Modelling as a Vehicle for Philosophical Inquiry in the Mathematics Curriculum

Lyn D. English

Philosophical inquiry in the teaching and learning of mathematics has received continued, albeit limited, attention over many years (e.g., Daniel, 2000; English, 1994; Lafortune, Daniel, Fallascio, & Schleider, 2000; Kennedy, 2012a). The rich contributions these communities can offer school mathematics, however, have not received the deserved recognition, especially from the mathematics education community. This is a perplexing situation given the close relationship between the two disciplines and their shared values for empowering students to solve a range of challenging problems, often unanticipated, and often requiring broadened reasoning.

In this article, I first present my understanding of philosophical inquiry as it pertains to the mathematics classroom, taking into consideration the significant work that has been undertaken on socio-political contexts in mathematics education (e.g., Skovsmose & Greer, 2012). I then consider one approach to advancing philosophical inquiry in the mathematics classroom, namely, through modelling activities that require interpretation, questioning, and multiple approaches to solution. The design of these problem activities, set within life-based contexts, provides an ideal vehicle for stimulating philosophical inquiry.

Philosophical Inquiry in the Mathematics Classroom

On re-reading my copy of Splitter and Sharp's (1995) book, *Teaching for Better Thinking*, I first turned to the back cover and became excited again by their ideas, which I have incorporated in my research over many years (e.g., English, 2010a). The author's pose the significant question, "What would happen if existing classrooms were transformed into communities of philosophical inquiry?" Their response, adeptly illustrated in the book, indicates the opportunities afforded for students to grapple with the "big questions," to "think for themselves," and to appreciate that they *can* make a difference to the world. Such a transformation, however, raises many questions itself. Two such questions come to mind: Do students see their learning of mathematics as a means of improving their life and that of others, both now and in the future? Do their classroom experiences in mathematics engender such a perception? These are long-standing issues that require greater attention and continue to be the subject of debate by researchers in the broad field of social justice (e.g., Sriraman, 2007). Establishing a community of philosophical inquiry can broaden students' mathematical learning to encompass issues of social justice.

From my perspective as a mathematics educator, I envision communities of philosophical inquiry in a similar vein to Splitter and Sharp (1995). That is, a mathematics classroom that embraces philosophical inquiry is one that evokes a spirit of co-operation, trust, and ease—one in which there is a willingness to share, respect, question, and critique one another's ideas on issues that are relevant, meaningful, and considered worthy of investigation. Furthermore, I argue that for such communities to thrive, both students and teachers must be open and committed to the sharing of alternative ideas, to the critical questioning of mathematical and contextual assumptions, and to the continued enrichment of their thinking.

Although mathematics educators have been emphasizing for many years the centrality of problem solving and the range of thinking skills needed in dealing with “real-world” problems (e.g., Jackson, Shahan, Gibbons, & Cobb, 2012; Lester & Charles, 2003; National Council of Teachers of Mathematics, 2000), research has shown that students still struggle with problem solving (e.g., English & Sriraman, 2010; Lesh, & Zawojewski, 2007). Again, communities of philosophical inquiry offer considerable promise in alleviating some of these difficulties because they can empower students to draw more extensively on their previous experiences and understandings both inside and outside of school.

Research that has explored philosophical inquiry in the mathematics classroom has drawn on the rich foundations set by Matthew Lipman’s *Philosophy for Children* (P4C e.g., Lipman, 1988). The parallels between the conceptual underpinnings of his program and students’ mathematical learning have been well documented (e.g., Daniel, 2000; De la Gaza, 2000; English, 1994; Lafortune et al., 2000; Stoyanova Kennedy, 2012a). For example, both P4C and mathematics education value students’ problem posing and solving, in which problem goals must be interpreted and defined, hidden assumptions identified, alternative courses of action considered, tentative solutions generated, and the reasonableness of conclusions assessed (Kennedy, 2012b).

Further, the myriad thinking skills that lie at the heart of philosophy are essential to effective mathematics learning and problem solving. Such skills, which have been repeatedly cited in the literature and do not need elaborating here, include creativity and innovation, critical and reflective thinking, deductive and inductive reasoning, investigative inquiry, and drawing informal and formal inferences, to name but a few. Splitter and Sharp (1995) provide a very comprehensive list of “strategies” that may be identified under the rubric of “thinking” (p. 9).

Philosophical Inquiry and Critical Mathematics Education

As previously noted, issues raised in social justice studies of mathematics learning are at home in classrooms that embrace philosophical inquiry. More specifically, the increasingly important field of critical mathematics education targeting concerns of social justice, encompasses philosophical inquiry, yet specifically establishing such communities seems to be rarely addressed. Nevertheless, the groundwork for these communities has been laid as evident in recent publications (e.g., Skovsmose & Greer, 2012; Greer, Mukhopadhyay, Nelson-Barber, & Powell, 2009; Gutstein, 2006) and in earlier works (e.g., Mellin-Olsen, 1987).

In their introduction to their 2012 edited book, *Opening the Cage: Critique and Politics of Mathematics Education*, Greer and Skovsmose outline core elements of critical mathematics education. These include the observation and analysis of mathematics as it occurs in multiple socio-political contexts and the making and critiquing of value judgements, with the ultimate desire to generate change in accordance with these judgements. Of particular relevance to this article is Greer and Skovsmose’s point that:

> Within the field, there is heightened cultural and historical awareness, both within and beyond academic mathematics, and an increased acknowledgement of the ubiquity and importance of “mathematics in action” and the implications for mathematics education, including more curricular prominence for probability, data handling, modelling, and applications. (pp. 3-4)

Philosophical inquiry can heighten students’ awareness and understanding of these increasingly important content areas, content that cannot take second place to the inquiry processes being nurtured. Rather, in a community of inquiry, concepts and processes are developed concomitantly in ways that stimulate and challenge students’ thinking about, and beyond, the content. At the same time, however, communities of inquiry are complex communicative systems (Kennedy, 2012c) and engaging students in productive discourse that facilitates the development of content and processes is an evolving endeavour; one that should begin in the earliest years of schooling (Splitter & Sharp, 1995). One approach to establishing these communities is through mathematical modelling experiences set within interdisciplinary, authentic contexts. I now consider some features of the modelling problems I have implemented from the early grades through the middle school and illustrate how philosophical inquiry is an inbuilt component.

Mathematical Modelling and Philosophical Inquiry

Modelling is increasingly recognized as a powerful tool for not only promoting students' understanding of a wide range of core mathematical and scientific concepts, but also for helping them appreciate the potential of mathematics as a critical tool for analyzing important issues in their lives, communities, and society in general (Greer, Verschaffel, & Mukhopadhyay, 2007; Romberg, Carpenter, & Kwako, 2005). For example, in Tate's study (1995) cited by Greer et al. (2007) students in a predominantly African American urban middle school were asked to pose a problem negatively affecting their community, to investigate the problem, and to develop and implement strategies for solution. One particularly interesting problem that was posed addressed the presence of 13 liquor stores within 1000 feet of the students' school. The students devised a plan to move the stores away and carried out their plan through various means including lobbying the state senate. Mathematical modelling was an important tool in solving this real-world problem, as was evident in the students' analysis of the local tax and other codes that led to financial advantages for the liquor stores. The students subsequently reconstructed this incentive system to protect their school community.

Students' development of powerful models should be regarded as among the most significant goals of mathematics education, yet its appearance in the curriculum is still limited especially in the elementary and middle schools. My research and that of others, however, has shown that elementary school children are indeed capable of developing their own models and sense-making systems for dealing with complex problem situations (e.g., English, 2008; English, 2010b; Lehrer & Schauble, 2005).

Numerous interpretations of models and modelling have appeared in the literature, including with reference to completing word problems, conducting mathematical simulations, constructing representations and explanations of problem situations, and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp 1999; English & Halford, 1995; Greer 1997; Lesh & Doerr 2003). The definition of mathematical models that I have adopted in a good deal of my research is that of "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some other familiar system" (Doerr & English, 2003, p. 112). Within this perspective, I view modelling problems as realistically complex situations set within life contexts that engage student groups in mathematical content and thinking that extend beyond the usual classroom school experience.

Many of the modelling experiences my colleagues and I have created, such as the engineering-based *Water Shortage Problem* addressed here, are realistic, open-ended problems where a client requires a team of workers to generate a product (a model) for solving a given problematic situation. In developing their models, students have to identify a process that the client can use to solve not only the given problem, but also similar problems, as indicated in the design principles I describe next. The structure of the modelling experiences reflects features of Lipman's P4C program in that the context and data presented are intriguing and often ambiguous with an element of uncertainty, and where multiple interpretations and solutions are possible. Importantly, the problem must evoke the desire to question, debate, and challenge ideas, together with a keenness to work collaboratively in resolving the problem.

Design Principles

In creating such modelling experiences, I have been guided by a number of design principles advanced by

Lesh and his colleagues (e.g., Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). These principles include those pertaining to model construction, documentation, personal meaningfulness, self-assessment, and generalization (e.g., English & Mousoulides, 2011). With respect to the first principle, modelling problems require students to develop an explicit mathematical or scientific construction, explanation, description, or prediction of a meaningful complex system. Such models need to focus on the underlying structural characteristics (key ideas and their relationships), rather than the surface features, of the system being addressed, and should be shared with others for constructive discussion and refinement.

Documentation of students' deliberations and resultant models is a core component of the modelling problems. They are required to externalize their thinking and reasoning as much as possible and in a variety of ways, revealing insights into their conceptual development and understanding. More than a brief answer is required here; descriptions and explanations of the steps and decisions taken in constructing their models are to be included, together with the creation of various representations conveying their findings (e.g., lists, tables, graphs, diagrams, and drawings).

As the term implies, the personal meaningfulness principle highlights the importance of selecting authentic contexts that are relevant, enticing, and of genuine concern to students. In my research, I have used interdisciplinary contexts in which opportunities for socio-political discussion are embedded. Importantly, these contexts and the associated disciplinary content need to align with the teacher's curriculum programs such as those involving mathematical, scientific, societal, and environmental understandings. Students' modelling experiences are thus not viewed as "add-ons" to an already crowded curriculum but as enriching existing learning and providing valuable links across disciplines.

The design of these modelling problems also incorporates inbuilt criteria for self-assessment, with "self" referring to both an individual's and group's critical analysis of the models being generated. Such analysis is akin to "reflective thought as an ongoing process of reconstruction," which Lipman adopted from Dewey (1933) in the P4C program (Kennedy, 2012a, p. 83). As illustrated in the modelling activity I describe next, self-assessment engages students in determining whether their final model is an effective one and adequately meets a fictitious client's needs in dealing with the given problematic situation. Such criteria also enable students to progressively assess and revise their models as they work the problem.

The generalization design principle is an important feature of modelling experiences in that students' models should be applicable to related problem situations, that is, problems that share similar underlying structural features. Not only being able to recognize the structural commonalities between problem situations but also knowing to look for such similarities and how to apply one's learning to these new situations is a powerful reasoning process across disciplines (English & Sriraman, 2010). The importance of generalization, where students progress from particular issues to a "higher level of generality," is a core feature of the communities of inquiry promoted by Lipman's program (Kennedy, 2012a, p. 83).

Further supporting the underpinnings of philosophical communities of inquiry, these modelling problems are designed so that multiple solutions of varying mathematical and scientific sophistication are possible, enabling students with a range of personal experiences and knowledge to tackle them without fear of failure or ridicule. The opportunities for multiple feedback points during model development encourage students to rethink their models (e.g., through "what-if" questioning) and to discuss freely the strengths and weaknesses (with respect to the client's criteria for success). This reflection is shared within a classroom that has generated a range of alternative models, with no one "correct" model. Rather, follow-up discussion engenders students' thinking about the conceptual and pragmatic understandings that have been generated by different models.

The Water Shortage Problem

One of the engineering-based modelling activities my colleague and I developed and implemented in middle school Cyprus classrooms with 11 year-old students (English & Mousoulides, 2011) is the Water Shortage problem (presented in the appendix).

The problem begins with students being “sent” a letter from a client, the Ministry of Transportation, who needs a means of (model for) selecting a country that can supply Cyprus with water during the next summer period. The letter asks students to develop such a model using the given data, as well as search for additional information using available tools such as Google Earth, maps, and the Web. The quantitative and qualitative data provided for each country include water supply per week, water price, tanker capacity, and ports’ facilities. Students can also obtain data about distance between countries, major ports in each country, and tanker oil consumption. After students develop their model, they are to write a letter to the client detailing how their model selects the best country for supplying water. As an extension of this problem, students are given a second letter from the client including data for two more countries and are asked to test their model on the expanded data and improve their model, if needed.

The environmental engineering context of the Water Shortage problem is an authentic one for the students in Cyprus, where water has been rapidly drying up since the 1970’s. The lack of drinkable water in Cyprus is a major problem, with water supply to homes limited. The water issue features prominently in the Cypriot media and for all members of the community, including students, this is an authentic problem whose solution appears hindered by conflicting political agendas.

As indicated in the following examples, this problem generated substantial discussion that resonated with that of a community of philosophical inquiry. In particular, the students were concerned about the socio-political and environmental issues that they drew from the problem context.

Philosophical Inquiry and Conceptual Development in the Water Shortage Problem

Consideration is given here to the discussion of two student groups whose model development was influenced by a consideration of the above issues. The first student group, like a few other groups, initially decided to exclude some of the data in particular water supply per week and port facilities. Using the provided data, the students calculated total oil cost per trip by multiplying oil cost per 100 km by distance and then dividing by 100. They then calculated water cost per tanker by multiplying water price by tanker capacity, and next calculated the total cost per trip by adding oil cost and water cost. Finally, they calculated the water price per ton by dividing the total cost by tanker capacity (as shown in Table 1). Using their data, the group decided that Greece was the most appropriate country for the purchase of their water.

Table 1: One Group’s Interim Model

Country	Distance (km)	Oil cost	Water cost per tanker	Total cost	Average water cost per ton
Egypt	420	€ 84000	€ 120000	€ 204000	€ 6.80
Greece	940	€ 235000	€ 100000	€ 335000	€ 6.70

Lebanon	260	€ 52000	€ 156000	€ 208000	€ 6.94
Syria	280	€ 56000	€ 150000	€ 206000	€ 6.87

Not satisfied with this model, however, the group then extensively discussed sea pollution. Based on the newspaper article that they had worked on during the first session of the modelling activity, one student raised the question of whether it would be wise to buy water from Greece. He mentioned that the distance from Pireus to Limassol was more than three times greater than the distance from Lebanon and Syria, and proposed to buy water from Egypt or Syria, the second and third country in distance ranking. An extract from the students' discussion appears below:

Student A: It is much better to buy water from a country close to Cyprus.

Student B: Why?

Student A: It is good to minimize oil consumption. That's why.

Student C: Yes, you are right, but we decided to use water cost per ton in our solution, not only oil consumption.

Student A: I agree. I am not saying to focus only on oil cost. But I believe that we need a solution that takes into account that it is better to buy water from a country near Cyprus, like Lebanon, as to avoid more oil consumption.

Student B: Exactly, especially since oil is getting more and more expensive.

Student A: It is not the only reason. We need to think of the environment. Especially in our case, we need to minimize Mediterranean's pollution from oil and other waste.

The students also documented in their reports that all countries in the Mediterranean Sea should be fully aware of sea pollution and therefore try to minimize ship oil consumption. Another student suggested buying water from Syria, since water price is not that expensive (compared to the price from Greece and Egypt). The students finally ranked countries in the following order: Syria, Egypt, Lebanon, and Greece and decided to propose to the local authorities to buy water from Syria.

Another group of students became concerned about the port facilities factor, a component that some student groups chose to ignore in the models they generated. This group decided to quantify the factor and integrate their calculations within the port facilities data. A subsequent discussion focused on the amount of money necessary for improving the ports' facilities and how this amount of money would change the water price per ton. To assist them here, the students asked for more information about the amount of money necessary for improving port facilities in Syria, Lebanon, and Egypt. They were surprised when they learned that improving the ports' facilities would cost from five to ten million euro. This feedback prompted concerns regarding socio-economic considerations:

Student C: Ten million euro? That's huge. The government cannot pay so much money.

Student D: It is obvious that we will buy water from Greece. It is more expensive than water from the other countries, but at least we will not pay for improving port facilities.

Researcher: I agree with you that ten million euro is not a small amount of money. But, don't you have to think of that amount in terms of the whole project of importing water in Cyprus?

Student D: Do you mean thinking of this amount in comparison to how much importing water in Cyprus will cost?

Researcher: Exactly!

During their second round of model improvement, the students debated issues related to tanker capacity and oil cost, and how these factors might relate to their solution, not only in terms of the mathematical relations. They were aware of energy consumption issues, discussing in their group that oil consumption should be kept as minimum as possible. When their teacher prompted them to decide which factor was more important, water price or oil consumption, the students replied that it would be better to spend a little more money and to reduce oil consumption. The group also made explicit that it was not only oil consumption but also other environmental issues, like the pollution of the Mediterranean Sea, which needed to be considered. Finally, this group proposed Syria as the best place from which to buy water, since its costs were quite reasonable and it is the closest to Cyprus.

Discussion and Concluding Points

In this article, I have tried to show how philosophical inquiry can enhance students' mathematical learning, with a focus on modelling experiences set within thought-provoking contexts. However, as I indicated many years ago (e.g., English, 1993; 1994) and as Kennedy (2012a) reiterated recently, philosophical inquiry is not just restricted to problem solving per se. Rather, such inquiry can and should become a natural component of all mathematics learning, with students being at ease in sharing and questioning one another's ideas and creations.

Implementing philosophical inquiry in the mathematics classroom is a multi-pronged approach. Consideration needs to be given to the concepts and understandings to be developed, the context in which these will be housed, the thinking and reasoning processes to enhance this learning, and the features to be developed in the classroom community. Addressing each of these components in detail is of course beyond the scope of this article. However, it is worth emphasizing again the importance of selecting task contexts that engender learning of a critical and reflective nature, and of the need to nurture a classroom community that is open to, and values, such learning.

With respect to context, Mukhopadhyay and Greer's (2007) "education for statistical empathy" (p. 169) provides an excellent example of how we might address the two questions I posed at the beginning of this article. With the avalanche of data students face on a daily basis from various sources, they need more than ever before to be critical consumers of information. Mukhopadhyay and Greer refer to mass gun killings as a case in point when mathematics is a powerful tool for analyzing and questioning issues pertaining to contemporary life. A classroom community of philosophical inquiry can help students appreciate how they can apply their mathematics learning to reveal the "untold messages" behind data-laden media reports, and to question what might be done to address these problematic issues.

Nurturing reflective and critical learning in the mathematics classroom is receiving increased attention (e.g., Cengiz, 2013; Barlow & McCrory, 2011) but substantially more is needed. As Prediger (2005) stressed, students should not only be given opportunities to be reflective, but also to develop the skills and disposition to do so. Being able to question and research, reflect on findings, describe and justify conclusions, and communicate and evaluate end-products is fundamental here. Mukhopadhyay and Greer's (2007) "cycles of discussions to foster statistical empathy" (p. 175) provide a valuable framework for generating and sustaining such communities.

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Appendix

There is Trouble in Paradise: Severe Water Problem Shortage in Cyprus

Nicosia. Alex Chris, a landscape gardener working for several foreign embassies and private estates in Nicosia, said many of the capital's boreholes are now pumping mud. "I installed one expensive garden with 500 meters of irrigation pipe in Nicosia a few months ago," he said. "Last week they called to tell me the system had stopped and their trees and lawns were dying. I found that sludge had been pumped through the pipes and then solidified in the heat. It was like cement".

Last week, people all over Cyprus received a water conservation advisory via mail, reporting "extremely dry conditions in Cyprus as a result of a lack of rain." The mail suggested several measures the Cyprus Water Board would take to conserve water. Water shortage in Cyprus is among the country's most important problems. However, water shortage, is common in much of the world. In fact, half the planet's population is expected to face an insufficient water supply by 2025.

Emergency water rationing as well as a request to import water from nearby countries was ordered as a result of a severe water shortage due to a drought over the last four years.

Reservoir reserves have plunged dangerously low and desalination plants cannot keep up with a growing demand for water. Cyprus has two desalination plants running at full capacity, with a third due to come on stream in June. The island's reservoirs are now 10.3 percent full and there has been little rainfall since 2003. "Cuts are essential to cover the needs of the population. This is an extremely grave situation," said a government spokesman. The island is increasingly relying on desalinization plants for water, but they can only provide 45% of demand, and their operation is energy heavy. The head of the Cyprus Water Board said: "We don't desalinate lightly, without being aware of the consequences. It is energy-consuming ... and this causes (greenhouse gas) emissions Cyprus that has to pay fines for."

Water has always been a valuable commodity on the Mediterranean island, which has one of the world's highest concentrations of reservoirs. The country is used to regular periods of drought due to its location and climate, but there has been a sharp decrease in rain in the past 35 years. Since 1972, rainfall has decreased 20% and runoff into reservoirs has decreased by 40%. The demand for water in Cyprus, for now, outstrips supply. Experts estimate the island will need almost 5,09 million cubic meters of water until the new year. Kouris, one of the island's largest and most important dams, currently stands less than 2.5% full, with 3.23 million tons of water.

Cypriot officials decided to sign a contract with a nearby country, to import more than 12 million cubic metres over the summer period starting at the end of June. Officials will also sign a contract with a shipping company to use oil tankers for supplying Cyprus with water. The tanker supply program will continue until a permanent solution to the problem has been reached. The water will be pumped directly into the main water supply pipelines with any surplus going to the reservoirs.

Readiness Questions

1. Who is Alex Chris?

2. What is Kourris?

3. How many desalination plants for water are currently in Cyprus?

4. Why does the Cyprus government not build more desalination plants to cover country's water needs?

5. Which solution did the Cyprus Water Board decide to adopt for solving the water shortage problem?

The Problem

Cyprus Water Board needs to decide from which country Cyprus will import water for the next summer period. Using the information provided, assist the Board in making the best possible choice.

Lebanon, Greece, Syria, and Egypt expressed their willingness to supply Cyprus with water. The Water Board has received information about the water price, how much water they can supply Cyprus with during summer, oil tanker cost and the port facilities. This information is presented below.

Write a letter explaining the method you used to make your decision so that the Board can use your method for selecting the best available option not only for now, but also for the future when the Board will have to take similar decisions.

Country	Water supply per week (metric tons)	Water Price (metric tons)	Tanker Capacity (metric tons)	Tanker Oil Cost (per 100 km)	Port Facilities for Tankers
Egypt	3 000 000	€ 4.00	30 000	€ 20 000	Average
Greece	4 000 000	€ 2.00	50 000	€ 25 000	Very Good
Lebanon	2 000 000	€ 5.20	30 000	€ 20 000	Average
Syria	3 000 000	€ 5.00	30 000	€ 20 000	Good

Address Correspondences To:
Lyn D. English
Queensland University of Technology, Australia
l.english@qut.edu.au