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The improved DGR analytical model of electron density height profile and total electron content in the ionosphere

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Abstract
Tests of the analytical model of the electron density profile originally proposed by G. Di Giovanni and S.M. Radicella (DGR model) have shown the need to introduce improvements in order to obtain a model able to reproduce the ionosphere in a larger spectrum of geophysical and time conditions. The present paper reviews the steps toward such progress and presents the final formulation of the model. It gives also a brief review of tests of the improved model done by different authors.

Key words *ionosphere – electron density profile*

1. Introduction

Basic criteria that should be taken into account to judge the usefulness of empirical models of electron density height profile in the ionosphere have been described by Dudeney and Kressman (1986) and can be summarized as follows: 1) the profile shape obtained by ionogram inversion or other techniques should be well reproduced, without gradient discontinuities in the first and second derivatives; 2) input data required should be limited ideally to those available through routine scaling of the ionograms; 3) mathematical formulations should be simpler than the ionogram inversion methods. These criteria are difficult to achieve in each given model.

The model introduced originally by Di Giovanni and Radicella (1990) [from now on: DGR] represents an attempt to model the electron density profile in such a way as to fulfil, to a reasonable extent, the three criteria given above by using simple analytical expressions.

It is known (Gulyaeva, 1984; Serafimov, 1985; Radicella and Mosert de Gonzalez, 1988; Mosert de Gonzalez and Radicella, 1990) that the electron density profile shows at least two characteristic points below the maximum of electron density in the F region in terms of the variation with height of the function *dN/dh.* One is defined by a maximum of this function – always present at the base of the F_2 layer – and the other by a minimum or null that is seen only when a clearly developed F_1 ledge is present. Mosert de Gonzalez and Radicella (1987, 1990) and Radicella and Mosert de Gonzalez (1991) [from now on: MG-R 87 and 90 and R-MG 91] have shown that the height and electron density of these points can be calculated with reasonable precision using simple empirical relations involving parameters routinely scaled from the ionograms. For the characteristic point at the base of the F_2 layer the value of the gradient *dN/dh* can also be calculated using a simple empirical relation (MG-R 90). The

DGR model has proved able to reproduce the existence of these characteristic points observed in the experimentally deduced profiles. Moreover, the model is built on the assumption that the height, electron density and electron density gradient (dN/dh) at these characteristic points are known or can be computed from routinely scaled data.

The model is based on the Epstein layer introduced by Rawer (1983) represented by the following analytical expression when only the F_2 layer is present

$$N(h) = \frac{4\,NmF_2}{\left(1 + \exp\left(\frac{h - hmF_2}{B_2}\right)\right)^2} \exp\left(\frac{h - hmF_2}{B_2}\right) \tag{1.1}$$

where: $N(h)$ is the electron density at height h; NmF_2 is the peak electron density in the F region; hmF_2 is the height of the peak electron density; $B_2 = 0.385\ Nm/(dN/dh)m$.

NmF_2 is computed from f_0F_2 with the well known expression

$$Nm\,[10^{11}/\mathrm{m}^3] = 0.124\ (f_0F_2\,[\mathrm{MHz}])^2$$

and hmF_2 can be estimated from scaled ionospheric characteristics by using one of the simple methods published in the literature. Dudeney (1983) gives a critical review of the accuracy of these methods. The expression of B_2 is obtained through derivation of function (1.1) and is a parameter related to the thickness of the layer. $(dN/dh)m$ is the value of the gradient of $N(h)$ at the characteristic point at the base of the F_2 layer and can be calculated by using the empirical relation given by MG-R 90:

$$\ln\left((dN/dh)_{\max}\,[10^9\,\mathrm{m}^{-3}\mathrm{km}^{-1}]\right) =$$

$$= -3.467 + 0.857\ln(f_0F_2\,[\mathrm{MHz}])^2 + 2.0\ln(M(3000)F_2). \tag{1.2}$$

When E and F_1 layers are present in the ionograms the model is constructed as the sum of two or three Epstein layers that are formally identical. In the original DGR model, the value of the B_1, the thickness parameter corresponding to the F_1 layer, was found empirically to be related to the height hmF_1 of the F_1 characteristic point. This height was calculated in that version of the DGR model by using the empirical expressions given in MG-R 87.

It must be noted that both the original DGR model and the improved one do not attempt to calculate the electron density profile at D region heights.

2. Improvement of the model

After a series of tests done by the authors of the original DGR model and the group of the Geophysics Institute of Beograd it was found that in order to obtain better results from the model in a wider range of conditions it is necessary to introduce the asymmetry between the thickness of the bottom and top parts of each layer in a given profile of the electron density calculated with the model. In the case of the F_1 layer it was shown theoretically that the value of the B_1 must also be related to the F_2 peak electron density and height in order to eliminate unwanted effects of the presence of the F_1 layer. It was proven that the height and electron density of the F_2 base point in the calculated profile are insensitive to the presence of the F_1 layer. The study done by Radicella and Di Giovanni (1991) described these results and gave an improved version of the model.

Subsequent tests have shown that an apparently excessive valley was computed between the E and F_1 layers by the model, particularly when f_0F_2 and hmF_2 values are low. An additional problem was found when f_0F_1 is not scaled in the ionogram, in which case the region between the E and F_2 layers in the computed model also show a too large valley.

Both problems were solved to a large extent in the work done by Di Giovanni *et al.* (1992). The problem of the valley between E and F_1 layers has been eliminated by introducing a value for the thickness parameter B_E of the E layer linked to the thickness parameter of the F_1 layer. The large valley computed by the model when no f_0F_1 is scaled in the ionogram was partially solved by introducing in such a case a fictitious f_0F_1 with a value $f_0F_1 = f_0E + 0.5$ [MHz]. For nighttime conditions, however, the presence of a fictitious F_1 introduces problems in the calculation of $N(h)$ at low heights. Such problems are overcome by eliminating the fictitious F_1 when the value of f_0E is below 2 MHz. Figure 1 shows examples of model calculation and digisonde ionogram ARTIST inversion profiles for nighttime conditions. The value of f_0E used in the model calculation is the same assumed by the ionogram inversion method.

A long series of median total electron content (TEC) data obtained from radio signals of geostationary satellites at Florence and median ionosonde data recorded at Rome were used to check ad improve the ability of the DGR model to reproduce or predict the electron density profile above the F_2 peak. In the study done by Zhang *et al.* (1991) it was possible: to simulate the «effective» shape of the topside ionosphere with a reasonable degree of accuracy by introducing a topside «shape parameter» k that was calculated from the combination of experimental TEC values and ionosonde data by making use of the analytical expression of TEC derived from the Epstein layer expression (1.1) and given by:

$$\mathrm{TEC} = 2\,(1+k)\,NmF_2 \times B_2 \qquad (2.1)$$

when only the F_2 layer is considered. The factor $2\,(1+k)$, when $k \neq 1$, take into account the asymmetry between bottomside ad topside of the F_2 layer.

The time and seasonal variation of the «shape parameter» k of the topside F_2 layer, calculated by considering

$$\mathrm{TEC}_{\mathrm{DGR}} = \mathrm{TEC}_{\mathrm{EXP}},$$

where $\mathrm{TEC}_{\mathrm{DGR}}$ is the value of total electron content calculated by the DGR model ad $\mathrm{TEC}_{\mathrm{EXP}}$ is the experimental value, was investigated and the results are given in fig. 2. In order to improve the profile simulation above the electron density peak the values of k obtained were correlated with different parameters searching for a way to predict its behaviour as a function of known bottomside parameters. The results obtained by Zhang *et al.* (1991) were further developed in the work of Di Giovanni *et al.* (1992). The empirical relations of k found empirically and incorporated in the improved DGR model are – for the Northern Hemisphere –:

$k = -7.77 + 0.097\,(hmF_2/B_2)^2 + 0.153\,NmF_2$
(October to March)

$k = 6.705 - 0.014\,R_{12} - 0.008\,hmF_2$
(April to September).

(2.2)

Where NmF_2 is given in $10^{11}/\mathrm{m}^3$ and R_{12} is the 12 month running mean sunspot number.

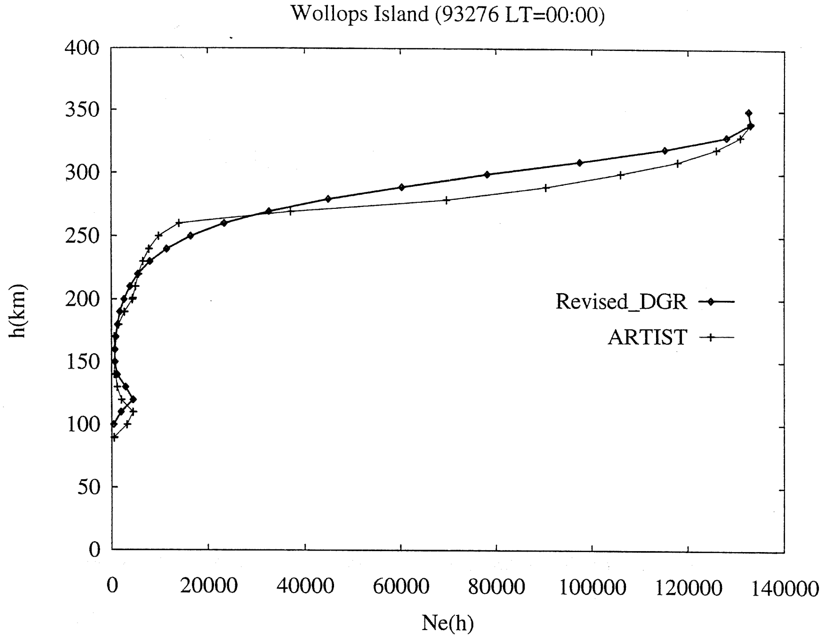
By making use of the studies mentioned above the improved version of the DGR model is described by the following equations:

$$N(h) = NF_2(h) + NF_1(h) + NE(h)$$

$$N(h) = \frac{4\,Nm*F_2}{\left(1+\exp\left(\frac{h-hmF_2}{B_2}\right)\right)^2}\exp\left(\frac{h-hmF_2}{B_2}\right) + \frac{4\,Nm*F_1}{\left(1+\exp\left(\frac{h-hmF_1}{B_1}\right)\right)^2}\exp\left(\frac{h-hmF_1}{B_1}\right) + \frac{4\,Nm*E}{\left(1+\exp\left(\frac{h-hmE}{B_E}\right)\right)^2}\exp\left(\frac{h-hmE}{B_E}\right) \qquad (2.3)$$

where:

$Nm*F_2 = NmF_2 - 0.1\,NmF_1$



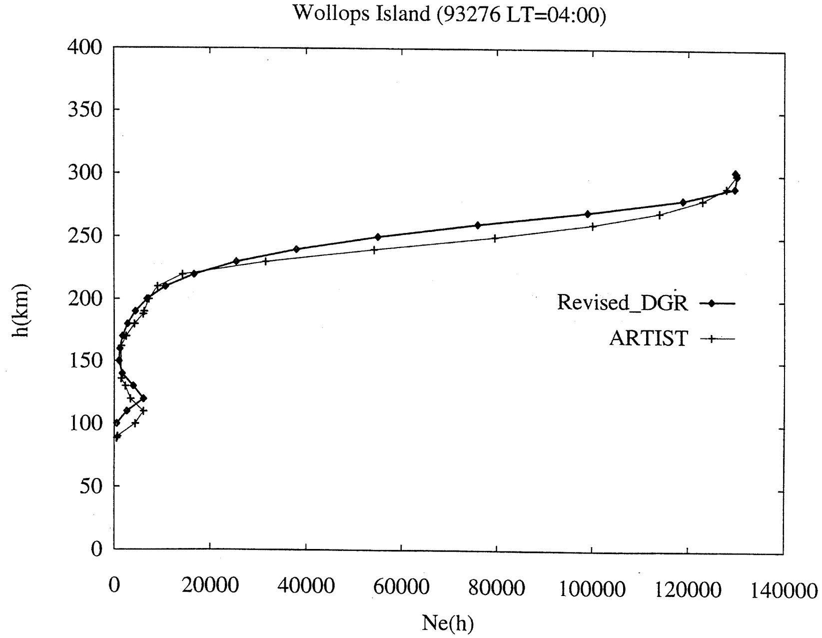


Fig. 1. Examples of improved DGR model calculations and digisonde ionogram ARTIST inversion electron density profiles for nighttime conditions.

$$
\begin{aligned}
Nm * F_1 &= NmF_1 - NF_2(hmF_1) \\
Nm * E &= NmE - NF_1(hmE = 120\text{ km}) - \\
&\quad - NF_2(hmE = 120\text{ km}).
\end{aligned}
$$

The various parameters involved in (2.3) are calculated with the following expressions:

$$hmF_2 = \frac{1470\,MF}{M + DM} - 176$$

$$DM = \frac{0.253}{(f_0F_2 / f_0\,E - 1.215)} - 0.012$$

$$MF = M\sqrt{\frac{0.0196\,M^2 + 1}{1.2967\,M^2 - 1}}$$

$$M = M(3000)\,F_2$$

$$hmF_1 = 1108.8 + 0.14\,NmF_1\ [10^9/\text{m}^3] + \\ +0.71\,|\,\text{Dip}\,|[^0]$$

$$B_2\ (h \leq hmF_2) = \frac{0.385\,NmF_2}{(dN/dh)_{\max}}$$

$$B_2\ (h > hmF_2) = kB_2\ (h \leq hmF_2)$$

$$B_{1\text{top}} = \frac{hmF_2 - hmF_1}{\ln\left(\dfrac{4\,(NmF_1 - NF_2\,(hmF_1))}{0.1\,NmF_1}\right)}$$

$$B_{1\text{bot}} = 0.7\,B_{F1\text{top}}$$

$$B_{E\text{top}} = 0.5\,B_{F1\text{top}} \quad (\text{if } F_1 \text{ present})$$

$$B_{E\text{top}} = 7\,\text{km} \qquad (\text{if } F_1 \text{ not present})$$

$$B_{E\text{bot}} = 5\,\text{km}\,.$$

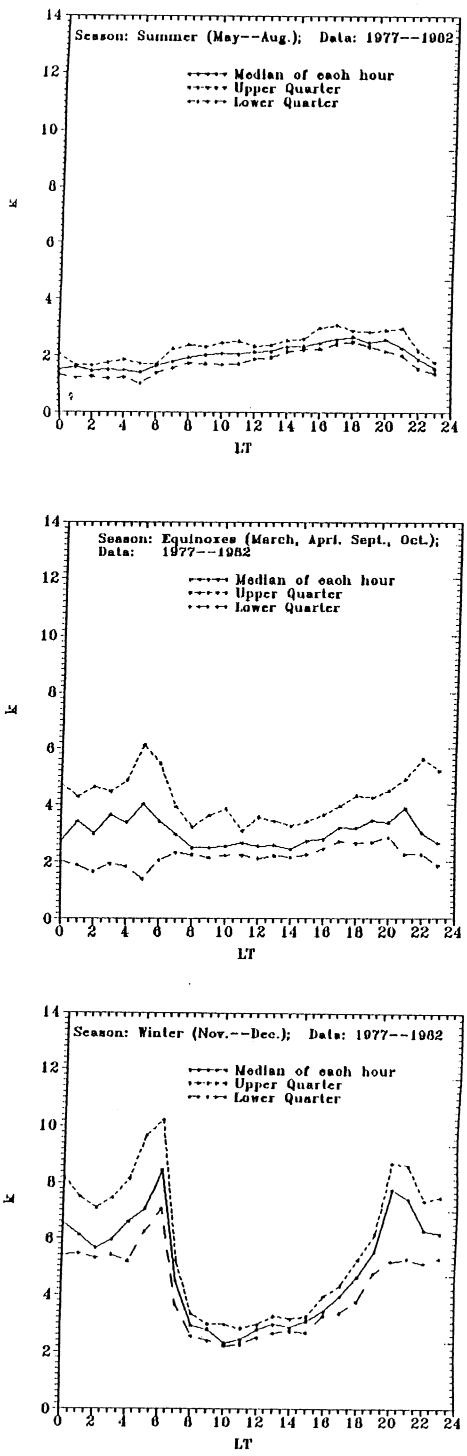


Fig. 2. Diurnal and seasonal variation of the «shape parameter» k of the topside ionosphere.

The formula used for the calculation of hmF_2 is the one given by Dudeney (1975).

The expression used to calculate TEC is the following:

$$\mathrm{TEC}_{\mathrm{DGR}} = 2Nm * F_2B_2(1+k) + 2Nm * F_1(B_{1\mathrm{top}} + B_{1\mathrm{bot}}) + 2Nm * E(B_{E\mathrm{top}} + B_{E\mathrm{bot}}) \,. \tag{2.4}$$

Electron content up to any given height is calculated by an analytical expression obtained from eq. (2.3).

3. Test of the improved model

Several authors have tested the improved DGR model to reproduce electron density profiles ad TEC values.

The results of the profile tests done by Di Giovanni *et al.* (1992) have shown that a potential limitation of the model lies in the intrinsic difficulties of the calculation of hmF_2 from routinely scaled ionospheric characteristics.

Radicella *et al.* (1992) made an extensive analysis of the median and interquirtile range of bihourly TEC experimental data from the geographical area around Rome, Italy, for the period 1978-1982 and calculated values obtained with the improved DGR model using the proper ionogram median scaled data. The analysis also included a comparison with the TEC empirical model developed by Feichter and Leitinger (1987) [from now on: GRAZ] and the TEC obtained by numerical integration of the IRI 90 electron density profile model (Bilitza, 1990). The result of this study shows that the DGR and the GRAZ models appear to be in better agreement with the experimental data than the IRI 90 model. As an example, the average percentage relative difference between experimental data ad model calculation found were as follows:

Model	Average difference (%)
IRI 90	58.4
GRAZ	31.1
DGR (improved)	26.3

Singer *et al.* (1994) compared both the electron density profiles calculated by instantaneous ionogram inversion and TEC values obtained experimentally around Juliusruh, Germany, at different hours, season and solar activity, with both the improved DGR and IRI 90 models. They found that both models gave results reasonably in agreement with the experimentally derived profiles. However the DGR model gave better agreement than the IRI 90 model for TEC values, particularly at high solar activity.

Kecic *et al.* (1994) compared both the improved DGR and the IRI 90 models with the profiles obtained by ionogram inversion using data from South Uist, United Kingdom, for both geomagnetically quiet and disturbed conditions. Their results show that both models also reproduced the experimental profiles that correspond to geomagnetic disturbances.

Soler (1994) compared electron density profiles obtained from Tortosa, Spain, digisonde instantaneous ionograms for different hours, seasons and solar activity with the IRI 90 and improved DGR models. He found that both models reproduce reasonably well the experimentally deduced profiles during the daytime particularly for low solar activity and always during the nighttime and low solar zenith angle conditions.

Ortiz de Adler *et al.* (1994) compared instantaneous values of experimental TEC data for Tucumán, Argentina, with the IRI 90 and improved DGR models for different seasons and hours for the period 1982-1983. Taking into account the need to invert the season for the value of the topside shape parameter k when considering Southern Hemisphere conditions, these authors found that the DGR model gives better agreement with the experimental values than the IRI 90 model.

Mosert de Gonzalez (1994) compared electron density profiles obtained by ionogram inversion using noon data from Tucumán and San Juan, Argentina, for different seasons and solar activity with both improved DGR and IRI 90 models. The results show that the DGR model reproduces better the experimentally derived profiles particularly at F_1 region heights.

4. The model computer program

A computer program in FORTRAN 77 language was prepared with the final version of the improved DGR model. It calculates the electron density and the value of *dN/dh* as a function of height below and above the peak of the F_2 layer, ad the total electron content. Input parameters for each given time and location are: f_0F_2, f_0F_1, f_0E, $M(3000)F_2$, geomagnetic Dip angle of the location and the R_{12} sunspot number. The values of *dN/dh* are calculated using the analytical expression of the first derivative of eq. (2.3). The source program is available through: E. mail ZHANG M @ ICTP, TRIESTE.IT.

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