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Tidal analysis of data recorded by a superconducting gravimeter

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Abstract
A superconducting gravimeter was used to monitor the tidal signal for a period of five months. The instrument was placed in a site (Brasimone station, Italy) characterized by a low noise level, and was calibrated with a precision of 0.2%. Then tidal analysis on hourly data was performed and the results presented in this paper; amplitudes, gravimetric factors, phase differences for the main tidal waves, M_2, S_2, N_2, O_1, P_1, K_1, Q_1, were calculated together with barometric pressure admittance and long term instrumental drift.

Key words *Earth tides – tidal analysis*

1. Introduction

The tidal parameters which describe the elastic response of the Earth to the gravitational attraction of moon and sun, are in general dependent on the tidal family (diurnal, semi-diurnal, etc.), the tidal frequency and location of stations; oceanic load, barometric pressure, topographic and geological features, induce regional anomalies of these parameters with respect to the more recent global models of the Earth. For example, the gravimetric factor (amplitude factor) of a region, defined as the Earth transfer function between the tidal force and the tidal gravity change, is not easily determinable without direct measurements using modern gravimeters. For this reason we recorded continuously for five months the tidal signal at a station located in the Apennines near Bologna (Italy, 44.11°N, 11.10°E) by means of a superconducting gravimeter designed by «GWR Instrument», in the frame of an experimental program to verify the validity of Newton's law (Achilli *et al.*, 1990, 1991).

The superconducting gravimeter differs from the traditional instruments with mechanical spring suspension because it uses magnetic levitation of a sphere, realized using persistent currents superconducting field coils. The inherent stability of supercurrents is used to produce a gravimeter of high stability and precision. The theoretical instrumental sensitivity is of the order of 0.001 μGal; geophysical and environmental noise reduce the effective accuracy to 0.1 μGal or better (Goodkind, 1991).

The gravimeter signal corresponds to a feedback force used to hold the levitating sphere in a fixed position when the gravity force changes; in fact the levitating force from the field coils is constant in time, and the position of the sphere is sensed by an imbalance in a capacitance bridge formed by plates above and below the spherical mass and a ring around its center.

The feedback force is provided by a variable magnetic field generated by a small coil underneath the sphere, the voltage applied to the feedback system is a measure of the force required to hold the sphere at the central position and is linearly dependent on the gravity variations.

For this reason a calibration of the gravimeter is necessary in order to obtain accurate determination of the scale factor.

Up to now the most accurate approach to obtain calibration consists in comparing the superconducting gravimetric tidal signal with the corresponding one of an absolute gravimeter; in this way a calibration at 1% level was obtained (Hinderer *et al.*, 1991).

In our case we adopted a method which consists in moving a mass which produces a known change of the gravity field (6.729 μGal).

The design of the apparatus, an annular mass of about 273 kg placed around the meter, reduces systematic errors (*e.g.* due to an erroneous determination of the mass position) well below the statistical accuracy of the method, which is 0.2% (Achilli *et al.*, 1995).

2. Data analysis

The tidal data analyzed were measured at the Brasimone station (4O°07′N, 11°07′E) from 1992 August 1 to 1992 December 31.

To generate the data we used the instantaneous signal recorded at a rate of 0.5 s filtered by means of a Low-Pass electronic Butterworth filter to eliminate the high frequency noise (Warburton and Goodkind, 1978). This generated a signal with a 20 s sampling interval; a standard format modification with detection and flagging of interruptions was applied and 1 min sampled data were generated.

In a second step we computed hourly sampled data by adjusting a polynomial on a 23

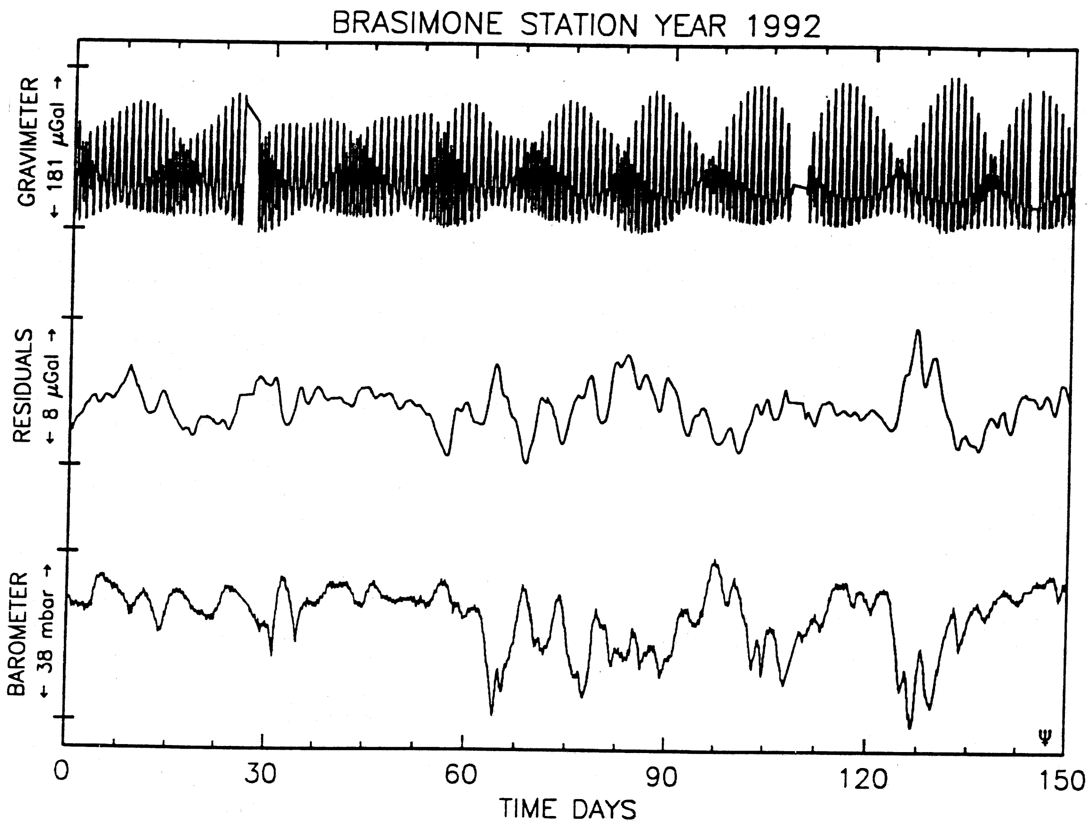

Fig. 1. Gravimeter signal, residual after removal of the tides and atmospheric pressure.

min time interval, centered around the full hour. Then a least square fitting of synthetic tidal signal was performed to recover from gaps when the interruptions did not exceed a few hours. Remaining anomalies in the tidal curve, like offsets or spikes, were detected applying a Lecolazet filter, and eliminating the erroneous data manually (fig. 1).

The tidal analysis itself consisted in two steps according to the technique developed by Venedikov (Venedikov, 1966; Ducarme, 1993), starting from the harmonic formulation which describes the reading $l(T)$ at the epoch T:

$$l(T) = \sum_j H_j \cos\left[\omega_j (T - T_0) + \phi_j\right] + C + D \qquad (2.1)$$

where H_j is the observed amplitude of wave of frequency ω_j, ϕ_j is the observed phase of the same wave at the conventional fixed epoch T_0, C is a constant depending upon the choice of the reference and D represents the instrumental drift, dependent on time (Cartwright and Tayler, 1971).

Different band pass filters were applied to the hourly sampled data to separate the three main tidal bands (diurnal (D), semi-diurnal (SD) and ter-diurnal (TD)), as well as the drift; in particular an even filter C and an odd filter S were applied on the sequences of 48 hourly readings around the epochs T_i; introducing for any sequence the time scale t, the amplification factors of these filters are given by:

$$c_j^{(\tau)} = \sum_{t=-23.5}^{t=+23.5} C_t^{(\tau)} \cos \omega_j t \qquad \tau = 1, 2, 3. \qquad (2.2)$$

$$s_j^{(\tau)} = \sum_{t=-23.5}^{t=+23.5} S_t^{(\tau)} \sin \omega_j t \qquad \tau = 1, 2, 3. \qquad (2.3)$$

The diurnal filters, for example, are harmonic functions with period $T = 24$ h and are thought to amplify the diurnal components and to eliminate the others. In this case the amplification factors range between 3.64 and 4.50 for the diurnal species and are exactly 0 for the N_2, M_2 and S_2, while they are smaller than 0.01 for all the other bands.

The properties of the semidiurnal wave filters amplification factors, are opposite: 0 for O_1, P_1 and S_1 components and 0.0005 for the K_1 (Melchior, 1983).

Introducing the time scale t in eq. (2.1), we can write:

$$l_{i,t} = \sum_j H_j \cos\left[\omega_j t + \phi_j (T_i)\right] + C + D \qquad (2.4)$$

$$\phi_j (T_i) = \phi_j + \omega_j (T_i - T_0).$$

The filters are applied to the hourly reading $l_{i,t}$ every continuous 48 h recording and then shifted from 48 h in order to obtain three new series of n independent data at epochs T_i.

$$M_i^{(\tau)} = \sum_{t=-23.5}^{t=+23.5} C_t^{(\tau)} l_{i,t} \qquad (2.5)$$

$$N_i^{(\tau)} = \sum_{t=-23.5}^{t=+23.5} S_t^{(\tau)} l_{i,t} \qquad (2.6)$$

$$i = 1, 2, \ldots n, \qquad \tau = 1, 2, 3.$$

According to eq (2.4), and adopting the Venedikov's approach which distributes the tidal development into p groups of waves with index numbers which range between α_k and β_k, the following system of $2n$ equations in $2p$ unknowns, are obtained:

$$M_i = \sum_{k=1}^{p} \left[\xi_k \sum_{j=\alpha_k}^{\beta_k} c_j h_j \cos \phi_j (T_i) + \right.$$

$$\left. + \eta_k \sum_{j=\alpha_k}^{\beta_k} c_j h_j \sin \phi_j (T_i)\right], \qquad (2.7)$$

$$N_i = \sum_{k=1}^{p} \left[\xi_k \sum_{j=\alpha_k}^{\beta_k} s_j h_j \sin \phi_j (T_i) + \right.$$

$$\left. + \eta_k \sum_{j=\alpha_k}^{\beta_k} s_j h_j \cos \phi_j (T_i)\right], \qquad (2.8)$$

where:

$$H_j = \delta_k h_j ,$$

$$\phi_j (T_j) = \bar{\phi}_j (T_j) + \Delta_{\phi_k} ,$$

$$\xi_k = \delta_k \cos \Delta_{\phi_k} ,$$

$$\eta_k = -\delta_k \sin \Delta_{\phi_k} ,$$

$$\alpha_k \leq j \leq \beta_k , \qquad k = 1, 2, \ldots, p.$$

where $\bar{\phi}$ and h are the theoretical phase and amplitude based on Cartwright-Tayler-Edden development (Cartwright and Tayler, 1971; Cartwright and Edden, 1973), δ_k represents the gravimetric factors, $\Delta_{\phi k}$ the phase differences of each tidal wave.

A least square adjustment resolves the unknowns ξ and η and then one obtains:

$$\delta_k^2 = \xi_k^2 + \eta_k^2, \qquad \Delta_{\phi_k} = \mathrm{arc\,tg}\,(-\eta_k / \xi_k). \qquad (2.9)$$

The residuals of this first solution are then analyzed for the computation of the influence of atmospheric pressure variation and the estimation of the long-term drift.

Generally the barometric fluctuations are the major cause of random fluctuation of gravity; this is due to the gravitational attraction from the air and to the distortion of the surface of the earth resulting from the pressure change (Warburton and Goodkind, 1978). Comparing the gravimeter residual, after removal of the tides, with the barometric pressure, we found a strong correlation (fig. 1) in the 1 – 0.1 cycle/day, and a mean pressure admittance of -0.201 ± 0.007 μGal/mbar; the instrumental drift was linear (2.709 ± 0.001 μGal/day).

Finally, using the data corrected for the pressure effect and drift, a final least square solution was computed. These results are shown in (table I), were the different tidal waves are classified adopting the Darwing symbols (Melchior, 1983); the following quantities are

Table I. Results of the least square analysis on the basis of the Cartwright-Tayler-Edden potential development; the data used are those corrected for the pressure effects. The first column gives the Darwin symbols (O_1 principal diurnal lunar wave; P_1 diurnal solar wave; M_2 semi-diurnal lunar wave, etc.).

Wave	H Estimated amplitude μGal	δ Amplitude factor	Δ_ϕ Phase differences (°)
Q_1	6.91 ± 0.03	1.1618 ± 0.0052	– 0.18 ± 0.26
O_1	35.99 ± 0.03	1.1590 ± 0.0010	– 0.05 ± 0.05
NO_1	2.82 ± 0.02	1.1537 ± 0.0093	0.38 ± 0.46
P_1	16.83 ± 0.03	1.1646 ± 0.0019	0.10 ± 0.09
S_1K_1	50.03 ± 0.03	1.1455 ± 0.0006	0.07 ± 0.03
J_1	2.83 ± 0.03	1.1578 ± 0.0118	– 0.41 ± 0.58
OO_1	1.50 ± 0.03	1.1210 ± 0.0193	1.17 ± 0.99
$2N_2$	1.37 ± 0.01	1.1607 ± 0.0102	0.48 ± 0.50
N_2	8.71 ± 0.01	1.1752 ± 0.0019	1.48 ± 0.09
M_2	46.00 ± 0.01	1.1883 ± 0.0004	0.94 ± 0.02
L_2	1.29 ± 0.02	1.1752 ± 0.0194	2.26 ± 0.94
S_2	21.44 ± 0.01	1.1905 ± 0.0008	– 0.01 ± 0.04
K_2	5.82 ± 0.01	1.1890 ± 0.0030	0.436 ± 0.14
M_3	0.59 ± 0.01	1.0722 ± 0.0153	– 1.63 ± 0.80

Table II. Comparison between the B vector and the oceanic loading vector L, giving the final residual X (Melchior and Ducarme, 1989); β, λ, χ represent the corresponding phase differences.

Wave	δ	Δ_ϕ (°)	B μGal	β (°)	L μGal	λ (°)	X μGal	χ (°)
Q_1	1.1618	– 0.18	0.05	– 23.0	0.04	– 126.7	0.07	8.1
O_1	1.1590	– 0.05	0.17	– 10.8	0.13	178.2	0.30	– 5.3
P_1	1.1646	0.10	0.24	7.1	0.05	98.9	0.24	– 4.7
K_1	1.1455	0.07	0.18	19.8	0.12	88.2	0.35	– 19.5
N_2	1.1752	1.48	0.26	60.1	0.29	76.4	0.08	– 44.0
M_2	1.1883	0.94	1.38	33.0	1.43	59.4	0.64	– 47.8
S_2	1.1905	– 0.01	0.58	– 0.4	0.48	32.7	0.31	– 56.3
K_2	1.1890	0.44	0.15	16.9	0.13	31.3	0.04	– 33.6

listed: amplitudes, gravimetric factors δ_k and phase differences $\Delta_{\phi k}$; the high quality of the tidal model obtained is due both to the high stability of the GWR gravimeter and to the good quality of site, characterized by a very low noise level.

The tidal parameters, compared with the Wahr-Dehant model of an ocean-free, elastic, rotating, isotropic, ellipsoidal Earth (Wahr, 1981; Dehant, 1987; Dehant and Ducarme 1987), give residual vector (B, β); table II lists the final values of δ_k and $\Delta_{\phi k}$, the vector B, the oceanic loading vector L derived from the cotidal charts of Schwidersky (Schwidersky, 1980; Melchior and Ducarme, 1989) and the residual $\overline{X} = \overline{B} - \overline{L}$, which partially may be described both to the strong lithospheric heterogeneities of the region, and to a perturbation by the Mediterranean and Adriatic Seas whose effect should not reach in any case a level higher than 0.25 μGal (Melchior, 1983).

3. Conclusions

Tidal gravity variation recorded by a superconducting gravimeter which worked for five months in a station located in the Apennines near Bologna (Italy), gave us the opportunity to define accurately the gravimetric factors, the phase differences of some tidal waves and the pressure admittance; the computed values may be used to reproduce a precise tidal correction for other epochs and positions in the Italian region.

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