

Genetic Algorithm Optimization of Gear Teeth Numbers for Six-Velocity Lepelletier Automatic Transmission

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ABSTRACT

This paper concentrates on the application of genetic algorithm optimization technique to find the gear teeth numbers for six-velocity Lepelletier automatic transmission with given approximate velocity ratios and a set of design constraints. MATLAB is employed to find the gear teeth numbers and the velocity ratios satisfying the design and geometric constraints.

Keywords: Automatic transmission, Epicyclic gear mechanism, Genetic algorithm, Lepelletier, Nomographs, Optimization, Ravigneaux, Velocity ratios

استخدام الخوارزمية الجينية لإيجاد عدد الاسنان الامثل لناقل حركة نوع Lepelleti ذو ست سرع

الخلاصة

تركز الدراسة الحالية على تطبيق تقنية الامثلة باستخدام الخوارزمية الجينية للحصول على اعداد الاسنان المثلى لتروس ناقل حركة من نوع ليبيلنديا Lepelletier ذو الستة وذلك بمعطيات تقريبية لنسب السرعة ومجموعة محددات التصميم. تم استخدام برنامج ماتلاب MATLAB لإيجاد عدد اسنان ناقل الحركة ونسب السرعة التي تلبي التصميم والقيود الهندسية.

INTRODUCTION

To achieve a set of desired velocity ratios, most automatic transmission mechanisms employ epicyclic gear trains (EGTs). The velocity ratio can define as the ratio of the velocity of the input link to the output link of a transmission mechanism. An epicyclic gear mechanism (EGM) employing a ten-link Lepelletier gear train as an automatic transmission is shown in **Figure (1)**.

Depending on the clutching condition, seven drives called first under-drive, second under-drive, third under-drive, fourth under-drive, first over-drive, second over-drive and reverse-drive are feasible. These seven clutching conditions are shown in **Table 1**, where an X indicates that the corresponding clutch C_i or brake B_i is activated on the i^{th} link of the gear train.

Lepelletier EGT can be analyzed as a compound of two FGEs. One FGE is a single-planet simple gear train and the second FGE is a double-planet Ravigneaux gear train. **Figures 2** shows a schematic drawing of these gear trains.

Simionescu et al. [2] proposed an optimization method to synthesis the gear-teeth number for a two under drive Ravigneaux type automatic transmission with a direct drive and a reverse. All possible assembly and interference avoidance requirements were considered as constraints to form an optimization problem. It was then solved with the aid of an estimation of distribution algorithm. Esmail [3-5] proposed an optimization technique to synthesis gear-teeth numbers of epicyclic gear trains. Conventionally available automatic transmissions are used to demonstrate the methodology. The proposed technique enables the designer to synthesize the number of teeth of all gears and satisfying all the design constraints in a single run. Hwang and Huang [6] proposed a methodology for the design of six-speed automatic transmissions. Six configurations of six-speed automatic transmissions were synthesized from the eight-link two-DOF Ravigneaux gear mechanism. Unfortunately, they did not include the included angle of planet gears into their consideration, resulting in an impractical and infeasible design. Hsu [7] presented an analytic method for the synthesis of the number of teeth of gears for an epicyclic gear mechanism with the clutching sequence table and the speed ratios. This approach is far more successful with simple gear trains having no design constraints. Hsu and Huang [8] synthesized the number of teeth of all gears of six speed Ravigneaux-type automatic transmissions by assigning three (out of seven) desired speed ratios in the analytic method proposed by Hsu [7]. Hsu's method becomes a trial and error method when applied to the six-speed Ravigneaux-type automatic transmission; a method of reaching satisfactory results by trying out certain desired speed ratios until other speed ratios and the design constraints are satisfied. It is lengthy and tedious. Hsu and Huang [8] concluded that the Ravigneaux gear mechanism could reach six forward speeds at most. An artificial neural network and a genetic algorithm are used by Shamekhi et al. [9] for the optimization of the gear ratios and gear teeth numbers of Simpson gear train with an error less than $\pm 0.3\%$.

DESIGN CONSTRAINTS

For the design to be feasible, the following requirements must be met:

1. Considering the driving performance of an automatic transmission, the absolute value of the velocity ratio of the reverse velocity R_{RD} should be at least between the first and the second under drive velocity ratios.

$$R_{UD2} \leq |R_{RD}| \leq R_{UD1} \tag{1}$$

2. The velocity ratio steps for forward velocities should be greater than 1.275.

$$1.275 \leq \frac{R_k}{R_{k+1}} \tag{2}$$

3. The difference in adjacent velocity ratio steps should be less than (± 0.25).

$$\left| \frac{R_k}{R_{k+1}} - \frac{R_{k+1}}{R_{k+2}} \right| \leq 0.25 \tag{3}$$

4. The highest overdrive velocity ratio must equal to or greater than 0.5.

$$0.5 \leq R_{OD \text{ highest}} \quad (4)$$

5. In order to facilitate load sharing and to make sure that there are four sets of combinations of planet gears arranged 90° apart from one another, the included angle of planet gears should be at least less than 84° . A space (S) of 6° is left to avoid contact between neighboring planet gear sets. Obviously, since the number of sets of combinations of planet gears (PGS) depend on the included angle, it can be written as

$$\theta^\circ - \frac{360^\circ}{PGS} + S \leq 0 \quad (5)$$

and

6. A practical gear train can't have too large or too small gear sizes. To avoid undercut, the minimum number of teeth on the planet gears is limited to 15. Considering the geometry relations of the Lepelletier gear mechanism, gear teeth number of ring gear-teeth should be 85 or less.

$$15 \leq Z_p \text{ for } p = 1, \dots, 8 \leq 85 \quad (6)$$

OPTIMIZATION OF GEAR RATIOS FOR EGTs

The optimization of gear teeth numbers of automotive automatic transmissions is a multi-objective task. Due to conflicts among objectives, it is impossible to obtain a single design that corresponds to optima of all the objectives. The optimization of gear teeth numbers can be solved using single-objective GA optimization method if all but one objective are converted into constraints. A genetic algorithm (GA) is a search and optimization method which works by mimicking the evolutionary principles and chromosomal processing in natural genetics.

To achieve a specific set of velocity ratios the designer has to choose a gear train, a set of clutches that are to be operated in a chosen sequence, and a set of gears that have a specific number of teeth. In practice, one wants to choose a set of gears to achieve a set of velocity ratios, i.e., the design variables are the gear teeth numbers. In the particular case of Lepelletier gear train, the design variables are eight ($Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7,$ and Z_8) and can only take integer values.

In what follows, a genetic algorithm MATLAB optimization method is proposed to complete this design problem. By assigning numerical values to the approximate velocity ratios, using the mechanism kinematic and geometric constraints and based on a set of design variables, MATLAB optimization can generate the possible gear ratios and their associated gear-teeth numbers in a single run (see Appendix).

FORMULATION OF THE OPTIMIZATION

Traditionally, the velocity ratio $R_{x,y}^z$ is used to study the velocity between links x and y with reference to link z where x, y and z are any three links in the EGT [10]. A methodology for expressing the overall velocity ratio of an EGM in terms of its FGEs was recently developed by Esmail [10]. In his approach, the concept of virtual planet gear ratio is applied for the kinematic analysis of EGMs. The EGM is decomposed into several EGTEs; one of them is considered as the primary EGTE. Then the virtual

planet gear ratios associated with various EGTEs are investigated. They are found in terms of the planet gear of the basic EGTE. This way, the velocity ratio of an EGM can be symbolically expressed in terms of the teeth number of each gear. The velocity ratio is written in terms of the planet gear ratios as

$$R_{x,y}^z = \frac{N_{p,x} - N_{p,z}}{N_{p,y} - N_{p,z}} \tag{7}$$

The term "planet gear ratio" refers to the ratio of the number of teeth on two meshing gears. It is defined by the ratio of a planet gear p with respect to a sun or ring gear x , where Z_p and Z_x denote the numbers of teeth on the planet and the sun or ring gear, respectively, and the positive or negative sign depends on whether x is a ring or sun gear.

$$N_{p,x} = \pm Z_p / Z_x \tag{7a}$$

The virtual planet gear ratio is defined as the planet gear ratio measured, with respect to the primary epicyclic gear train entity (EGTE), in an epicyclic gear mechanism.

$$\mathbf{N}_{p,u} = N_{p,b_1} + R_{u,b_2}^{b_1} \cdot (N_{p,b_2} - N_{p,b_1}) \tag{7b}$$

where N_{p,b_1} and N_{p,b_2} are associated with the basic EGTE and $R_{u,b_2}^{b_1}$ is associated with the secondary EGTE. Therefore, a link may have more than one virtual planet gear ratio depending on the connecting links that connect the primary EGTE to the EGTE to which the virtual link belongs. The virtual planet gear ratio is written in bold to differentiate it from the actual planet gear ratio which is written in *italic*.

The velocity ratio between links x and y with respect to a third link z may have more than one value depending on the connecting links that connect the primary EGTE to the EGTE to which a virtual link belongs.

OPTIMIZATION TECHNIQUE AND PROBLEM

A fitness function $F(Z)$ is defined in terms of the velocity ratios, R_k , for the gear train. The fitness function is the sum of squared residuals, a residual being the difference between the desired velocity ratio and the actual velocity ratio. The fitness function also ensures that a best set of gear teeth numbers is achieved that would give the desired velocity ratios without violating any of the design and geometric constraints. For minimizing the error between desired and optimized velocity ratios, the fitness function is written as

$$F(Z) = \sum_{k=1}^n (R_k - K_{kd})^2 \tag{8}$$

Where $F(Z)$ is the fitness function, Z is the vector of design variables (Z_1, Z_2, \dots, Z_m), n is the number of velocity ratios, R_k is the k^{th} optimized velocity ratio, K_{kd} is the k^{th} desired velocity ratio. The velocity ratios $R_1, R_2 \dots R_n$ depend on the topology of the gear train and the clutching sequence. The constraints under which the function is minimized will depend on the gear train being considered. The fitness function is subjected to the following constraints

$$g_i(Z) \leq 0 \quad \text{for } i = 1, \dots, v \tag{9}$$

$$h_j(Z) = 0 \quad \text{for } j = 1, \dots, w \tag{10}$$

$$\text{with } Z_p \in [l_p, u_p] \quad \text{for } p = 1, \dots, m \tag{11}$$

Where g_i and h_j represent inequality and equality constraints, l_p and u_p are the upper and lower bounds on the m^{th} design variable respectively.

FORMULATION FOR LEPELLETIER GEAR TRAIN

Lepelletier EGT can be analyzed as a compound of two FGEs. One FGE is a single-planet simple gear train and the second FGE is a double-planet Ravigneaux gear train. The clutching sequence, which includes the brake clutches, is assumed to be the same as those mentioned in Table 1. To relate the velocity ratios to the gear teeth numbers, the system of Eqs. (7) was solved seven times, each time for a different clutching condition. Some velocity ratios also have the same labeling, are different in value, depending on the common links between the EGTEs. Under certain clutching condition the Ravigneaux part of the gear train may act as a rigid body, in such a case the overall velocity ratio will be equal to the velocity ratio of the simple part of the transmission. The under-drives can be written as follows:

$$R_{UD1} = R_{3^1,4}^3 \text{ for common links } S_{3,8}^{2^1,2} = \frac{Z_4}{Z_2} \left(1 + \frac{Z_8}{Z_{3^1}} \right) \tag{12}$$

$$R_{UD2} = R_{3^1,4}^1 \text{ for common links } S_{1,8}^{2^1,2} = \frac{\left(1 + \frac{Z_8}{Z_{3^1}} \right) \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)}{\left(\frac{1}{Z_1} + \frac{1}{Z_4} \right)} \tag{13}$$

$$R_{UD3} = R_{3^1,2^1}^8 \text{ for common link } 2^1, 1, 2 = \left(1 + \frac{Z_8}{Z_{3^1}} \right) \tag{14}$$

$$R_{UD4} = R_{3^1,4}^8 \text{ for common links } S_{3^1,3}^{2^1,2} = \frac{1}{1 - \frac{Z_2 \cdot Z_8}{Z_4(Z_{3^1} + Z_8)}} \tag{15}$$

The over-drives are written as follows:

$$R_{OD1} = R_{3^1,4}^8 \text{ for common links } S_{3^1,3}^{2^1,1} = \frac{1}{1 + \frac{Z_1 \cdot Z_8}{Z_4(Z_{3^1} + Z_8)}} \tag{16}$$

$$R_{OD2} = R_{3^1,4}^1 \text{ for common links } S_{3^1,3}^{1,8} = \left(\frac{Z_4}{Z_1 + Z_4} \right) \tag{17}$$

The reverse-drive is written as

$$R_{RD} = R_{3^1,4}^3 \text{ for common links } S_{3,8}^{2^1,1} = -\frac{Z_4}{Z_1} \left(1 + \frac{Z_8}{Z_{3^1}} \right) \tag{18}$$

It should be realized that for an arbitrary combination of desired velocity ratios K_{kd} there may not be a set of solutions for the gear teeth numbers Z_m that satisfy Eqs. (12) to (18). In other words, the ideal minimum value of the fitness function ($F(Z) \rightarrow 0$) may or may not be achievable. The constraints under which the function $F(Z)$ is minimized for the Lepelletier gear train are now described.

Design constraints for Lepelletier gear train

As previously stated, R_{RD} and R_{UD1} are the reverse- and first under drive velocity ratios, respectively. Examining the first design constraint, it is convenient to rewrite it in the form

$$|R_{RD}| \leq R_{UD1} \tag{19}$$

Substituting equation (12) and (18) into equation (19) and dividing through by $Z_4 \left(1 + \frac{Z_8}{Z_{31}}\right)$ yields

$$\frac{1}{Z_1} \leq \frac{1}{Z_2} \tag{20}$$

This implies that for $|R_{UD1}|$ to be greater than $|R_{RD}|$ as required by the first design constraint, Z_2 must be smaller than Z_1 or

$$Z_2 - Z_1 \leq 0 \tag{21}$$

For the Lepelletier gear train, the velocity ratio steps for forward velocities should be greater than 1.275. Thus, the second design constraint can be written as

$$1.275 - \frac{R_{UD1}}{R_{UD2}} \leq 0 \tag{22}$$

$$1.275 - \frac{R_{UD2}}{R_{UD3}} \leq 0 \tag{23}$$

$$1.275 - \frac{R_{UD3}}{R_{UD4}} \leq 0 \tag{24}$$

$$1.275 - \frac{R_{UD4}}{R_{OD1}} \leq 0 \tag{25}$$

$$1.275 - \frac{R_{OD1}}{R_{OD2}} \leq 0 \tag{26}$$

The third design constraint is

$$\left| \frac{R_k}{R_{k+1}} - \frac{R_{k+1}}{R_{k+2}} \right| \leq 0.25 \tag{27}$$

As required by the fourth design constraint, the highest overdrive ratio is equal to or greater than 0.5. Rewriting the third design constraint in terms of the variables used in the governing equations,

$$0.5 \leq R_{OD2} = \left(\frac{Z_4}{Z_1 + Z_4} \right) \quad \text{and} \quad Z_1 \leq Z_4 \quad (28)$$

Prove

$$\begin{aligned} Z_1 &\leq Z_4 \\ Z_1 + Z_4 &\leq Z_4 + Z_4 \\ Z_1 + Z_4 &\leq 2 \times Z_4 \\ \frac{1}{2} &\leq \frac{Z_4}{Z_1 + Z_4} \\ 0.5 - \frac{Z_4}{Z_1 + Z_4} &\leq 0 \end{aligned}$$

Since the ring gear is always larger than the sun gear, then equation (28) implies that the second over drive velocity ratio R_{OD2} is always greater than the 0.5 value required by the third design constraint.

GEOMETRIC CONSTRAINTS

We can get the geometric constraints for the Lepelletier gear train from Figs. 1 and 2. The sum of the number of teeth of the sun gear and twice the number of teeth on the planet gear should equal the number of teeth on the ring gear.

Let d_m be the diameter of a gear element m , then $d_4 + 2d_6 = d_1$. If the diametral pitch P of all the gears is the same, then $d_m = P \cdot Z_m$ and $Z_4 + 2Z_6 = Z_1$

Also $d_3 + 2d_7 = d_8$ or $Z_3 + 2Z_7 = Z_8$.

By taking account of the presence of the planet gears between the sun and ring gears, it is obvious that d_{sun} should be smaller than d_{ring} , which can be represented equivalently as $Z_1 - Z_4 \leq 0$. Similarly, $Z_8 - Z_3 \leq 0$.

For planet gear 5 to be out of line with planet gear 6, the diameter of planet gear 5 should be at least as large as the difference between the radii of the large and small sun gears, which can be equivalently represented as

$$Z_1 - Z_2 - 2 \times Z_5 \leq 0 \quad (29)$$

In order to facilitate load sharing and to make sure that there are four sets of combinations of planet gears arranged 90° apart from one another, the included angle of planet gears should be smaller than 90° . The included angle of planet gears 5 and 6, shown in **Fig. 3**, is used as a design constraint in the optimization problem for its value to lie within the limit.

By applying the cosine law for triangle BAC and using the sine for triangles BAE and CAD, we get:

$$\theta = \cos^{-1} \left[\frac{(Z_2 + Z_5)^2 + (Z_1 + Z_6)^2 - (Z_5 + Z_6)^2}{2 \times (Z_2 + Z_5)(Z_1 + Z_6)} \right] + \sin^{-1} \frac{Z_5}{Z_2 + Z_5} + \sin^{-1} \frac{Z_6}{Z_1 + Z_6} \quad (30)$$

The first term in equation (30) must satisfy the following condition to give reliable values

$$\left[\frac{(Z_2 + Z_5)^2 + (Z_1 + Z_6)^2 - (Z_5 + Z_6)^2}{2 \times (Z_2 + Z_5)(Z_1 + Z_6)} \right] \leq 1 \quad (31)$$

Obviously, since the number of sets of combinations of planet gears (PGS) depend on the included angle, it would be advantageous to select an angle that would place all the sets in the range between θ_{min} and θ_{max} .

$$\theta^\circ - \frac{360^\circ}{4} + 6^\circ \leq 0 \quad (32)$$

$$\frac{360^\circ}{4} - 8^\circ - \theta^\circ \leq 0 \quad (33)$$

OPTIMIZATION RESULTS

The designer specifies the gear train, the clutching sequence and the design constraints on the velocity ratios to be achieved. The optimization problem starts at an arbitrary velocity ratios and at the end of its execution displays the optimized gear teeth numbers. Analytical or trial and error approaches for finding the gear teeth numbers of an automotive automatic transmission to satisfy the design constraints has been eliminated by formulating the problem as a GA optimization problem. The implementation of genetic algorithm was performed in MATLAB. The stopping condition is the number of generations. Starting with a population size of 200, number of generations as 1000, probability of crossover as 0.8 and probability of mutation as 0.2, the code was executed.

The optimization procedure was successfully applied to find those teeth numbers of the gear train for which the constraint are satisfying the original specification. The Optimized gear teeth numbers shown in **Table 2** satisfy the geometric and kinematic constraints given by Eqs. (19) to (33).

Carrying out a search to obtain the sets of optimized gear teeth numbers, yields the values shown in **Table 2**. In fact, GA provided different results from run to run with the same objective value of 1.89528×10^{-7} . The fundamental constraints regarding the included angle, reverse velocity ratio, velocity ratio steps, and the difference in velocity ratio steps are all satisfied. For example, the values for the velocity ratio steps are 1.7500, 1.5668, 1.3229, 1.3254, and 1.2751 and for the difference in velocity ratio steps are 0.1832, 0.2440, 0.0026, and 0.0503. The solutions found by the genetic algorithm are as good as the best solution in the literature.

Figure 4 shows a MATLAB plot to the optimization results for the gear teeth numbers given in the fifth row of **Table 2**.

By applying explicit enumeration of all possible $(85-15)^8 = 576480.1 \times 10^9$ gear teeth combinations, the GA solutions are found to be globally optimal. A single globally optimal solution is not always

sufficient when multiple global optima exist. One reason is that the design at this stage is far from complete and further steps in the design's analysis are needed to determine the teeth numbers based on the transmitted power. The existence of multiple global optima gives the designer the ability to quickly select a gear teeth combination, depending on how the gear train is intended to react to both the power requirements of the gear train and the velocity response of the planetary train.

The gear teeth combination shown in the ninth row of **Table 2** is identical to that reported in Lepelletier patent [1]. However, it seems that the included angle of planet gears is not considered in that work. Since the associated included angle is 90.0634° , then three sets of combinations of planet gears arranged 120° apart from one another are used.

It can be seen that the effect of gear teeth numbers on the included angle is always through planet gear 5. Obviously, the designer could select a high included angle, but this would fail the fifth design constraint, requiring that the included angle is only slightly smaller than 84° . Selecting any one of the gear teeth combination shown in the first to the fifth rows of **Table 2**, ensures that there will be four planet gear sets arranged 90° apart from one another.

The approach used to find the gear teeth numbers must be weighed against the changes that may have to be made in the velocity ratios during the optimization process. Approximate velocity ratios that are assumed in the beginning of the optimization process, may have to be modified with respect to the optimized values of the gear teeth numbers. The modification depends on how the gear train is intended to react to the fundamental constraints regarding the included angle, reverse velocity ratio, velocity ratio steps, and the difference in velocity ratio steps. Hence, an iterative procedure between the values achieved by the optimization process and those required to satisfy the design constraints may have to be done.

References

- [1] P. A. G. Lepelletier, "Multispeed automatic transmission for automobile vehicles," US patent no. 106352, 1992.
- [2] Simionescu PA, Beale D, Dozier GV (2006) Teeth-number synthesis of a multispeed planetary transmission using an estimation of distribution algorithm. *ASME J Mech Des* 128(1):108–115. doi:10.1115/1.2114867
- [3] Esmail EL, Salih KH (2011) Synthesis and optimization of epicyclic-type automatic transmissions based on nomographs. *Al-Qadisiya Journal Engineering Science*, 4(3):168–185.
- [4] Esmail, E. L. , 2013, "Nomographs and Feasibility Graphs for Enumeration of Ravigneaux-Type Automatic Transmissions ", *Advances in Mechanical Engineering Hindawi, Publishing Corporation*, Volume 2013 (2013), Article ID 120324, 15 pages
<http://dx.doi.org/10.1155/2013/120324>.
- [5] Esmail, E. L. , 2013, "Nomographs for synthesis of epicyclic-type automatic transmissions", *Meccanica*, October 2013, Volume 48, Issue 8, pp 2037-2049. DOI 10.1007/s11012-013-9721-z.
- [6] Hwang WM, Huang YL (2005) Configuration design of six-speed automatic transmissions with two-degree-of freedom planetary gear trains. *Trans Can Soc Mech Eng* 29(1):41–55.

[7] Hsu CH (2002) An analytic methodology for the kinematic synthesis of epicyclic gear mechanisms. ASME JMech Des 124:574–576.

[8] Hsu CH, Huang RH (2009) Systematic design of six speed automatic transmissions with an eight-link two-DOF Ravigneaux gear mechanism. ASME J Mech Des. 131:011004-1–011004-8

[9] Amir H. Shamekhi, Abbas Bidgoly, Ebrahim N. Noureiny, 2014, Optimization of the gear ratios in automatic transmission systems using an artificial neural network and a genetic algorithm, Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering 0954407014528887.

[10]Esmail, E.L., 2007, "Kinematic Nomographs of Epicyclic-Type Transmission Mechanisms", Emirates Journal for Engineering Research, volume 12, issue 3, pp. 47-55.

Table (1): Clutching sequence for Six-velocity Lepelletier automatic transmission [1].

	C ₁	C ₂	C ₃	B ₁	B ₃	Velocity ratio
UD ₁		X			X	4.135
UD ₂		X		X		2.363
UD ₃	X	X				1.508
UD ₄		X	X			1.140
OD ₁	X		X			0.860
OD ₂			X	X		0.675
RD	X				X	-3.127

Table (2): Gear teeth-numbers of the six-velocity Lepelletier gear train for different values of included angle with the same objective value of 1.89528×10^{-7} ..

Achieved gear-teeth numbers								Included angle (in degrees)
Z ₁	Z ₂	Z ₃	Z ₄	Z ₅	Z ₆	Z ₇	Z ₈	
41	31	61	85	15	22	15	31	75.0147
41	31	61	85	16	22	15	31	77.2695
41	31	61	85	17	22	15	31	79.3944
41	31	61	85	18	22	15	31	81.4034
41	31	61	85	19	22	15	31	83.3083
41	31	61	85	20	22	15	31	85.1188
41	31	61	85	21	22	15	31	86.8435
41	31	61	85	22	22	15	31	88.4896
41	31	61	85	23	22	15	31	90.0634
41	31	61	85	24	22	15	31	91.5704
41	31	61	85	25	22	15	31	93.0157
41	31	61	85	26	22	15	31	94.4036
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41	31	61	85	33	22	15	31	102.7958°
41	31	61	85	35	22	15	31	104.8470

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41	31	61	85	51	22	15	31	117.6214
41	31	61	85	53	22	15	31	118.8818
41	31	61	85	54	22	15	31	119.4906
41	31	61	85	55	22	15	31	120.0860

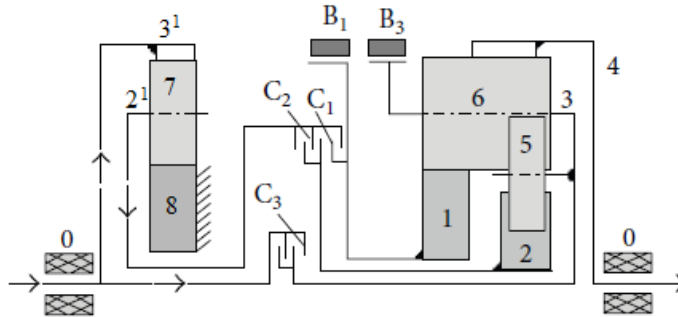


Figure (1): Six-velocity Lepelletier automatic transmission [1].

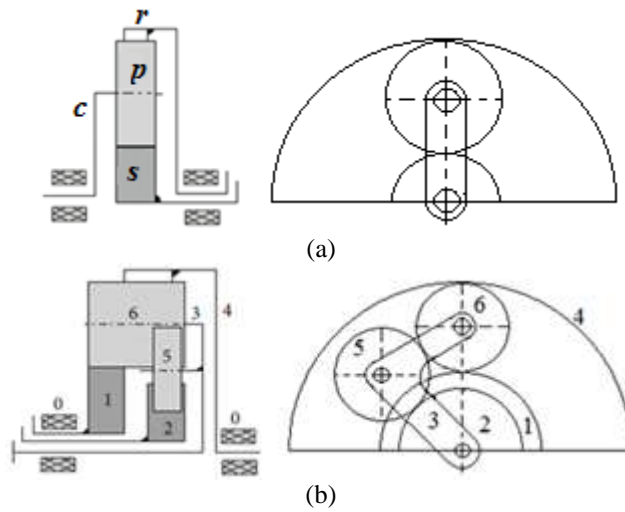


Figure (2): The functional schematics of (a) simple gear train, and (b) Ravigneaux gear train.

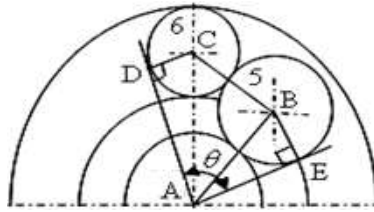


Figure (3): The included angle θ of planet gears 5 and 6 shown in Fig. 1

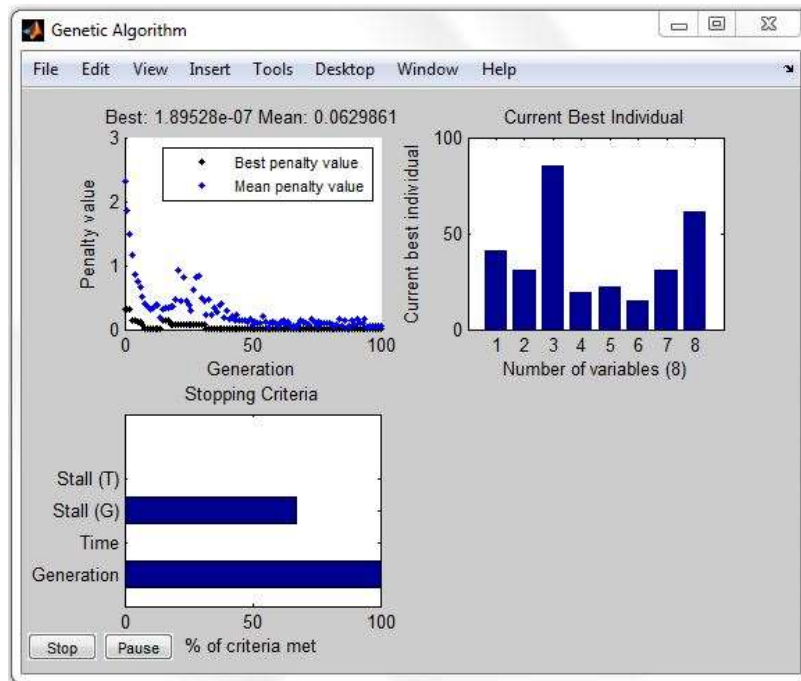


Figure (4): A MATLAB plot to the optimization results for the gear teeth numbers given in the fifth row of **Table 2**.

Appendix :

(1) MATLAB objective function m.file (Lepelletier.m)

```
function f=Lepelletier(z)
R1=z(3)*(1+(z(7)/z(8)))/z(2);
R2=((z(7)/z(8))+1)*((1/z(1))+1/z(2))/((1/z(3))+1/z(1));
R3=1+(z(7)/z(8));
R4=(1/z(2))*(1+z(8)/z(7))/((1/z(2))*(1+z(8)/z(7))-1/z(3));
R5=(1/z(1))*(1+z(8)/z(7))/((1/z(1))*(1+z(8)/z(7))+1/z(3));
R6=z(3)/(z(1)+z(3));
R7=-(z(3)/z(1))*(1+z(7)/z(8));
K1=4.135;K2=2.363;K3=1.508;K4=1.1401;K5=0.86019;K6=0.6746;K7=-3.1267;
f=((K1-R1)^2+(K2-R2)^2+(K3-R3)^2+(K4-R4)^2+(K5-R5)^2+(K6-R6)^2+(K7-R7)^2);
```

(2) MATLAB Nonlinear constraints m.file (Lepelletier_confune_GA.m)

```
function [c, ceq] =Lepelletier_confune_GA(z)
```

```

Thetaupper=84;
Thetalower=82;
Theta=acosd(((z(2)+z(4))^2+(z(5)+z(1))^2-
(z(5)+z(4))^2)/(2*(z(5)+z(1))*(z(2)+z(4))))+asind(z(5)/(z(1)+z(5)))+asind((z(4)/(z(4)+z(2)
)));
R1=z(3)*(1+(z(7)/z(8)))/z(2);
R2=((z(7)/z(8))+1)*((1/z(1))+1/z(2))/((1/z(3))+1/z(1));
R3=1+(z(7)/z(8));
R4=(1/z(2))*(1+z(8)/z(7))/((1/z(2))*(1+z(8)/z(7))-1/z(3));
R5=(1/z(1))*(1+z(8)/z(7))/((1/z(1))*(1+z(8)/z(7))+1/z(3));
R6=z(3)/(z(1)+z(3));
R7=-(z(3)/z(1))*(1+z(7)/z(8));
K1=4.135;K2=2.363;K3=1.508;K4=1.1401;K5=0.86019;K6=0.6746;K7=-3.1267;
E1=(R1/R2);E2=(R2/R3);E3=(R3/R4);E4=(R4/R5);E5=(R5/R6);
S1=abs(E1-E2);S2=abs(E2-E3);S3=abs(E3-E4);S4=abs(E4-E5);
OPTIMAL_INCLUDED_ANGLE_C=[Theta]
Velocity_Ratio_steps=[E1 E2 E3 E4 E5]
Difference_in_velocity_ratio_steps=[S1 S2 S3 S4]
Velocity_Ratios=[R1 R2 R3 R4 R5 R6 R7]
Velocity_Ratios_d=[K1 K2 K3 K4 K5 K6 K7]
c = [2*z(6)+z(7)-z(8);-2*z(6)-z(7)+z(8);2*z(5)+z(1)-z(3);-2*z(5)-z(1)+z(3);
z(1)-z(2)-2*z(4);z(7)-z(8);z(2)-z(3);z(1)-z(3);z(2)-z(1);R2-abs(R7);
1.275-(R1/R2);1.275-(R2/R3);1.275-(R3/R4);1.275-(R4/R5);1.275-(R5/R6);
abs(E1-E2)-0.25;abs(E2-E3)-0.25;abs(E3-E4)-0.25;abs(E4-E5)-0.25;
Theta-Thetaupper;
-Theta+Thetalower;
abs(((z(2)+z(4))/z(5))^2+(1+(z(1)/z(5)))^2-(1+(z(4)/z(5)))^2)-
abs(2*(1+(z(1)/z(5))*(z(2)+z(4))/z(5)))]];
ceq = [];

```

(3) MATLAB optimization m.file (Lepelletier_GA.m)

```

function [x,fval,exitflag,output,population,score] =
Lepelletier_GA(nvars,lb,ub,intcon,Generations_Data,StallGenLimit_Data)
nvars=8;
lb=[15 15 85 15 15 15 15];
ub=[85 85 85 85 85 15 85 85];
intcon=[1 2 3 4 5 6 7 8];
Generations_Data=100;
StallGenLimit_Data=100;
options = gaoptimset;
options = gaoptimset(options,'Generations',Generations_Data);
options = gaoptimset(options,'StallGenLimit',StallGenLimit_Data);
options = gaoptimset(options,'Display','final');
options = gaoptimset(options,'PlotFcns',{ @gaplotbestf @gaplotbestindiv @gaplotstopping
});
[x,fval,exitflag,output,population,score] = ...
ga(@Lepelletier,nvars,[],[],[],[],lb,ub,@Lepelletier_confune_GA,intcon,options);

```