# NONLINEAR FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE DEEP BEAMS WITH OPENING

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**Abstract**

This research deals with nonlinear analysis of reinforced concrete deep beams with openings by three dimensional finite element method under static load. The constitutive models of the material nonlinearity are adopted to take into account the nonlinear stress-strain relationships of concrete and steel, such as (cracking and crushing of concrete, and yielding of reinforcement). A twenty-noded isoparametric brick element with sixty degrees of freedom is employed to model the concrete while the reinforcing bars are modeled as axial members embedded through the brick element with perfect bond. Parametric study is considered to deal with the effect of opening location on the ultimate strength of deep beams. It was found that providing an opening at the shear zone causes sharp decrease in ultimate load by about (31%-56%) for simply supported deep beams. Therefore, if the designer has to provide an opening in a deep beam, he should keep it far away from the load path.

# التحليل اللاخطي للأعتاب الخرسانية المسلحة العميقة الحاوية على فتحات باستخدام طريقة العناصر المحددة

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**الخلاصة**

البحث يتناول التحليل اللاخطي للعتبات الخرسانية العميقة المحتوية على فتحات تحت تأثير الأحمال الساكنة باستعمال طريقة العناصر المحددة . تم اخذ اللاخطية لتصرف المادة بنظر الاعتبار عند تمثيل الخرسانة والحديد مثل تشقق وتهشم الكونكريت وحديد التسليح.العنصر الطابوقي ذو عشرين عقده و ستين درجة حرية تم توضيفه لتمثيل الخرسانة. قضبان حديد التسليح مثلت كعناصر محورية مطمورة ضمن العناصر الطابوقية مع ترابط تام بينهما. نظمت دراسة لاختبار تأثير موقع الفتحة على تصرف العتبات العميقة وتم التوصل إلى إن موقع الفتحة هو من أهم العوامل التي يجب آن تؤخذ بنضر الاعتبار عند التصميم، فعندما تكون الفتحة في منطقة القص فأنها سوف تحدث انخفاضا كبيرا بقيمة الحمل الأقصى بمقدار (٣١%-٥٦%) للعتبات العميقة البسيطة الاسناد.

***Notation***

| | | |
|---|---|---|
| $\xi, \eta, \zeta$ | | Natural coordinate system |
| u, v, w | | Displacement components in x, y and z – direction respectively |
| $N_i$ | | Shape function at the ith node |
| $u_i$, $v_i$, $w_i$ | | Nodal displacement |
| x, y, z | | Global or cartisian coordinates |
| [B] | | Strain – displacement matrix |
| [J] | | Jacobian matrix |
| [D] | | Constitutive matrix |
| f'c | | ltimate compressive strength of concrete |
| $f_t$ | | Maximum tensile strength of concrete |
| $\alpha_1$ | | The rate of stress release as the crack widens |
| $\alpha 2$ | | Sudden loss of stress at instant of cracking |
| $\gamma_1$ | | Rate of decay of shear stiffness as the crack widens |
| $\gamma_2$ | | Sudden loss in shear stiffness at the instant of cracking |
| $\gamma_3$ | | Residual shear stiffness due to the dowel action |
| H' | | Iardening parameter |

## Introduction

The classical definition of a deep beam is the member which has a depth much greater than the normal in relation to its span, while the thickness in the perpendicular direction is much smaller than either the span or the depth. Deep beams occur in engineering structures such as in bunkers and water tanks where the walls act as vertical beams spanning between column supports [Khalaf,(1986), Mahmoud,(1992)]. In some multistory buildings, it is often desirable to have the lower floors free of columns, therefore; these beams may be designed as beams spanning across the column free space. Almost, these structures may include elements in the form of deep beams provided with openings for electrical cables, mechanical ducts and water and sewerage pipes (EL-Hashimy et al.,(1989)). ACI-building code classified deep beams as those with span to depth ratio about (4) or less, or a shear span less than about twice the depth.

### Shear and Flexural behavior of Deep Beam

The previous studies showed that reinforced concrete deep beams have behavior more complex and differ from that of shallow beams in many items:

1. In deep beams, the transverse sections which are plane before bending do not remain plane after bending (Winter and Nilson,(1978)).
2. The neutral axis does not usually lie at mid-depth and moves away from the loaded face of the member as the span to depth ratio decreases as shown in **Figure (1)**
3. Flexural stresses and strains are not linearly distributed across the beam depth (Winter and Nilson,(1978)).

The flexural strength can be predicted with sufficient accuracy using the classical methods employed for beams of normal proportions. The equivalent rectangular stress block and associated parameters can be employed without change. Experimental studies showed that shear strength of deep beams may be as much as (2-3) times greater than that predicted by using the expression for normal members (Winter and Nilson,(1978)). It is well known that shear transfer of diagonally cracked concrete beams of normal proportions takes place by four mechanisms:

1.Direct transfer in the uncracked concrete compression zone.
2. Aggregate interlocking.
3.Dowel action of the flexural main reinforcement.
4.Direct tension of the web reinforcement.

For deep beams, however in addition to the items above, a significant amount of load is carried to the supports by compression thrust joining the load and the reaction (Sanad and Saka (2001)). Diagonal cracks, which form roughly in a direction parallel to a line from load to support, isolate a compression strut, which acts with the horizontal compression in the concrete and the tensile force in the main reinforcement as a truss to equilibrate the loads (Winter and Nilson,(1978)).

## Finite Element Analysis of Reinforced Concrete

### 1 Three–Dimensional Brick Element

The quadratic twenty-node brick element shown in **Figure (2)** are adopted to represent concrete. This type of element is popular due to its superior performance. A major advantage of the quadratic twenty–node brick element over the eighty–node brick element, when studying complex cases, is that less number of elements can be used, as well as it may have curved sides and therefore provides a better fit to curved sides of an actual structure [Cook,(1974), Moaveni,(1999)].

### 2 Shape Functions

The element has twenty nodes and sixty degrees of freedom and bounded by planes with $\xi$, $\eta$, and $\zeta = \pm 1$ in $\xi$, $\eta$, $\zeta$ space. The starting point for the stiffness matrix derivation is the element displacement field. The isoparametric definition of displacement components is:

$$\left.\begin{aligned} u(\xi,\eta,\zeta) &= \sum_{i=1}^{n} Ni(\xi,\eta,\zeta)ui \\ v(\xi,\eta,\zeta) &= \sum_{i=1}^{n} Ni(\xi,\eta,\zeta)vi \\ w(\xi,\eta,\zeta) &= \sum_{i=1}^{n} Ni(\xi,\eta,\zeta)wi \end{aligned}\right\}$$

where $N_i(\xi, \eta, \zeta)$ is the shape function at the i-th node and $u_i$, $v_i$ and $w_i$ are the corresponding nodal displacements with respect to global x, y, and z coordinates. The shape functions of the quadratic twenty -node brick element are shown in **Table (1).**

**3 Element Stiffness Matrix**

The tangential stiffness matrix of the three-dimensional isoparametric solid element is given by:

$$[K]^e = \int_{v^e} [B]^T.[D].[B]\, dv^e \qquad \dots (2)$$

by using the transformation product rule, the stiffness matrix becomes:

$$[K]^e = \int_{-1}^{+1}\int_{-1}^{+1}\int_{-1}^{+1} [B]^T [D][B]|J|d\xi d\eta d\zeta \qquad \dots(3)$$

**4 Reinforcement Idealization**

In developing a finite element model for reinforced concrete members, at least three alternative representations of reinforcement have been used:

*a) Distributed Representation*
*b) Discrete Representation*
*c) Embedded Representation*

**5 Concrete Model Adopted in the Analysis**

In this study, a plasticity-based model is adopted for the nonlinear analysis of three-dimensional reinforced concrete structures under static loads. In compression, the behavior of concrete is simulated by an elastic-plastic work hardening model followed by a perfectly plastic response, which is terminated at the onset of crushing. The plasticity model in compression state of stress requires the following characteristics (Chen,(1982)):

1.Yield criterion
2.Hardening rule
3.Flow rule
4.Crushing condition

In tension, linear elastic behavior prior to cracking is assumed. A smeared crack model with fixed orthogonal cracks is adopted to represent the cracked concrete. The model will be described in terms of the following:

1.Cracking criterion
2.Post-cracking formulation
3.Shear retention model

**6 Modeling of Reinforcement**

Compared to concrete, steel is a much simpler material to represent. Its stress-strain behavior can be assumed to be identical in tension and compression. In reinforced concrete members, reinforcing bars are normally long and relatively slender and therefore they can be assumed to be capable of transmitting axial forces only. In the current study, the uniaxial stress-strain behavior of reinforcement is simulated by an elastic-linear work hardening model.

## Simply Support Reinforced Concrete Deep Beams

Simply support reinforced concrete deep beams are analyzed using the computer program (P3DNFEA). Details of three deep beams tested experimentally by Ramakrishan and Anathanarayana (1968) are shown in **Figure (3).** The first beam (A4) was subjected to two point loads located at the third portion of the beam, while the second (K2) was loaded with a uniform load and the thrid beam (K'1) was analyzed under one concentrated load applied at mid-span. Due to symmetry of loading and geometry, only one half of the beam is analyzed using sixteen 20-node brick elements for the first beam and twelve 20-node brick elements for the second and third as shown in **Figure (4).** The steel reinforcement is represented by embedded bar along span length. Material properties of concrete and steel are given in **Table (2). Figure (5) to Figure (7)** show the load-deflection response at mid-span of the beams. The computed response of load-deflection refers to good agreement with experimental result for most loading levels with difference in ultimate load about (3%), (2%) and (5%) for beams (A4), (K2), and (K'1), respectively.

The cracking pattren of the beam for different load levels are shown in **Figure(٨).** The first crack initiates at bottom surface of the middle zone of the beam at load (21.4%), (22.9%), and (22.1%) of ultimate load for the beam $A_4$, $K_2$, and $K'_1$, respectively. The cracks develop in all three directions of the beam, throughout the increase in the loading levels.

## Parametric study of Simply Supported Reinforced Concrete Deep Beams with openings

Simply supported reinforced concrete deep beams with openings are considered. All the openings with position and size are indicated by symbols, which are ranging from B0 to B10 as explained in **Figure (٩)** and **Table (3).** Two openings are provided in each beam symmetrically about the mid-span except (B6, B7, B8, B9, B10) that has only one opening at mid-span, and B0 that is solid without opening. Area of each opening is equal to (4%) of the side view area of the beam. Due to symmetry of loading and geometry, only one half of the beams is analyzed as shown in **Figure(١٠).** Material properties of the concrete and steel are given in **Table (4).** A Parametric study is presented including the influence of opening location along the beam span and through the depth .

To explain the effect of openings on the behavior of deep beams, the openings are provided at different positions along beam span and throught depth. These figures illustrate that the effect of an opening on the ultimate load depends on the extent to which it interrupts the load path joining the bearing block at the load and reaction points.

Presence of openings in the shear zone of a deep beam leads to reduce considerably the ultimate load as shown in **Figure (1١).** The ultimate load in beams B1, B2, and B3 is about (31%, 49%, and 38%) less than that of the case of no opening of similar solid beam B0, respectively. In beams B4 and B5, the reduction in ultimate load is about (40% and 56%) of the solid beam B0, respectively, where the openings completely interrupted the load path, therefore; serious strength reduction occurred as shown in **Figure (1٢).**

Ultimate load in beams B6 and B7, at which there is one opening made in the beam center and far away from the load path , is about (8% and 19%) less than that of B0, respectively. On the other hand, for B8 the opening piercing the critical shear zone, ultimate load is about (39%) less than that of B0 as shown in **Figure (1٣).**

In beams B9, and B10, the openings are provided in mid-span and are reasponsably clear from the load path, **Figure (1٤)** shows that their ultimate loads were comparable to that of solid beam B0 with a difference about (13%).

## Conclusions

On the basis of the analysis carried out by using three-dimensional nonlinear finite element method with P3DNFEA computer program, the following conclusions can be made:

1. The effect of opening on the ultimate load of deep beams depend primarily on the extent to which it intercepts the 'load path' and on the location at which this interception occurs. Therefore, if the designer has to provide an opening in a deep beam, he should keep it far away from the load path.
2. Providing an opening at the shear zone causes sharp decrease in the ultimate load by about (31%-56%) for simply supported deep beams
3. Presence of openings in the critical shear zones of deep beams leads to reduce considerably the ultimate load more than that in flexural zone by about (34%).

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**Table (1): Shape Functions of the Quadratic 20-Node Brick Element (Cook,(1974))**

| Location | ξ | η | ζ | Ni (ξ, η, ζ) |
|---|---|---|---|---|
| Corner nodes | ±1 | ±1 | ±1 | $(1+\xi\xi_i)(1+\eta\eta_i)(1+\zeta\zeta_i)(\xi\xi_i+\eta\eta_i+\zeta\zeta_i-2)/8$ |
| mid – side nodes | 0 | ±1 | ±1 | $(1-\xi^2)(1+\eta\eta_i)(1+\zeta\zeta_i)/4$ |
| mid – side nodes | ±1 | 0 | ±1 | $(1-\eta^2)(1+\xi\xi_i)(1+\zeta\zeta_i)/4$ |
| mid – side nodes | ±1 | ±1 | 0 | $(1-\zeta^2)(1+\xi\xi_i)(1+\eta\eta_i)/4$ |

**Table (2): Material Properties and Additional Parameters of Deep Beams**

| | Material properties and material parameters | Symbol | $A_4$ | $K_2$ | $K_1$ |
|---|---|---|---|---|---|
| Concrete | Young's modulus | $E_c$(N/mm$^2$) | 23460 | 17836 | 18503 |
| | Compressive strength | $f'_c$(N/mm$^2$) | 27.12 | 14.2 | 15.5 |
| | Tensile strength | $f_t$(N/mm$^2$) | 2.5 | 1.6 | 1.6 |
| | Poisson's ratio | $\nu$ | 0.2 | 0.2 | 0.2 |
| Reinforcement | Young's modulus | $E_s$(N/mm$^2$) | 200000 | 200000 | 200000 |
| | Yield stress | $f_y$(N/mm$^2$) | 317 | 317 | 317 |
| | Hardening parameter | H | 0.0 | 0.0 | 0.0 |
| Tension stiffening parameterS | Rate of stress release | $\alpha_1$ | 20.0 | 20.0 | 20.0 |
| | Sudden loss of tension stiffness at the instant of cracking | $\alpha_2$ | 0.5 | 0.5 | 0.5 |
| Shear retention parameters | Rate of decay of shear stiffness | $\gamma_1$ | 10.0 | 10.0 | 10.0 |
| | Sudden loss of shear stiffness at the instant of cracking | $\gamma_2$ | 0.5 | 0.5 | 0.5 |
| | Residual shear stiffness due to the dowel action | $\gamma_3$ | 0.1 | 0.1 | 0.1 |

**Table (3): Open Notations**

| Beam No. | Open No. | Size | | Position | |
|---|---|---|---|---|---|
| | | $a_1$ | $a_2$ | $k_1$ | $k_2$ |
| B0 | - | - | - | - | - |
| B1 | 1 | 0.2 | 0.2 | 0.2 | 0.1 |
| B2 | 2 | 0.2 | 0.2 | 0.4 | 0.1 |
| B3 | 3 | 0.2 | 0.2 | 0.6 | 0.1 |
| B4 | 4 | 0.1 | 0.4 | 0.45 | 0.05 |
| B5 | 5 | 0.4 | 0.1 | 0.3 | 0.15 |
| B6 | 6 | 0.2 | 0.2 | 0.4 | 0.4 |
| B7 | 7 | 0.1 | 0.4 | 0.45 | 0.3 |
| B8 | 8 | 0.075 | 0.53 | 0.481 | 0.235 |
| B9 | 9 | 0.4 | 0.1 | 0.3 | 0.45 |
| B10 | 10 | 0.53 | 0.075 | 0.235 | 0.481 |

**Table (4): Material Properties and Additional Parameters of Simply Supported Deep Beams with Openings**

| | Material properties and material parameter | Symbol | value |
|---|---|---|---|
| Concrete | Young's modulus | $E_c (N/mm^2)$ | 244210 |
| | Compressive strength | $f'_c (N/mm^2)$ | 27.8 |
| | Tensile strength | $f_t (N/mm^2)$ | 2.6 |
| | Poisson's ratio | $v$ | 0.17 |
| Steel | Young's modulus | $E_s (N/mm^2)$ | 200000 |
| | Yield stress | $f_y (N/mm^2)$ | 350 |
| | Hardening parameter | H | 0.0 |
| Tension stiffening parameter | Rate of stress release | $\alpha_1$ | 20.0 |
| | Sudden loss of shear stiffness at the instant of cracking | $\alpha_2$ | 0.5 |
| Shear retention parameters | Rate of decay of shear stiffness | $\gamma_1$ | 10.0 |
| | Sudden shear stiffness at the instant of cracking | $\gamma_2$ | 0.5 |
| | Residual shear stiffness due to the dowel action | $\gamma_3$ | 0.1 |

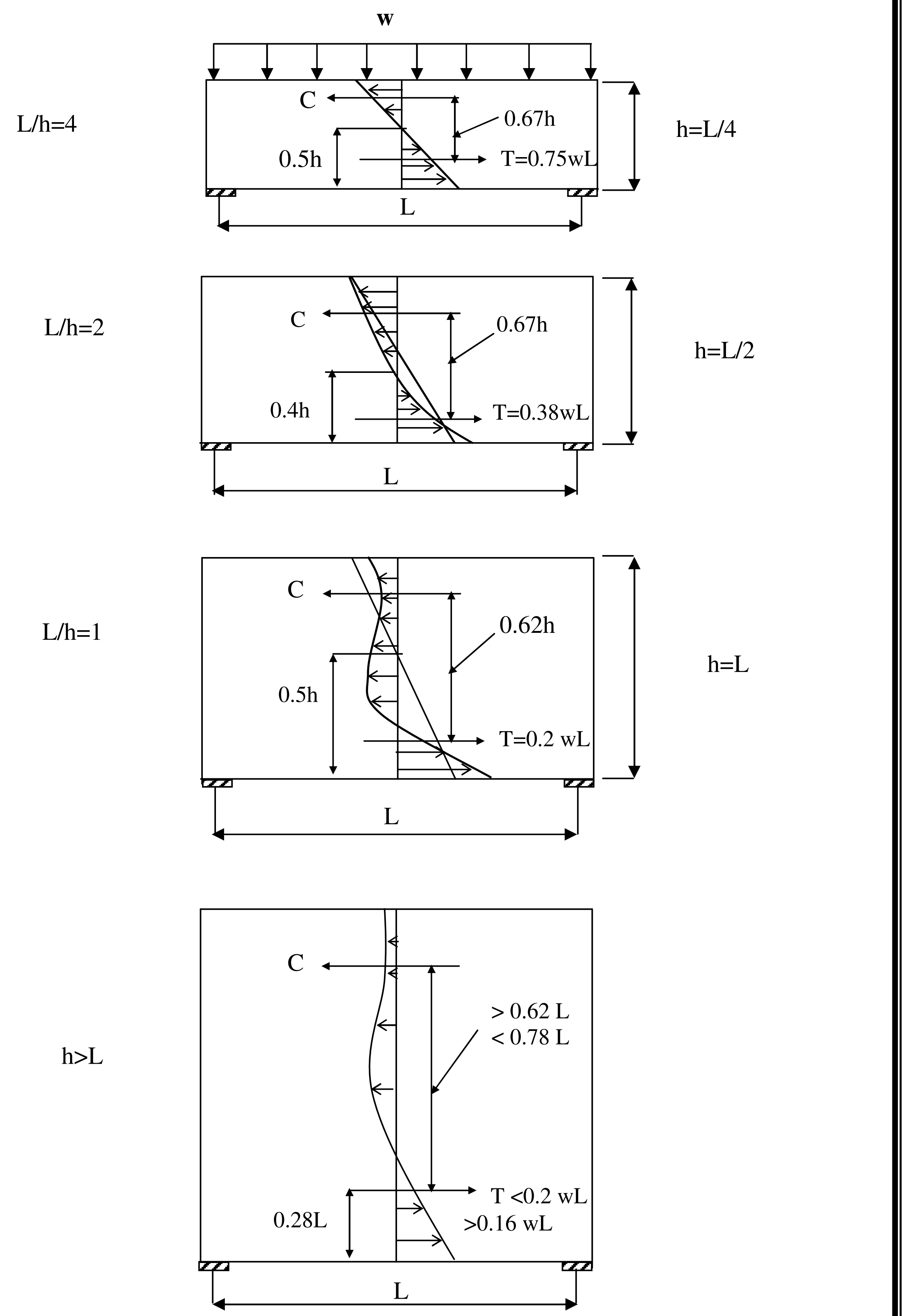



**Figure (1): Distribution of Flexural Stresses in Homogeneous Simply Supported Deep Beam**

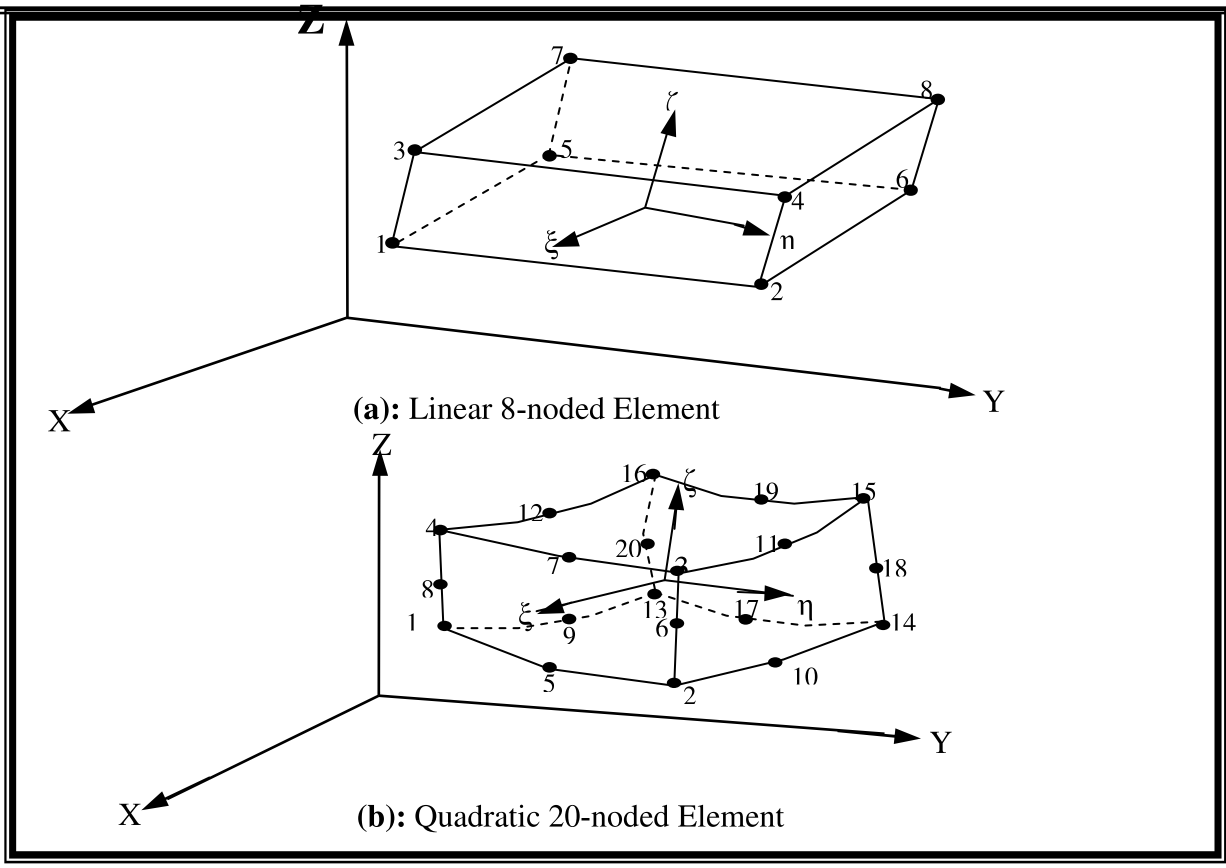
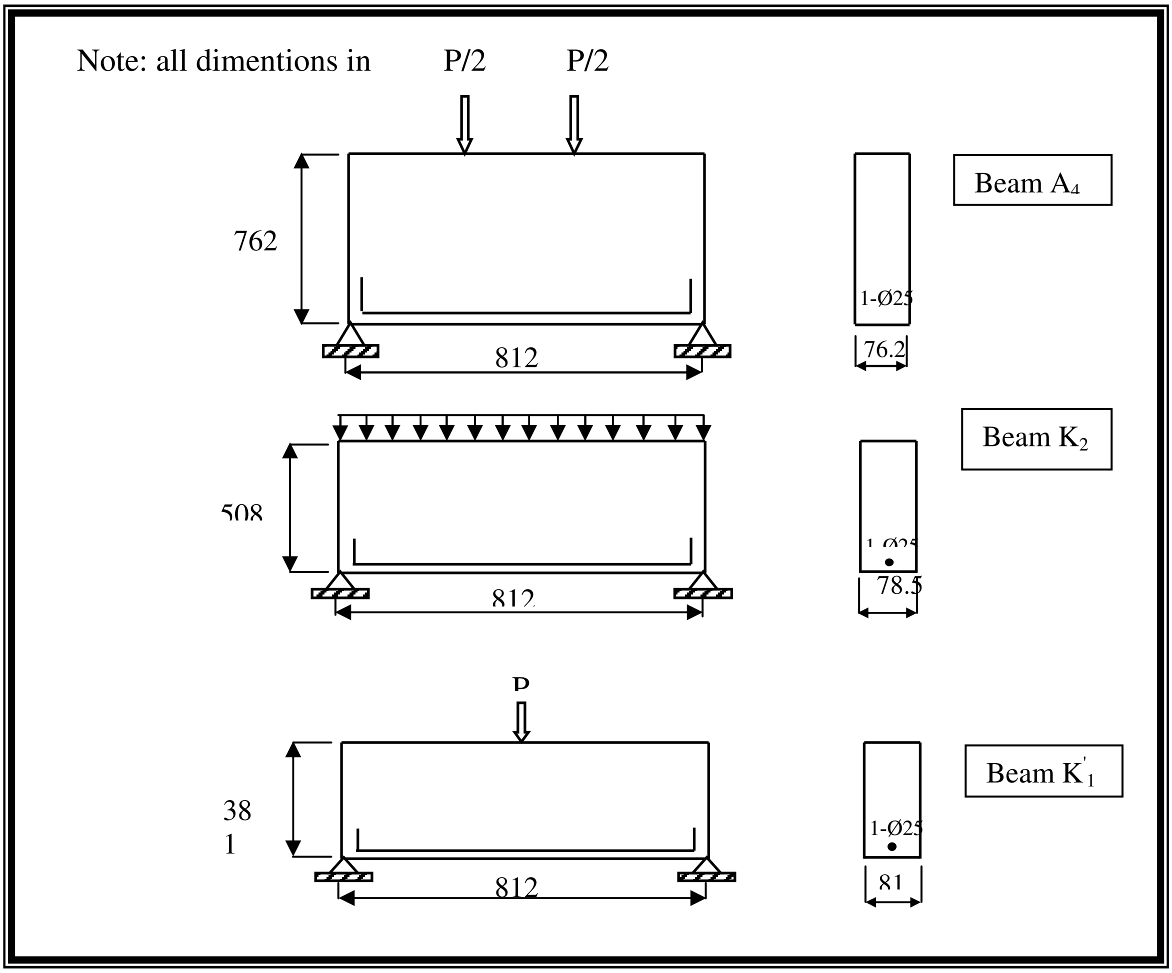



**Figure (2): Linear and Quadratic Isoparamatric Solid Element (Hinton,(1988)**



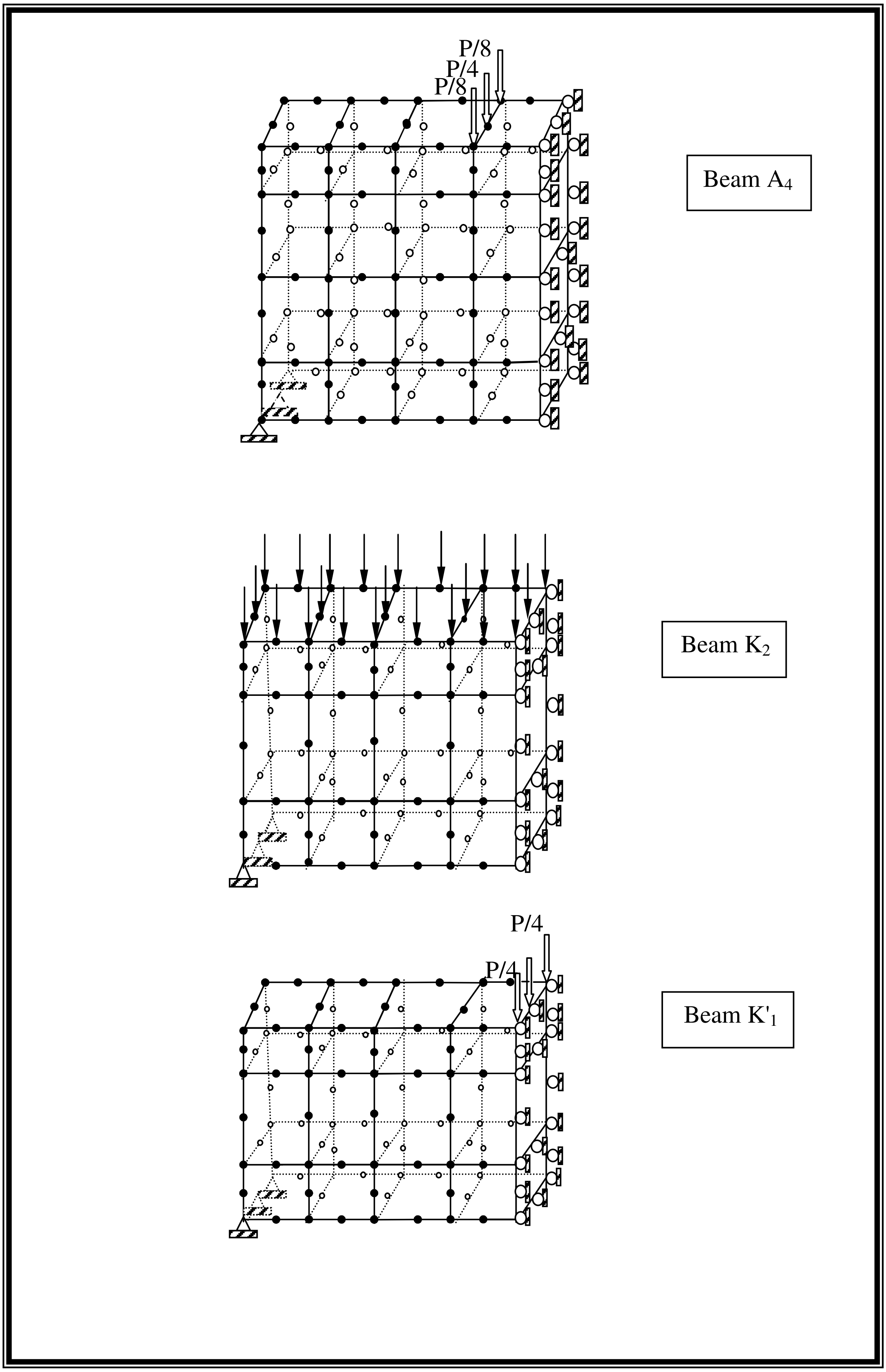



**Figure (4): Finite Element idealization of Half of the Simply Supported Deep Beams**

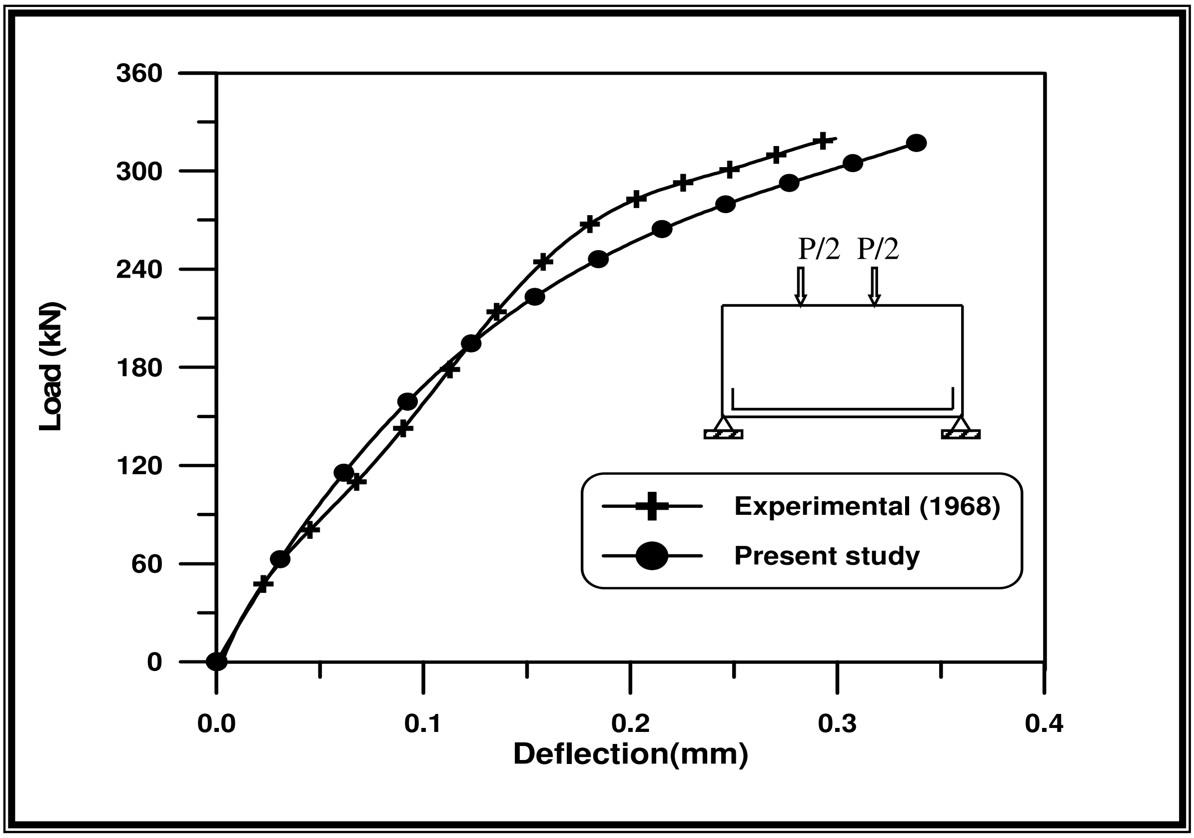



**Figure (5): Load-Deflection Curve at Mid-span for ($A_4$) Deep Beam**

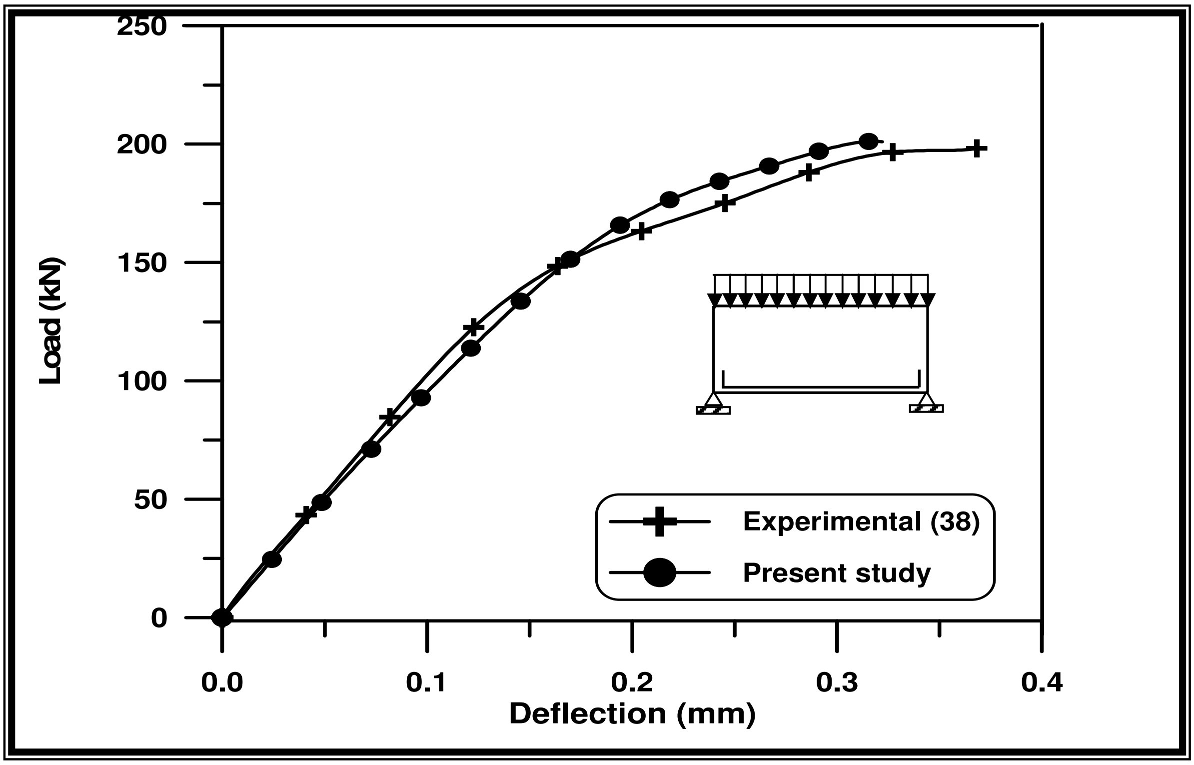



**Figure (6): Load-Deflection Curve at Mid-Span for ($K_2$) Deep Beam**

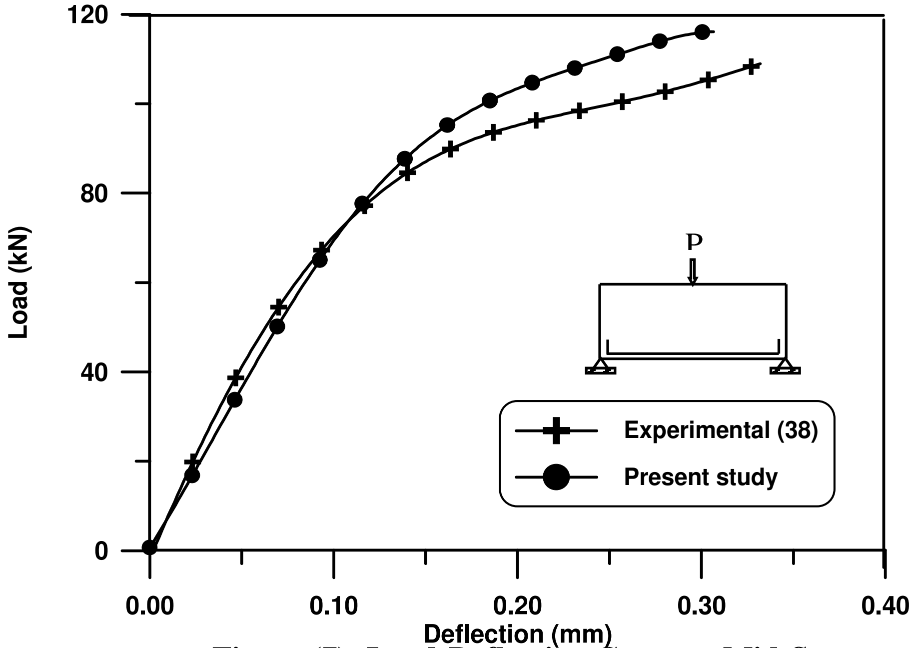



**Figure (7): Load-Deflection Curve at Mid-Span for ($K'_1$) Deep Beam**

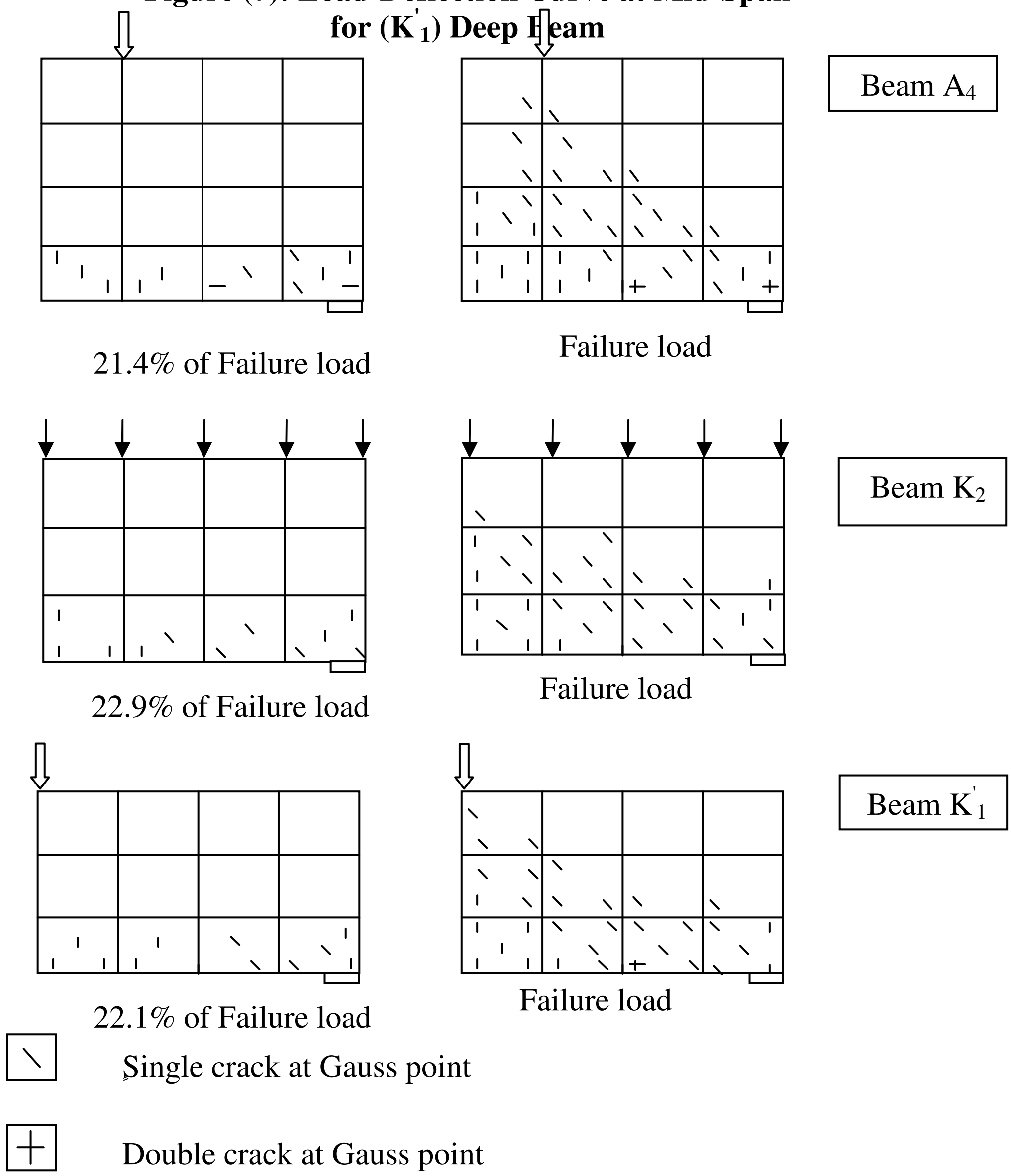



**Figure (8): Cracking Patterns of Simply Supported Deep Beams**

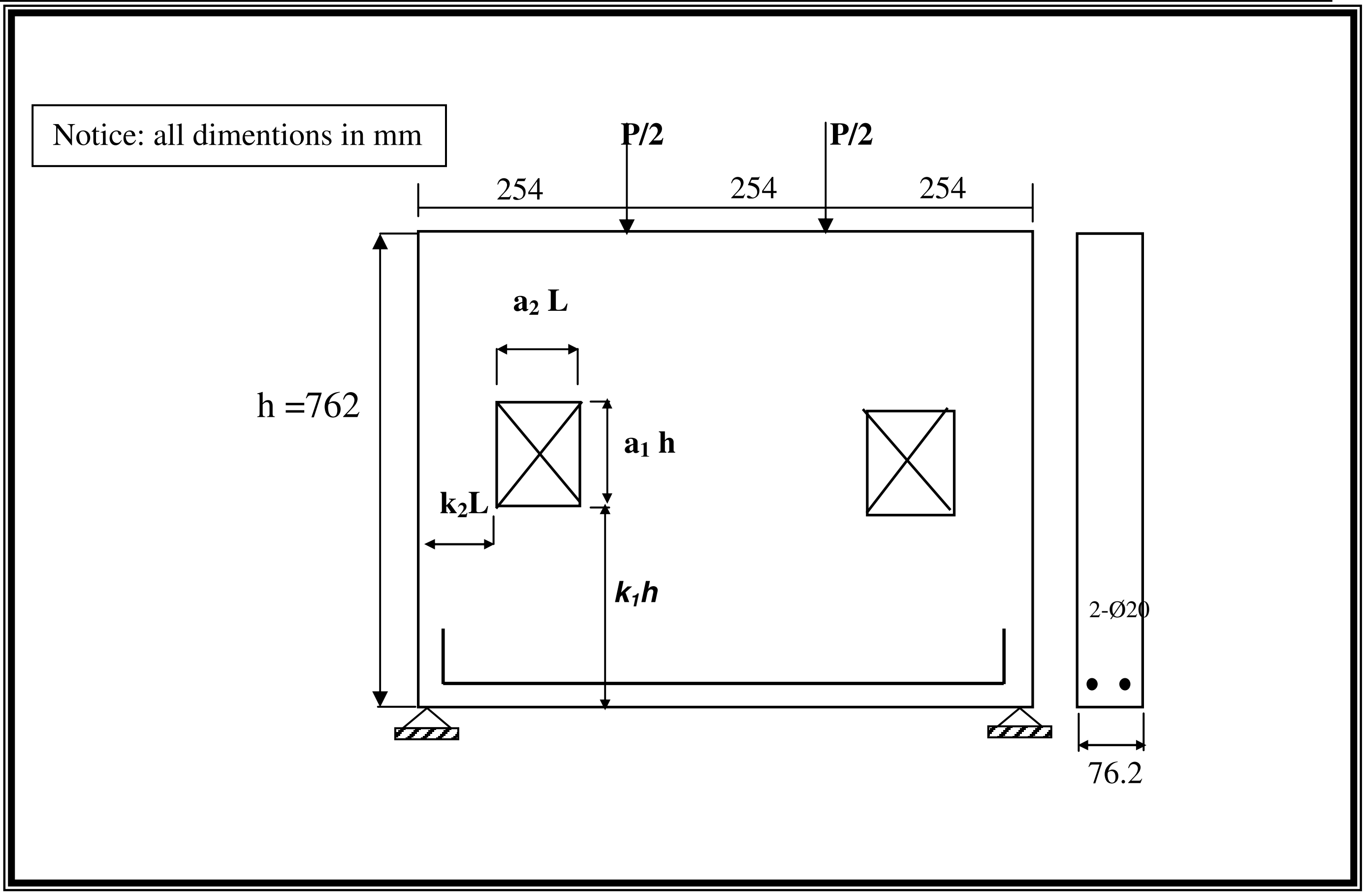



**Figure (9): Simply Supported Deep Beams with Openings**

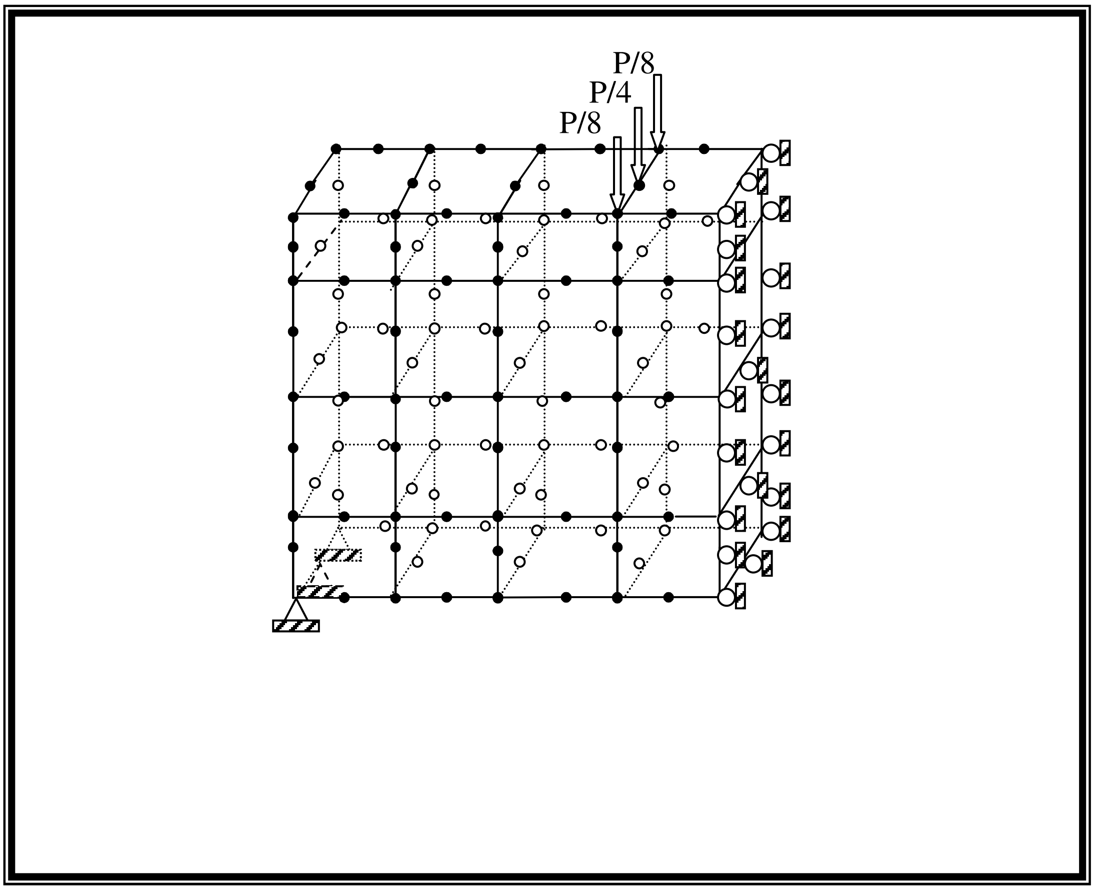



**Figure (10): Finite Element idealization of B0**

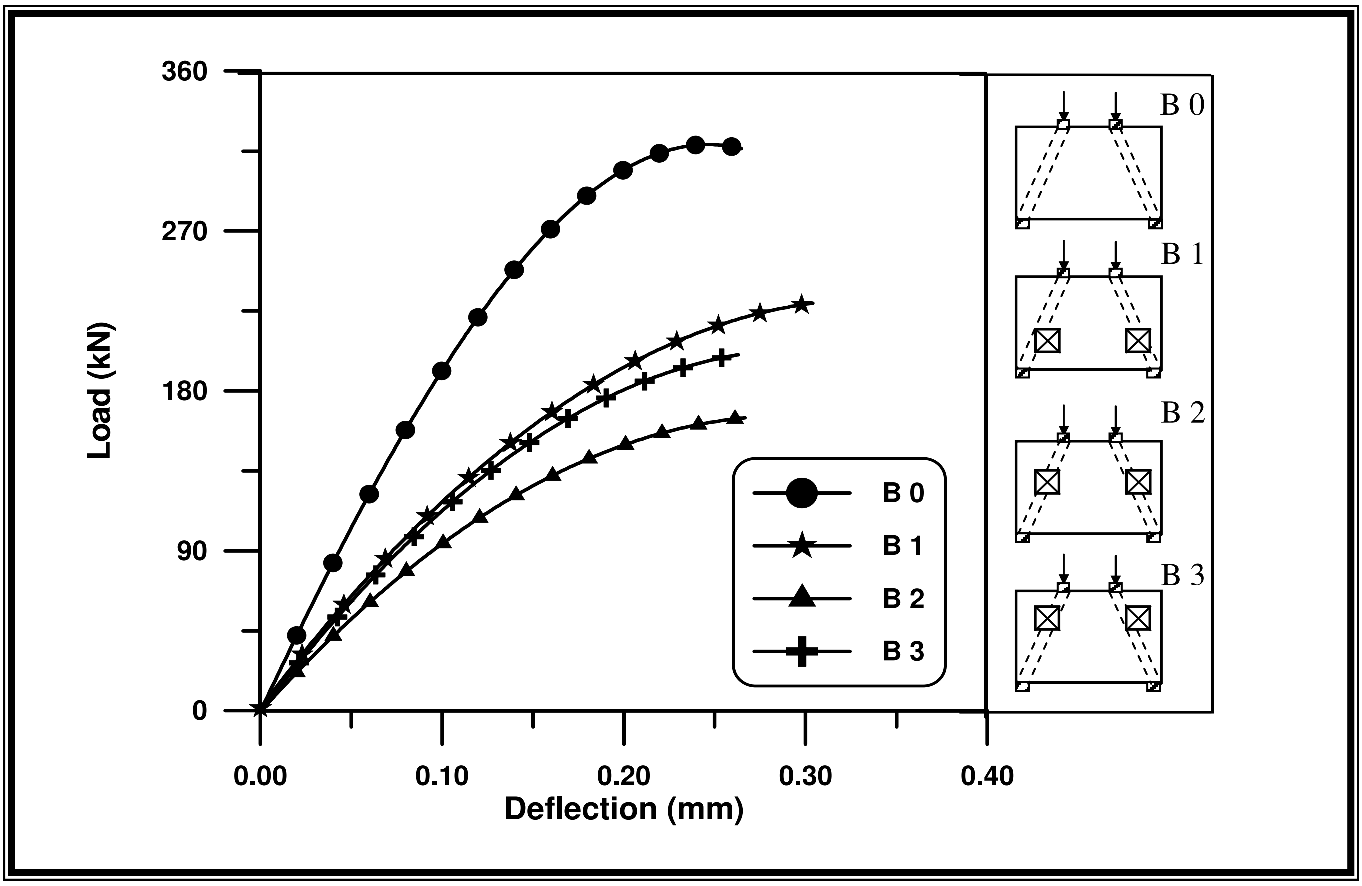



**Figure (11): Load-Deflection Curve for at Mid-Span for Deep Beams (B0, B1, B2, and B3)**

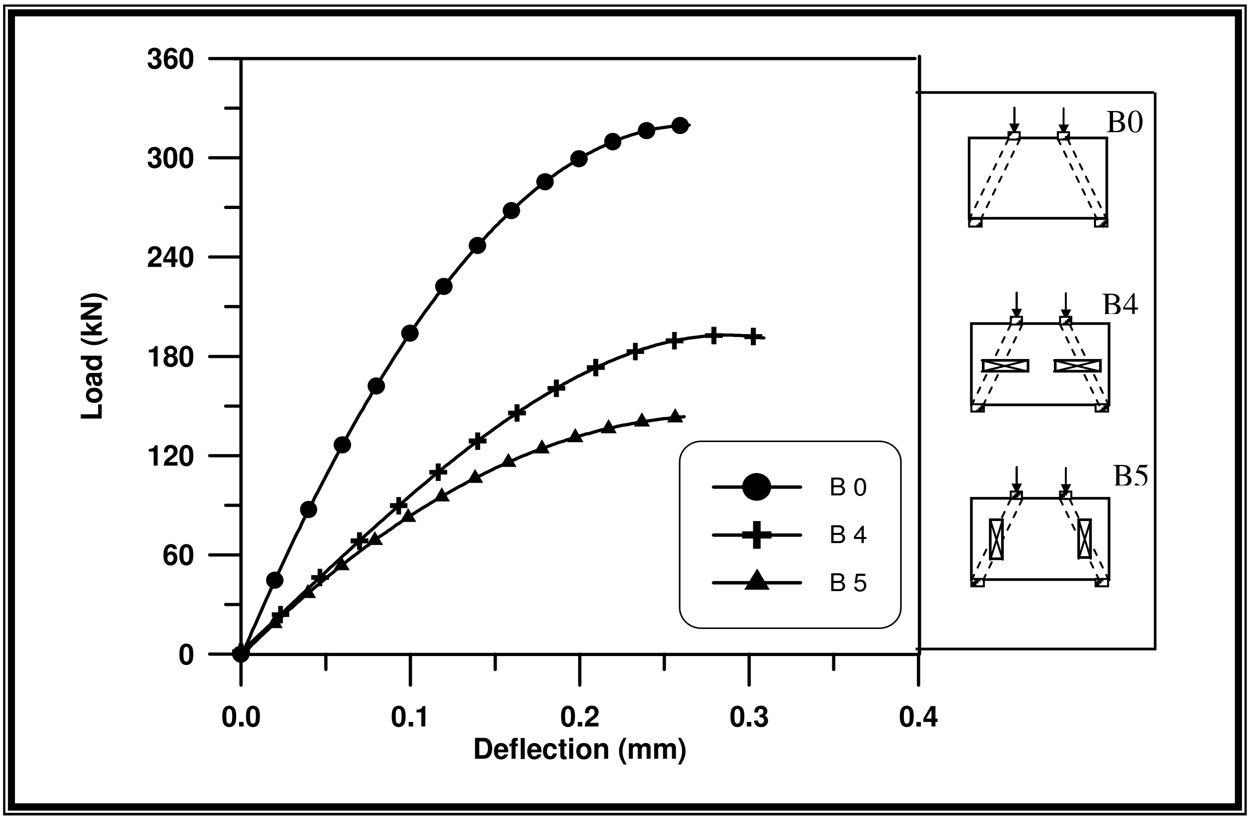



**Figure (12): Load-Deflection Curve at Mid-Span for Deep Beams (B0, B4, and B5)**

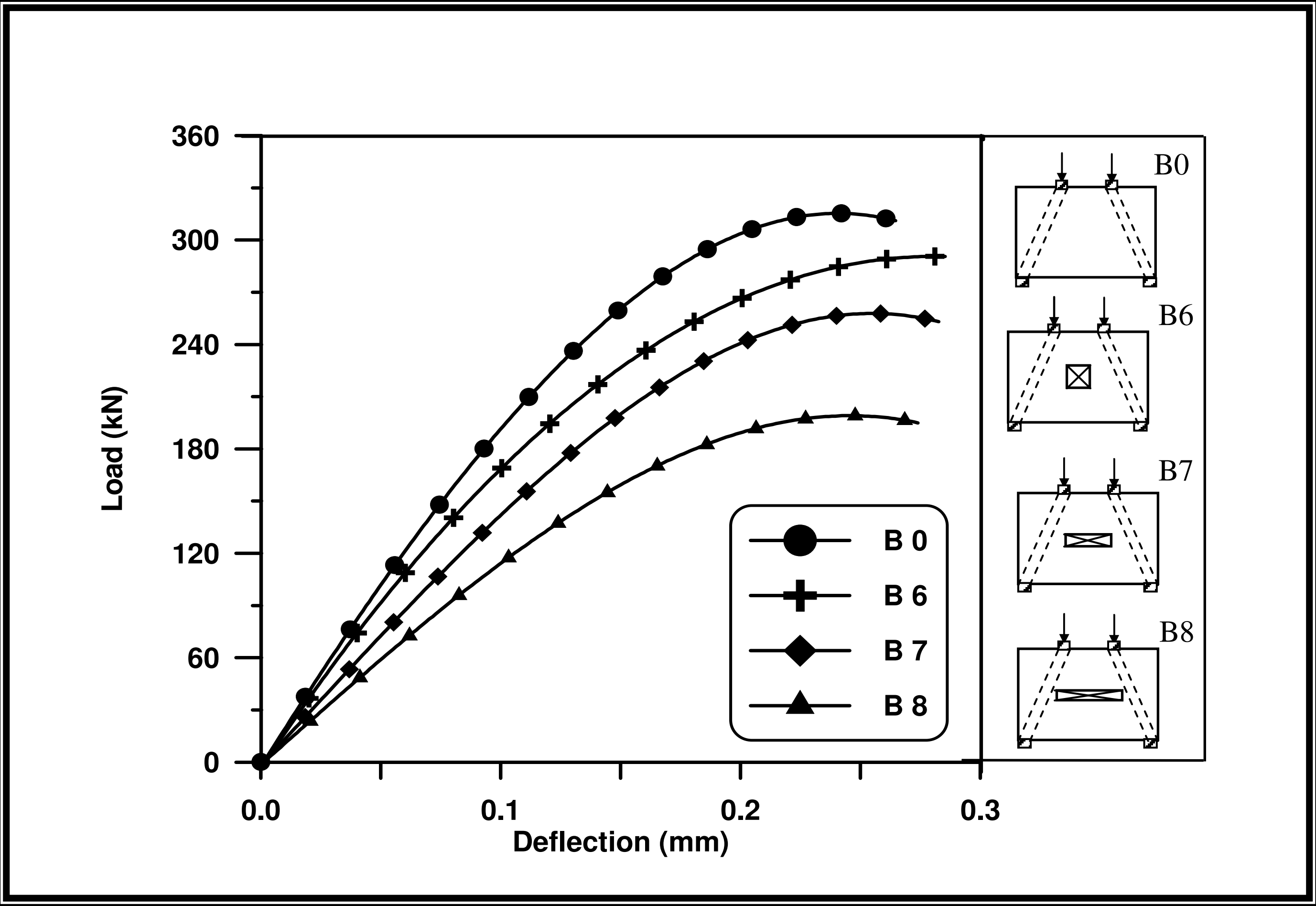



**Figure (13): Load-Deflection Curve at Mid-Span for Deep Beams (B0, B6, B7, and B8)**

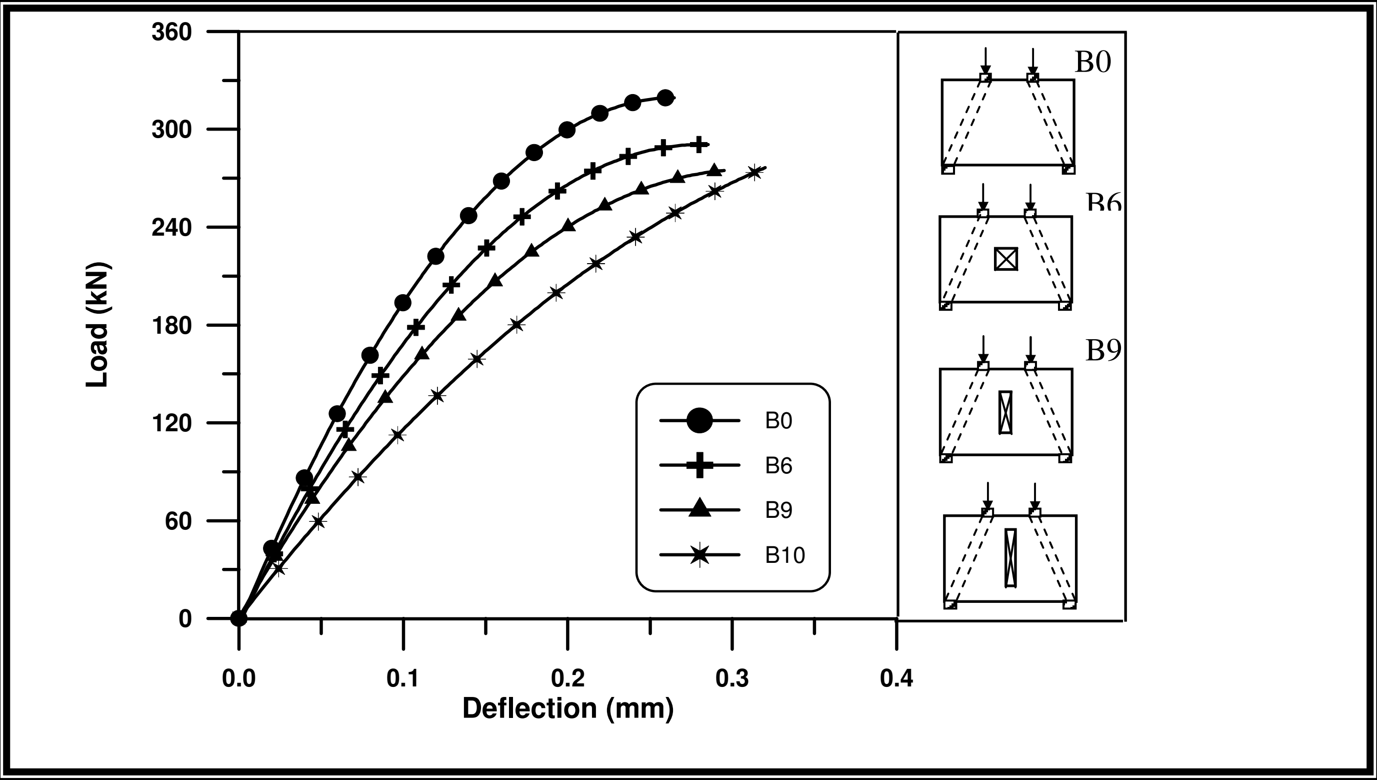



**Figure (14): Load-Deflection Curve at Mid-Span for Deep Beams(B0, B6, B9, and B10)**