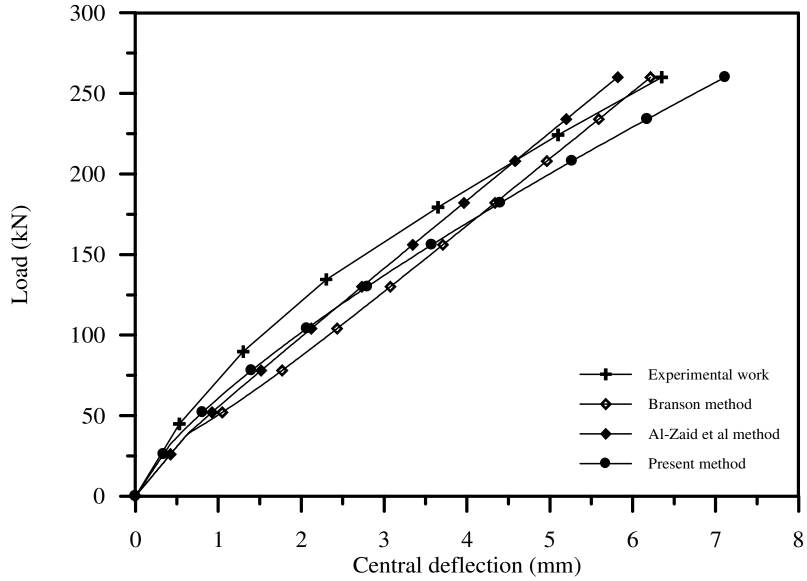


Figure (7): Load-deflection curve of reinforced concrete simply supported beam under concentrated load at mid span (kreshna)

$f_c' = 24.1$ *MPa*
$E_c = 37.85$ *GPa*
$f_r = 2.26$ *MPa*
$f_y = 345$ *MPa*
***L*=3660** mm
***b*=305** mm
***H*=560** mm

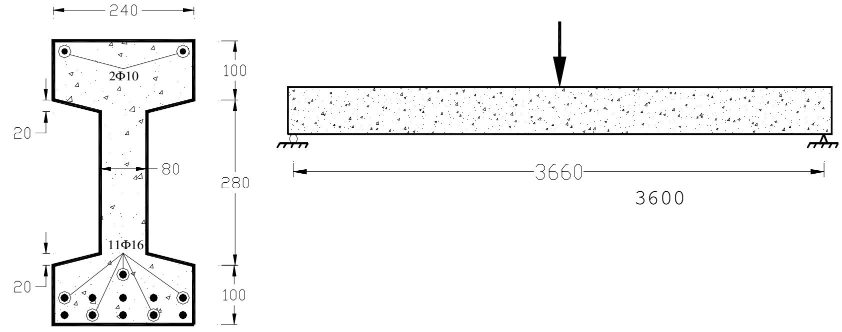


Figure (8): Details of a reinforced concrete simply supported beam under concentrated load at mid span with (*L/H*=7.5)(Nurnbergerova et al.)

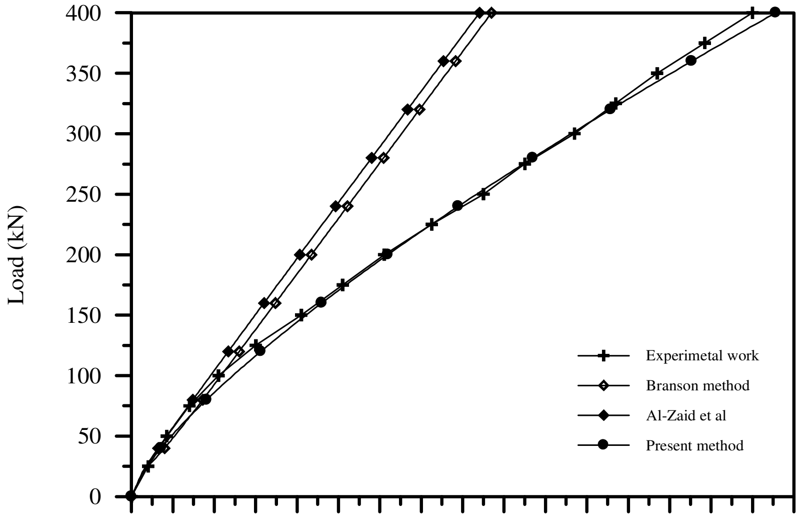


Figure (9): Load-deflection curve of reinforced concrete simply supported beam under concentrated load at mid span (Nurnbergerova, et al.)

$f_c' = 38.2\ MPa$

$E_c = 29.634\ GPa$

$f_r = 3.39\ MPa$

$f_y = 413\ MPa$

L=2500 mm

b=200 mm

H=200 mm

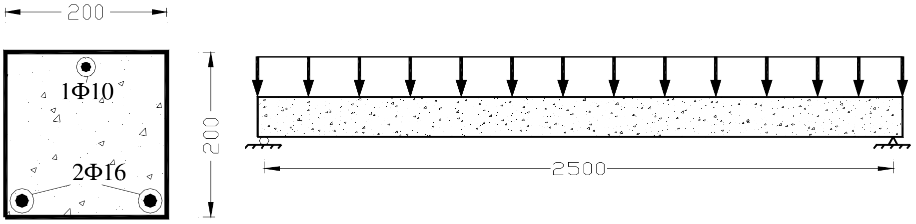


Figure (10): Details of a reinforced concrete simply supported beam under distributed load with (*L/H*=12.5)(Al-Zaid, et al.)

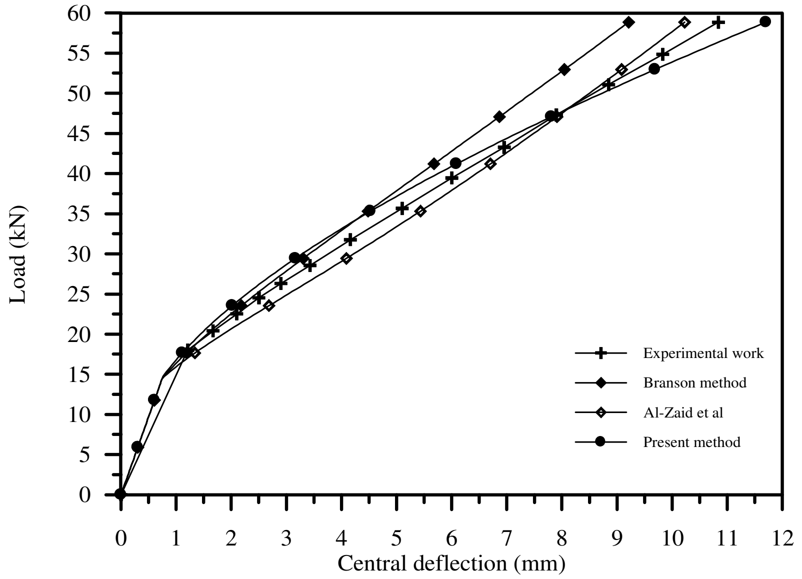


Figure (11): Load-deflection curve of reinforced concrete simply supported beam under distributed load (Al-Zaid et al.)

$f_c' = 38.2\ MPa$

$E_c = 29.634\ GPa$

$f_r = 3.39\ MPa$

$f_y = 413\ MPa$

L=2500 mm

b=200 mm

H=200 mm

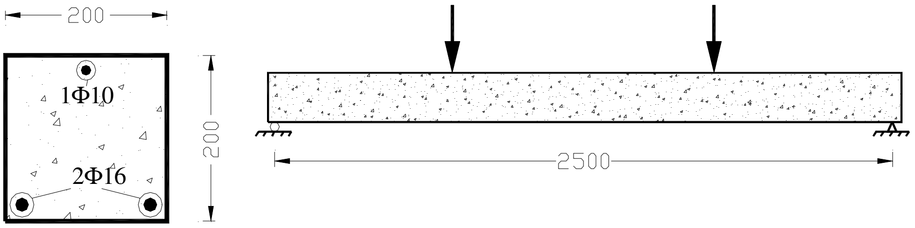


Figure (12): Details of a reinforced concrete simply supported beam under two point load with (*L/H*=12.5)(Al-Zaid, et al.)

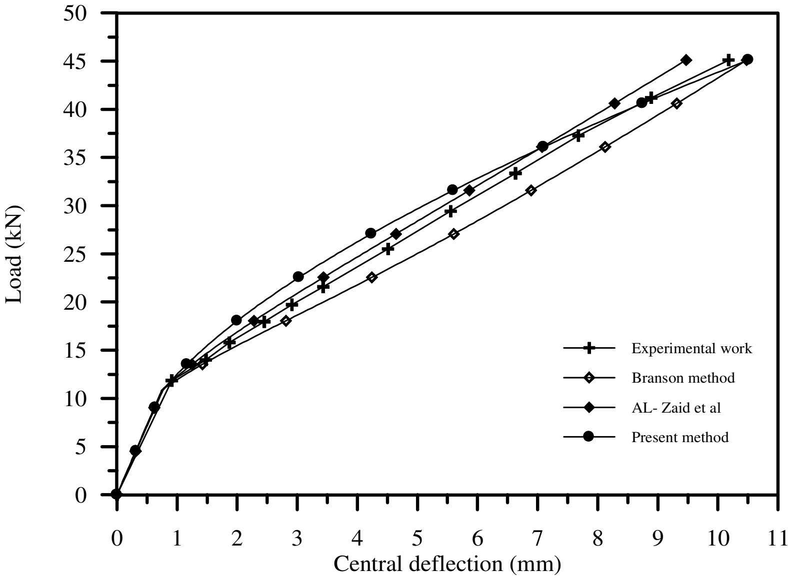


Figure (13): Load-deflection curve of reinforced concrete simply supported beam under two point load (Al-Zaid et al.)

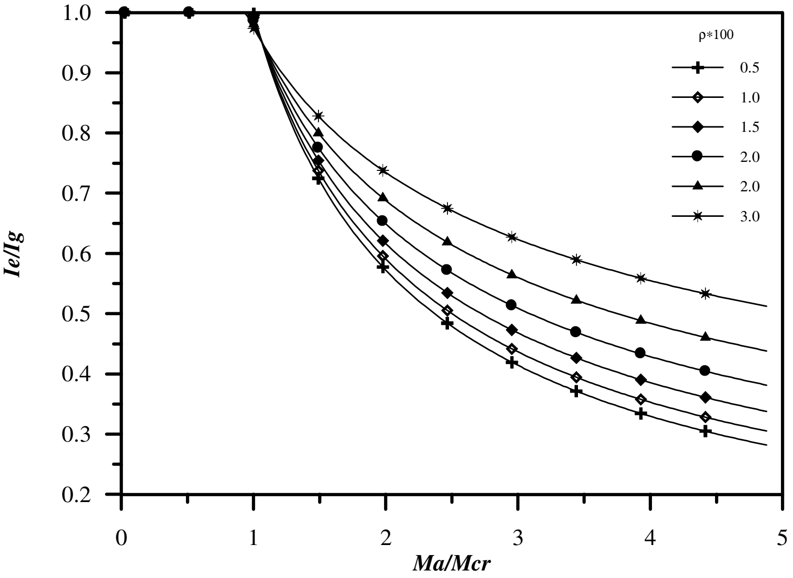


Figure (14): Effect of reinforcement steel ratio of flexural on the effective moment of inertia of simply supported concrete beam under concentrated load by present study

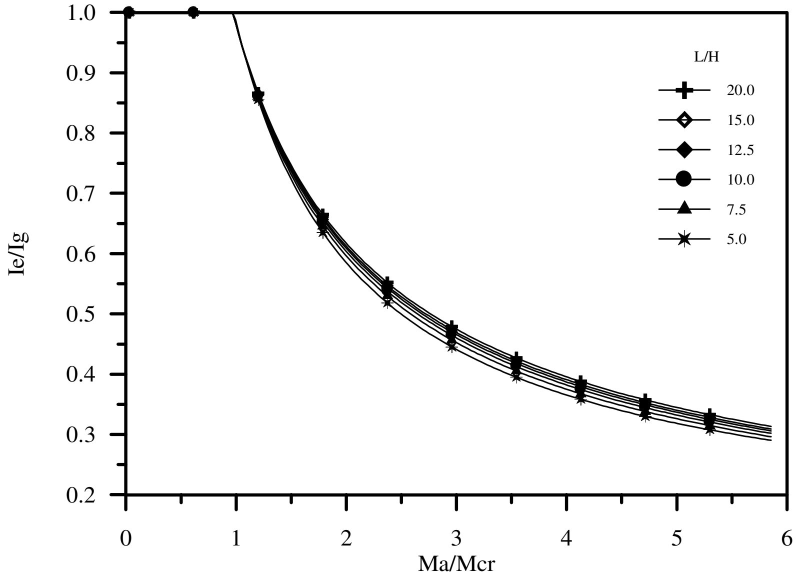


Figure (15): Effect of (span/depth) ratio of simply supported concrete beam under concentrated load (rectangular section)

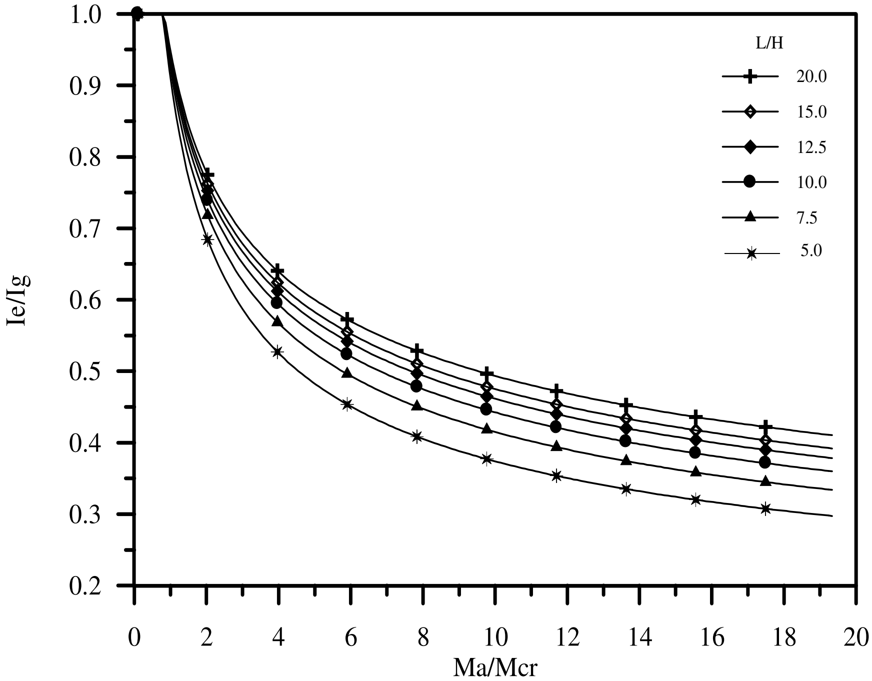


Figure (16): Effect of (span/depth) ratio of simply supported concrete beam under concentrated load (I-section)

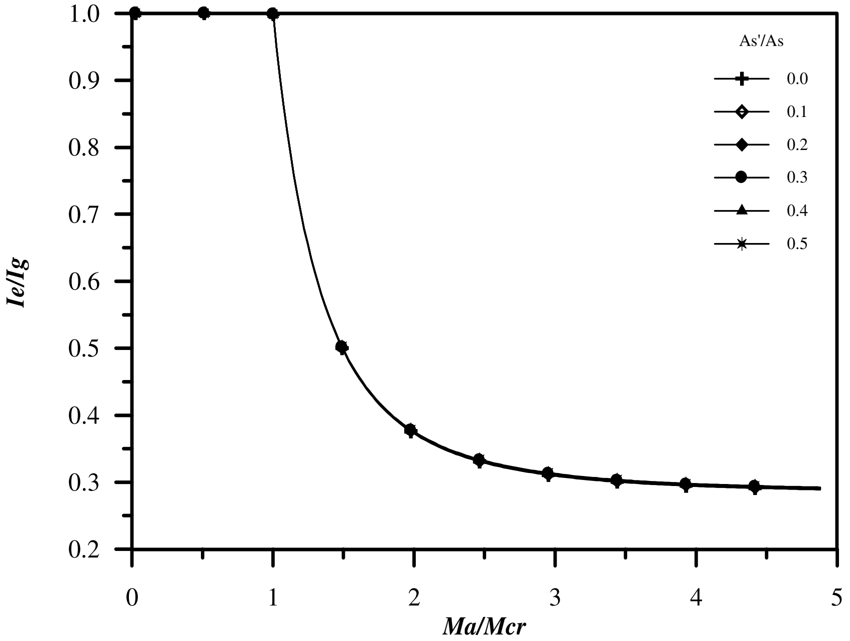


Figure (1٧): Effect of compression reinforcement steel ratio on the effective moment of inertia of simply supported concrete beam under concentrated load by Branson method

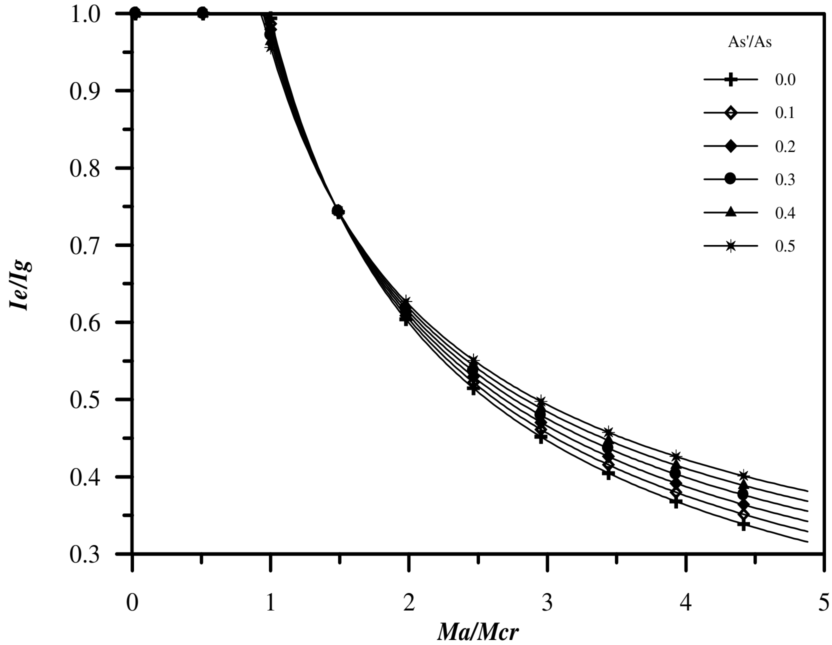


Figure (18): Effect of compression reinforcement steel ratio on the effective moment of inertia of simply supported concrete beam under concentrated load by present study