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Modelling the light-ion densities in the ionosphere

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Abstract
A steady-state theoretical model is used to obtain variations of the H^+/O^+ and He^+/O^+ density ratios in the upper ionosphere at middle latitudes. The model results are compared with the existing data from satellite measurements. Analytical functions are constructed approximating the latitude and altitude variations of these ratios.

Key words *mathematical modelling – light-ion density – improving empirical models*

1. Introduction

Modelling the densities of the light ions, H^+ and He^+, for different geographic locations and various ionospheric conditions is a problem presenting two main difficulties. *First*, the experimental data (satellite, rocket, or ground-based) are too few to completely embrace the altitude variations of the light-ion densities with respect to local time, solar and geomagnetic activity, season, and latitude. This shortage restrains the validity of some empirical models to narrow ranges of the mentioned parameters (Taylor *et al.*, 1978; Köhnlein, 1981). Moreover, the problems with the instrument calibration impede the use of data from different satellites and from different measurement techniques. *Second*, the purely theoretical approach is not very helpful. The problems with the existing theoretical models are in the exact specification of boundary values (especially the ion fluxes at the upper boundary), the complex nature of ion chemistry, the choice of adequate neutral atmosphere models, etc.

Here, a different approach to the modelling of the light-ion densities is proposed, combining the advantages of empirical and theoretical modelling. It is focused on modelling the ion composition (the individual ion densities normalized to the total ion density) rather than on the individual ion densities (Köhnlein, 1989a). On both sides of the O^+-H^+ and O^+-He^+ transition heights the corresponding density ratios $n(H^+)/n(O^+)$ and $n(He^+)/n(O^+)$ are calculated by using a theoretical ionospheric model and the solutions are compared with satellite measurement data. Analytical expressions are constructed giving the altitude variations of the density ratios depending on dipole latitude. The theoretical model might then use an empirical model of transition levels to obtain the variations of the above density ratios due to local time, longitude, season, etc. A possible implementation of the constructed formulae in existing empirical models is discussed, *e.g.* for improving the International Reference Ionosphere (IRI) model (Bilitza, 1992).

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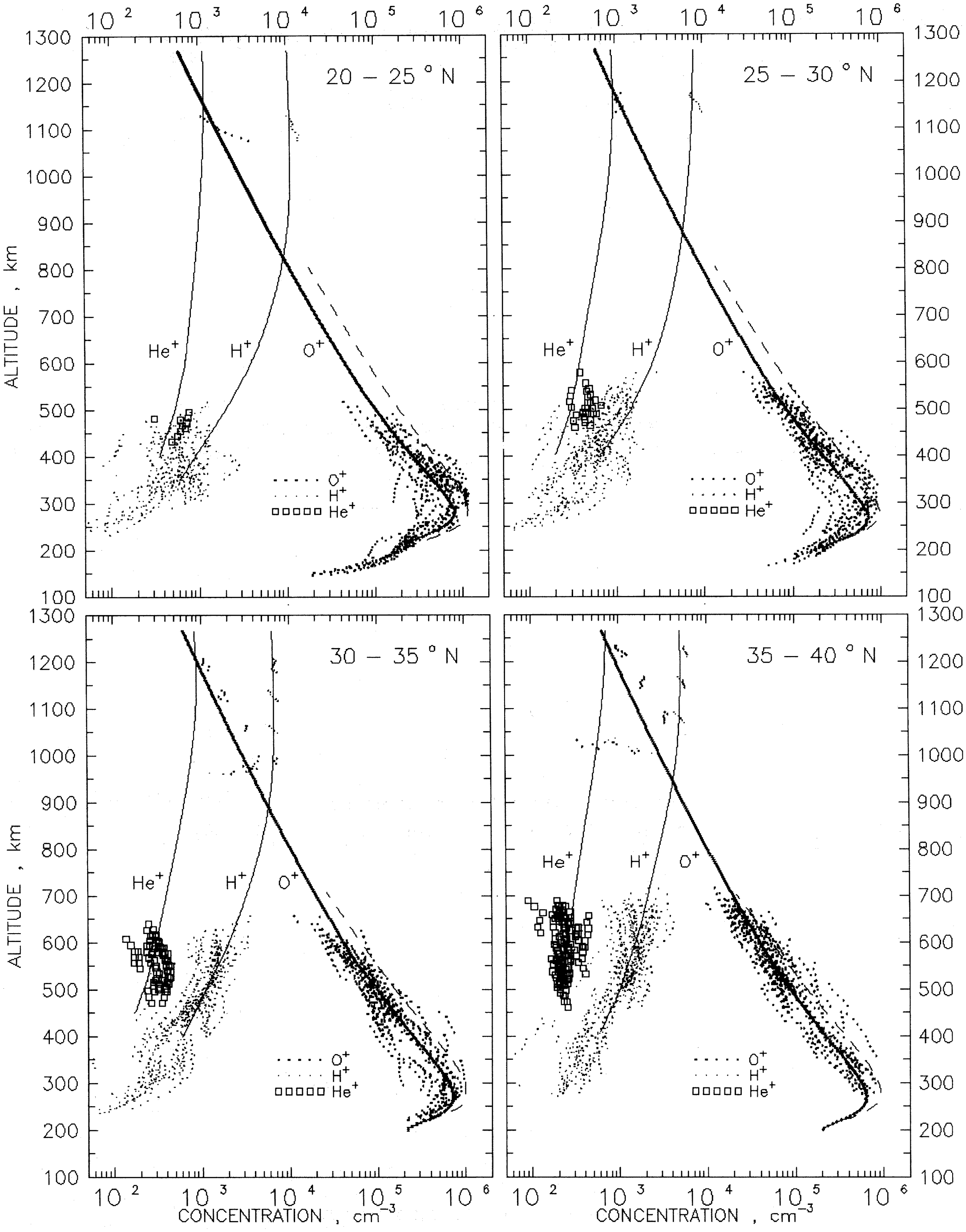

Fig. 1. Model results compared with AE-C measurements for different latitudes.

2. Mathematical model

The mathematical model (Stankov, 1996) used in this study is steady state and based on the equations (numerically solved) of continuity, momentum and energy balance for the basic ions in the upper ionosphere and the plasmasphere – O^+, H^+, and He^+. A centred dipole approximation to the Earth's magnetic field is used. The model calculates the ion concentrations and velocities, the ion and electron temperatures along a given magnetic field line from a start altitude (~ 200 km) to an arbitrary upper altitude. Being not self consistent, it requires a number of quantities as input parameters. The key parameters are the neutral atmosphere, the solar EUV flux, the neutral wind velocity, etc. The model is developed for an extensive use of satellite measurement data, so the inputs are given at a «base» altitude anywhere on the field line. Also, boundary (initial) values are required only at one «start» point and the length of the integration interval is freely chosen. As an extension of the steady-state model, an original numerical procedure is developed for determination of the boundary values in a self-consistent manner.

3. Data base

For the purpose of this study, ion composition data from the *Atmosphere Explorer* (AE-C) satellite is used. The O^+, H^+, He^+ ion concentrations, measured by a Bennet Radio Frequency Mass Spectrometer (BIMS), are collected for the autumn equinox period: 1/10/1974-31/10/1974. This period is characterized with low solar ($F_{10.7}$ ~ 90) and moderate geomagnetic (A_p ~ 20) activities. Noon conditions are considered, so measurements carried out in the local time period 10:00-14:00 LT are only extracted. The data from all the longitudes and within the invariant latitude range 20-40°N are sorted into four groups (fig. 1) according to latitude: 20-25°, 25-30°, 30-35°, and 35-40°N. Most of the data are in the altitude range 150 to 700 km. Due to satellite evolution, the data from higher latitudes cover higher altitudes. There are just a few data between 900 and 1300 km height, but they are very important for comparing the theoretical profiles with the measurements at and above the transition levels.

The noon O^+ profile is well described in the *F*-region. The peak O^+ density tends to slightly decrease at the higher latitudes. The H^+ and He^+ data are much more scattered than the O^+ data. The figure shows a pronounced decrease in the H^+ density with increasing latitude, especially at higher altitudes. The available He^+ data demonstrate a similar behaviour.

4. Comparison between model results and measurement data

The model was started along the field lines at $L = 1.20, 1.34, 1.49, 1.63$, at 0° longitude. Adopting the neutral atmosphere from the Mass Spectrometer and Incoherent Scatter (MSIS) model (Hedin, 1991) for day 274, the O^+, H^+, He^+ density profiles are calculated for every hour from 10:00 to 14:00 LT. The averaged results are given in fig. 1. The model tends to overestimate the O^+ density, especially at lower altitudes (the dashed lines in fig. 1). The O^+ profiles had to be adjusted to match the measurements. New profiles (the solid lines) were obtained after a 5-10 % decrease in the initial value of the electron temperature, T_e, at the base height of 300 km. The theoretical H^+ and He^+ profiles are also corrected by reducing the H and He neutral densities. This reduction is different for each of the four cases and varies between 2 and 5 times.

5. Results: analytical functions for the H^+/O^+ and He^+/O^+ density ratios

Here only the method for obtaining analytical expressions for the light-ion density ratios will be demonstrated by using the corrected results described above. First, the altitude variations of $n(H^+)/n(O^+)$ and $n(He^+)/n(O^+)$ are calculated for each of the latitude ranges. Thus, after smoothing, four altitude profiles of $n(H^+)/n(O^+)$ ratio (the left panel of fig. 2) and four profiles of the $n(He^+)/n(O^+)$ ratio (the right

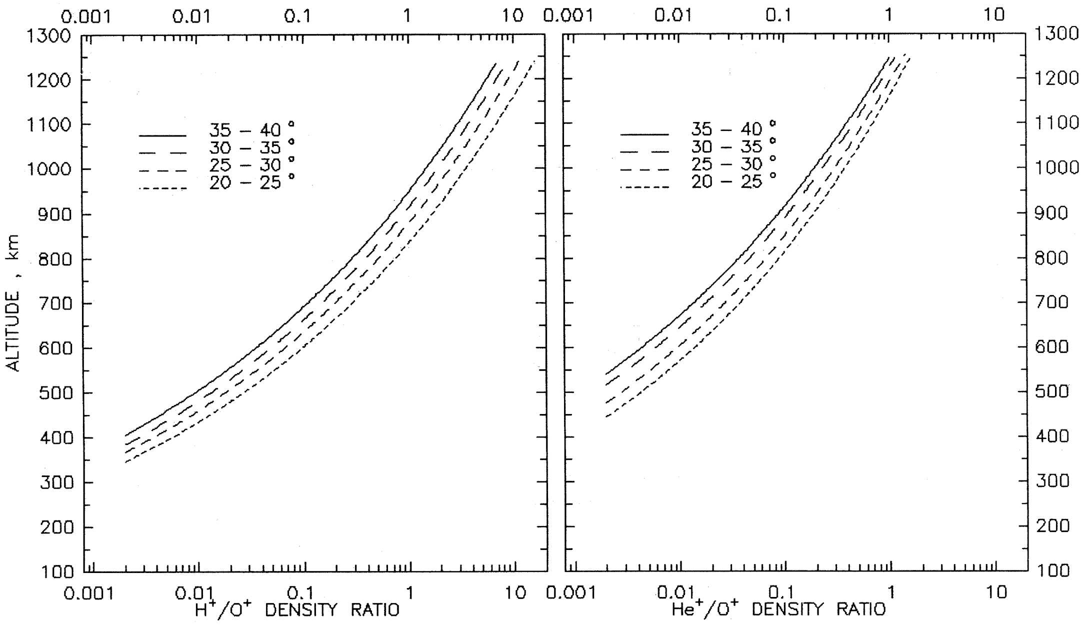

Fig. 2. Altitude variations of the modelled H^+/O^+ (left panel) and He^+/O^+ (right panel) density ratios for different latitudes.

panel of fig. 2) are obtained. The four ratio profiles were used to derive a latitude dependence.

Finally, the altitude and latitude ratio variations were approximated by the two-variable function

$$f_i\,(x,\,y) = a_i\,\exp\,(b_i\,y)\,x^{(c_i y + d_i)}, \qquad i = 1,\,2$$

where x is the altitude, y is the dipole latitude, $i = 1$ stands for $n(H^+)/n(O^+)$ and $i = 2$ stands for $n(He^+)/n(O^+)$. The coefficients are

$$a_1 = 2.05149\times10^{-19} \qquad b_1 = -0.184102$$

$$c_1 = 0.018306 \qquad d_1 = 6.59654$$

$$a_2 = 4.78135\times10^{-16} \qquad b_2 = -0.455081$$

$$c_2 = 0.059176 \qquad d_2 = 5.11627$$

which are valid for the conditions stipulated above, *i.e.* equinox, noon, low solar activity, 20-40°N dipole latitude. Additional satellite data are needed to obtain coefficients for other conditions.

6. Discussion

Analytical functions, approximating the $n(H^+)/n(O^+)$ and $n(He^+)/n(O^+)$ ratios, have been constructed for low solar activity using a theoretical model and satellite measurements. The formulae are very useful for calculating the H^+ and He^+ densities when the O^+ density is available, *e.g.* from an empirical model. Thus, they might be used with existing empirical models to improve their ion composition part.

The most adequate ionospheric model to date is the IRI. This empirical model provides the ion composition, which is preferable because the empirical models of the total ion density variations are well established (Danilov

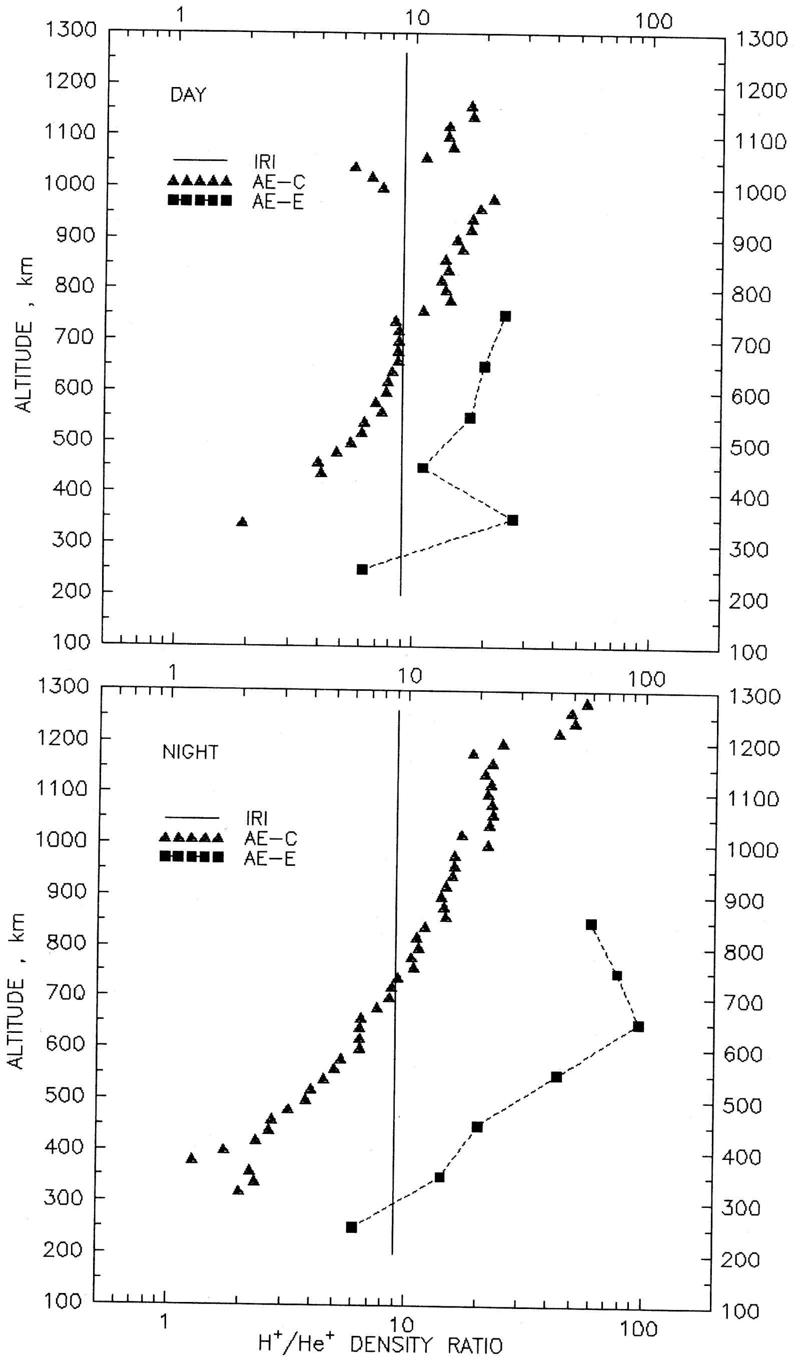

Fig. 3. Daytime (upper panel) and nighttime (lower panel) altitude variations of the H^+/He^+ density ratio as modelled by IRI (solid line) and measured at middle (▲) and equatorial (■) latitudes.

and Yaichnikov, 1985; Bilitza, 1990). However, the IRI ion composition needs further improvement (Hoegy *et al.*, 1991; Bilitza *et al.*, 1993b; Hoegy and Grebowsky, 1994). The basic difficulties encountered so far are: (i) the great variability of the light ion densities; (ii) the instrument calibration, and (iii) the limited data base (a huge amount of data from recent satellites are not considered). As a consequence, the percentage of light ions is overestimated. Also, the model shows the $n(H^+)/n(He^+)$ ratio to be almost constant with varying altitude, which in most of the cases is not valid. The latter is clearly demonstrated in fig. 3 for equinox conditions during low solar activity. On the upper panel daytime conditions are presented, and on the lower one – nighttime. The solid lines represent the $n(H^+)/n(He^+)$ ratio as modelled by the IRI, *i.e.* equal to 9. For comparison, satellite measurements by the AE-C satellite are also given (Stankov, 1996). The data (in fig. 3 denoted with triangles) are collected for equinox at middle latitudes and are averaged over 20 km in altitude. At lower heights, the $n(H^+)/n(He^+)$ ratio is small – about 2, but it increases with altitude; at 1300 km the values are 20 (during the day) and 60 (during the night). The ratio varies not only with altitude but also with latitude and solar activity. It becomes obvious if comparing the above AE-C data with measurements from AE-E in the equatorial ionosphere (Gonzalez *et al.*, 1992) for slightly lower solar activity. Much greater values at all altitudes are recorded (the squares in fig. 3). The discrepancy between the IRI values and AE-E measurements is more pronounced.

The use of results from a theoretical model calculation bound to satellite data is appropriate and would not change the empirical nature of IRI. The functions presented here depend on altitude and latitude. The variations due to solar activity and season should be considered as well. To reliably extend the formulae to other conditions, more data are required. However, the use of satellite data in the way described in this paper is not always possible – the ion composition measurements might be insufficient, difficult to deduce, or not available, the measurements might be in a narrow altitude range, etc. From the modelling aspect, valuable information can be obtained from the O^+-H^+ and O^+-He^+ transition heights if known. In fig. 2, the altitudes where $n(H^+)/n(O^+)$ and $n(He^+)/n(O^+)$ ratios are equal to 1 are exactly the modelled transition heights. Empirical models of O^+-H^+ and O^+-He^+ transition levels will be very helpful for two main reasons. *First*, the transition level will serve as an anchor point, when theoretically calculating the density profile. Thus, the uncertainty in the adopted neutral atmosphere model will be overcome. The influence of the adopted atmosphere on the model results (especially during daytime) was analyzed (Kutiev and Stankov, 1994) and a high sensitivity demonstrated. *Second*, the transition-level model provides the important variations in respect to solar activity, local time, month, latitude, and longitude.

An empirical model of the O^+-H^+ transition level has been already developed (Kutiev *et al.*, 1994). The global O^+-H^+ transition surface is based on OGO-6, IK(Bulgaria)-1300, Alouette-1, ISS-b, and other satellite measurement data. The transition-level variations are approximated by a fitting generalized polynomial using algebraic, trigonometric, or Tchebishev's basis.

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