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Probable values of rise and fall off time of solar cycles 23, 24, and 25

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Abstract
From an analysis of the rise and fall off time of solar cycles 4 to 22, a recurrence tendency of 7 cycles is observed in the rise time and, apparently, of 9 cycles in the fall off time. The envelope of these times presents a decreasing amplitude of oscillation. According to this behavior, the rise and fall length of future solar cycles until cycle 25 can be inferred qualitatively. These values are compared with those obtained with a multiple regression method showing a good agreement.

Key words *solar cycle length – solar activity – rise time*

1. Introduction

Dates and values of future maximum and minimum sunspot number are important for studies and predictions in the ionosphere, satellites' time life determination (air density varies with solar activity), and several aspects of the Sun-Earth relationship. The sunspot number, *Rz*, presents periodicities of 11 years, 22 years or magnetic Hale cycle, and 70-90 years or Gleissberg period. International Sun Services use, among others, Waldmeier (1961) and regression methods to predict monthly mean *Rz*. These methods can only predict 12 to 18 months in advance.

The Solar Cycle Length (SCL), is a parameter of long-term variation which has a periodicity of around 90 years. This is the Gleissberg period also observed in the magnitude of the maximum sunspot number. SCL varies with solar activity in such a way that high activity implies short solar cycles whereas long solar cycles correspond to low activity levels. Danish researchers (Friis-Christensen and Lassen, 1991) have found that long-term variations of solar activity represented by SCL correlate with global temperature indicating that SCL may be associated with a variation in the solar energy output. In this sense, SCL would become an important parameter in long-term climate forecasts.

SCL can be split into a rise time, T_R, from sunspot minimum to sunspot maximum and a fall off time, T_F, from the maximum to the next minimum. In this work periodicities and amplitude of the rise and fall off time of solar cycles are analyzed, and a forecast of their behavior until cycle 25 is presented.

2. Data

T_R and T_F values were taken from Xanthakis (1967) for cycles 4 to 19 and from McNamara (1991) for cycles 20 to 22 (table I). Data covering cycles previous to cycle 4 are not sufficiently accurate and much care is needed in taking into acount the corresponding values of

Table I. Year of first minimum and year of maximum of each solar cycle and the corresponding rise and fall off time values – T_R and T_F – in years. Values have been taken from Xanthakis (1967) and McNamara (1991).

Cycle	Year of minimum	Year of maximum	T_R (years)	T_F (years)
4	1784	1787	3.4	10.2
5	1798	1804	6.9	5.5
6	1810	1817	5.8	7.1
7	1823	1830	6.6	4.0
8	1833	1936	3.3	6.3
9	1843	1847	4.6	7.8
10	1856	1860	4.1	7.1
11	1867	1870	3.4	8.3
12	1878	1883	5.0	6.2
13	1889	1893	4.5	8.0
14	1901	1907	5.3	7.5
15	1913	1917	4.0	6.0
16	1923	1928	4.8	5.4
17	1933	1937	3.6	6.8
18	1944	1947	3.3	6.9
19	1954	1957	3.6	6.6
20	1964	1968	4.1	7.6
21	1976	1979	3.5	6.8
22	1986	1989	3.0	

T_R and T_F. Therefore these cycles are disregarded in this work.

Figure 1 plots T_R values for cycles 4 to 22 in terms of the cycle number. T_R of cycles 4 to 11 (solid line), have a sequence of upward and downward trends similar to those of cycles 11 to 18 (dotted line). A recurrence tendency of seven cycles, already suggested by King-Hele (1963), is observed. Cycles 18 to 22 (dashed line), except cycle 20, show the same pattern.

If the recurrence tendency of 7 cycles is maintained, the same 7-cycle behavior should be expected for cycles 18 to 25 as that of cycles 4 to 11 and 11 to 18.

Figure 2 shows T_R variation for cycles 4 to 22. The envelope has a periodicity of 7 cycles and its amplitude decreases after each period. T_R values which correspond to the minimum of the envelope, fluctuate slightly around 3.4 years.

Based on the hypothesis of a seven cycle recurrence tendency and taking into account the decreasing trend observed in T_R values (fig. 2), the rise time behavior could be inferred until cycle 25. Further cycles are not subject to prediction since the envelope amplitude trend could decrease still more or increase.

This argument leads to a qualitative estimation of T_R values of 4.0, 3.5 and 3.0 for cycles 23, 24 and 25 respectively (triangles in figs. 1 and 2).

Figure 3 shows T_F in terms of the cycle number. T_F for cycles 7 to 16 (solid line) presents the same sequence of slope signs as T_F for cycles 16 to 21 (dashed line), pointing out a recurrence tendency of 9 cycles in T_F.

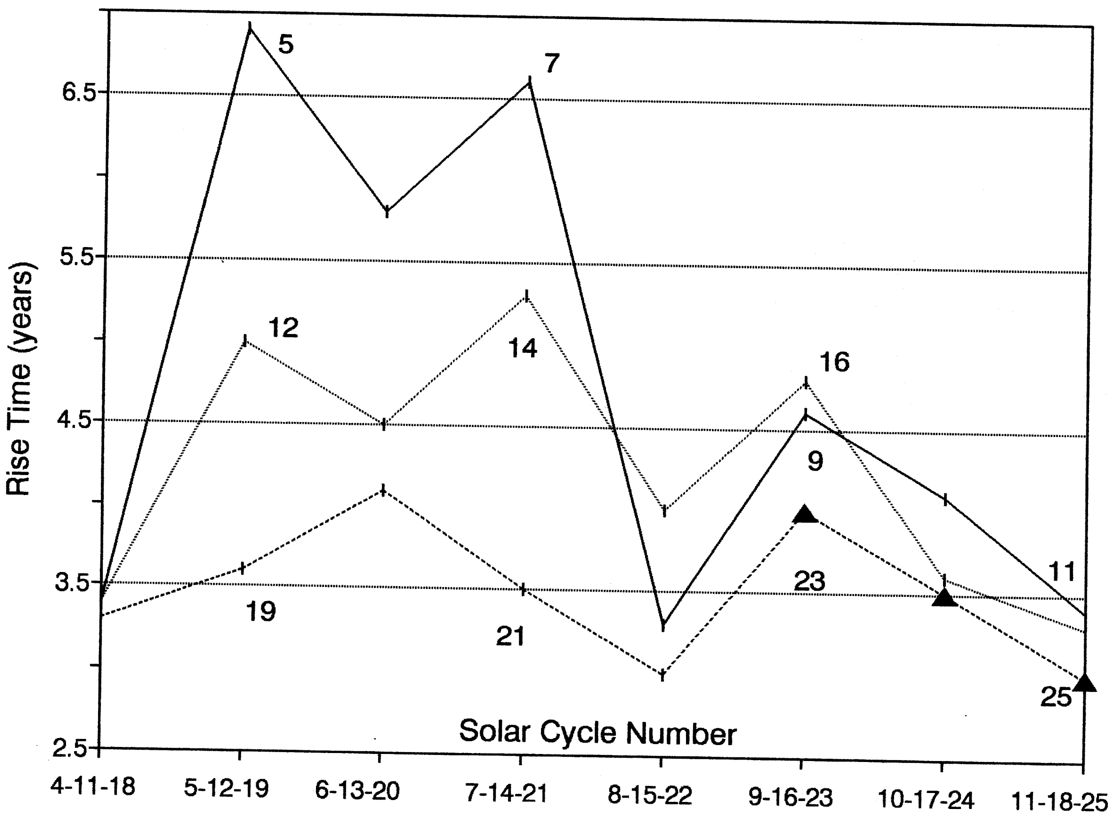

Fig. 1. T_R values in terms of the cycle number separated into: cycles 4 to 11 (solid line), cycles 11 to 18 (dotted line), and cycles 18 to 25 (dashed line). The triangles are the predicted values.

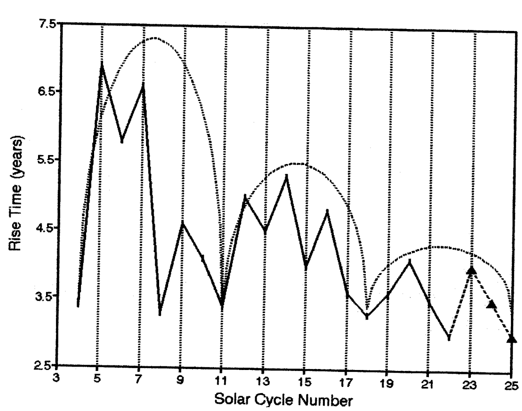

Fig. 2. T_R from cycle 4 to cycle 22 (solid line) and its envelope (dotted line). The triangles are the predicted values.

Figure 4 shows T_F for cycles 4 to 21, enveloped by a curve of decreasing amplitude and a 9 cycle period. Unlike T_R, T_F values corresponding to the minimum of the envelope do not seem to keep a constant value.

As in the case of T_R, T_F values for cycles 22 to 25 could be qualitatively estimated assuming a persisting 9 cycle recurrence tendency and an amplitude decreasing trend of the envelope.

Since T_F of cycle 22 must be smaller than T_F of cycle 13 (decreasing amplitude of the envelope curve), and greater than that corresponding to cycle 21 (to keep the slope), T_F is estimated to be about 7.2 years. T_F values for cycles 23, 24 and 25 (dashed line) are calculated in the same way. In figs. 3 and 4, triangles represent the predicted T_F values.

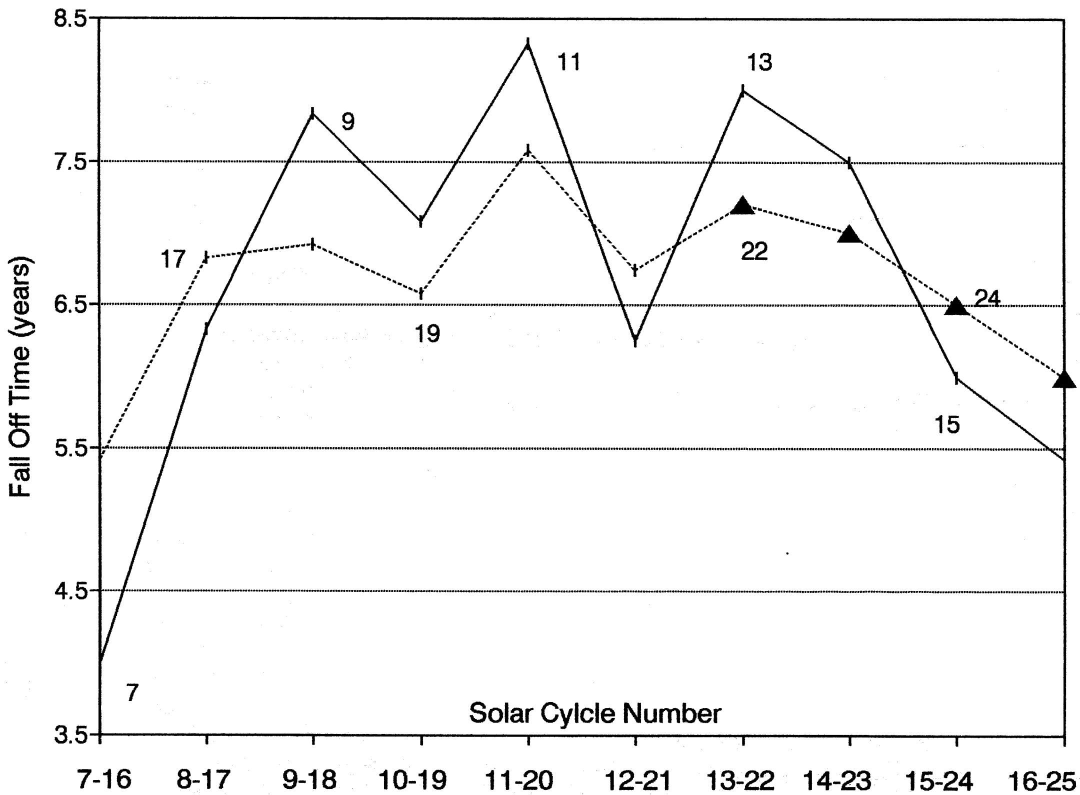

Fig. 3. T_F values in terms of the cycle number separated into: cycles 7 to 16 (solid line), and cycles 16 to 25 (dashed line). The triangles are the predicted values.

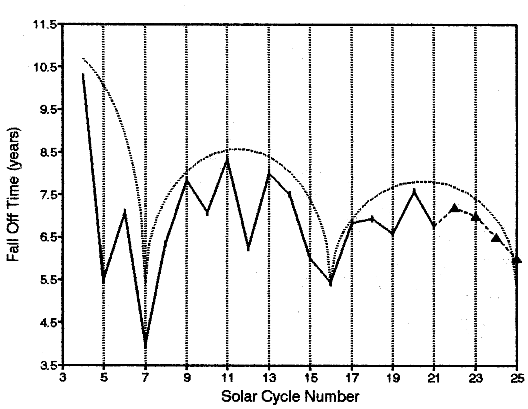

Fig. 4. T_F from cycle 4 to cycle 22 (solid line) and its envelope (dotted line). The triangles are the predicted values.

3. Multiple linear regression method

To obtain a quantitative estimation of future T_R and T_F values, a multiple linear regression method was applied to both data series. T_R and T_F values are considered as time series y_1, y_2, ..., y_N, where N is the number of data. $N=19$ (cycles 4 to 22) in the case of T_R and $N=18$ (cycles 4 to 21) in the case of T_F.

A multiple linear regression of the general form

$$y_t = \sum_{i=1}^{n} \alpha_i x_i + \beta \qquad (3.1)$$

where α_1, ..., α_n and β are constants, and x_i's are the independent variables, was fitted to T_R

and T_F data series. The independent variables x_i were chosen as lagged values of y_t, and so eq. (3.1) can be rewritten as follows:

$$y_t = \alpha_1 y_{t-1} + \ldots + \alpha_n y_{t-n} + \beta. \qquad (3.2)$$

To choose the lagged values to be considered in eq. (3.2), an autocorrelation analysis of each series was made. From this analysis the main periodicities resulted: 3, 7 and 12 cycles for T_R; and 3, 9 and 12 cycles for T_F. The corresponding lagged y_i values explain around 90% of the variability of the series in each case. So, the independent variables in the multiple regression to be considered in eq. (3.2) are: y_{t-3}, y_{t-7}, y_{t-12} for T_R, and y_{t-3}, y_{t-9}, y_{t-12} for T_F.

The regression coefficients were calculated by the least squares method.

To evaluate the method's performance, the regression coefficients were calculated with all the series data except the last three. As can be seen from the third column of tables II and III, there is a good agreement between the predicted values for the last three cycles and the corresponding measured values of T_R and T_F (second column of tables II and III).

Considering now the complete series, the regressions obtained are

$$y_t = 0.72\, y_{t-3} + 0.20\, y_{t-7} - 0.42\, y_{t-12} + 2.00 \quad \text{for } T_R \qquad (3.3)$$

$$y_t = 0.35\, y_{t-3} + 0.52\, y_{t-9} - 0.06\, y_{t-12} + 1.25 \quad \text{for } T_F. \qquad (3.4)$$

Figures 1 and 3 plott the predicted rise and fall off times obtained qualitatively (triangles),

Table II. First column: solar cycle number; second column: measured rise time, T_R; third column: predicted T_R from the regression equation without considering the last three cycles; forth column: T_R estimated from eq. (3.3). Shadowed values are the calculated ones.

Cycle	T_R measured	T_R estimated without the last three cycles	T_R estimated with eq. (3.3)
20	4.1	4.3 ± 0.6	4.1 ± 0.5
21	3.5	3.5 ± 0.6	3.4 ± 0.5
22	3.0	3.8 ± 0.6	3.6 ± 0.5
23			4.5 ± 0.5
24			3.0 ± 0.5
25			2.8 ± 0.5

Table III. First column: solar cycle number; second column: measured fall off time, T_F; third column: predicted T_F from the regression equation without considering the last three cycles; fourth column: T_F estimated from eq. (3.4). Shadowed values are the calculated ones.

Cycle	T_F measured	T_F estimated without the last three cycles	T_F estimated with eq. (3.4)
19	6.6	6.5 ± 0.6	6.6 ± 0.4
20	7.6	7.6 ± 0.6	7.6 ± 0.4
21	6.8	6.5 ± 0.6	6.4 ± 0.4
22			7.3 ± 0.4
23			7.3 ± 0.4
24			6.4 ± 0.4
25			6.1 ± 0.4

which agree fairly well with the corresponding values estimated from regression eqs. (3.2) and (3.3) (fourth column of tables II and III).

4. Conclusions

A recurrence tendency of 7 solar cycles in the rise time and of 9 cycles in the fall off time are pointed out. These patterns together with the envelope shape of each series, allow the values of T_R and T_F to be forecast until cycle 25. Both time values will decrease in average, thus predicting a decrease in SCL at least until solar cycle 25.

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