Fractal and chaotic analysis of the geomagnetic field

Angelo De Santis and Massimo Chiappini
Istituto Nazionale di Geofisica, Roma, Italy

Abstract
Geomagnetic field time variations can be analysed by means of fractal and chaotic techniques. This new kind of analysis can definitely help in studying geophysical signals even when their chaotic and/or fractal aspects are not so obvious. In this short paper some examples of this new kind of analysis are presented in the case of geomagnetic time variation data from L'Aquila Observatory.

Key words *chaos – fractals – geomagnetism*

This short note aims to briefly present to the geo-electromagnetism community a very recent approach to the analysis of the state of the magnetosphere, when considered as a whole. This new kind of analysis involves the use of fractal and chaotic concepts shedding new light on the magnetospheric phenomena. More details can be found in the papers listed in the references.

The knowledge of the state of the geomagnetic field is closely related to the measure of its level of activity. A conventional widely used indicator is the K index (Mayaud, 1980). It indicates the deviation of the geomagnetic signal from the normal diurnal variation, *i.e.* the level of perturbation due to solar particles trapped by the geomagnetic field within the magnetosphere. The estimate of K is manually made from the magnetograms, involving operations which are somewhat subjective. The disposal of digital data allows the automatic derivation of the K index by means of appropriate and efficient computer algorithms (Menvielle and Berthelier, 1991). A very new approach to this problem has been presented by De Santis and Chiappini (1992) and Chiappini *et al.* (1993). The Earth's magnetosphere-ionosphere system is a macroscopic, non-linear, open and dissipative system to which it is possible to apply fractal and chaotic analyses. Fractal analysis implies the self-similarity or, more exactly, self-affinity of the phenomenon under study. This can be crudely seen when some portions of the geomagnetic signal recorded at the Earth's surface are magnified without losing their typical irregular and fragmented aspect.

Fractal dimension D of a curve is related to its capacity to cover all plane. In the time domain, D gives a measure of the signal complexity. The idea of extending fractal concepts to the geomagnetic field to estimate its complexity, *i.e.* its level of perturbation, and, consequently for the estimation of K, was straightforward. Using the ruler method (*e.g.* Mandelbrot, 1983) directly on the magnetograms of L'Aquila Observatory, as applied by Vörös (1991) for Hurbanovo, De Santis and Chiappini (1992) found a reasonable agreement between manual K values and those computed by means of fractal analysis (fig. 1; at the abscissa there is the sum of logarithms of the apparent lengths for the rulers of 5, 6 and 9 min). A better agreement was finally found combining these results with those obtained by means of the harmonic analysis, reaching perfect accordance in around 70% of cases.

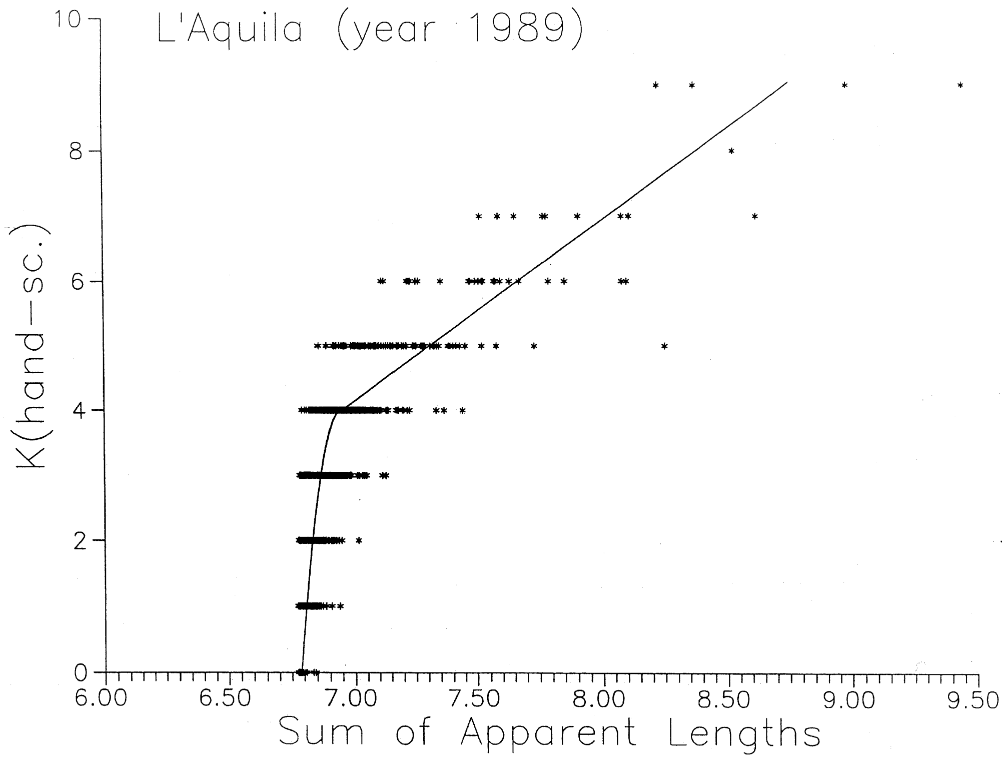

Fig. 1. Hand-scaled *K* values vs. sum of logarithms of apparent lengths of 5, 6 and 9 min ruler (figure from De Santis and Chiappini, 1992).

Having established the fractal character of the geomagnetic field, we wanted to investigate its behaviour in the phase space. This space can be easily reconstructed by means of consecutive delays of a single variable that characterizes the behaviour in time of the physical phenomenon (Takens, 1980). This approach is what we call «chaotic». The chaotic characteristic of the signal arises from its strong sensitivity to the initial conditions, simply explained by the fact that the attractor of the system is fractal (*e.g.* Baker and Gollub, 1991): in practice it is not possible to specify the true position of the dynamic system state exactly, because with any change of scale, new complexity and irregularities arise.

Here we simply present the different behaviour of the geomagnetic field in quite or very perturbed conditions in a two-dimensional phase space (figs. 2 and 3, respectively). The latter case shows the orbit of consecutive 30 min delays of the horizontal component, H, during the magnetic storm of 13-14 March 1989, which was probably the strongest geomagnetic storm of our century. Figure 4 shows the same latter case in a three-dimensional phase space. The increase in embedding dimension did not change the aspects of the orbit too much. It is possible to compute the fractal dimension of the signal finding a value close to 2: this is evident in the tendency of the curve of fig. 3 to cover the whole plane.

Figure 5 shows a fractal brownian motion in a plane, *i.e.* that of a dust particle on the surface of the water, with the same fractal dimension $D = 2$ of the curve of fig. 3; we see evident similarity between these two figures. Apparently the two behaviours, even due to such

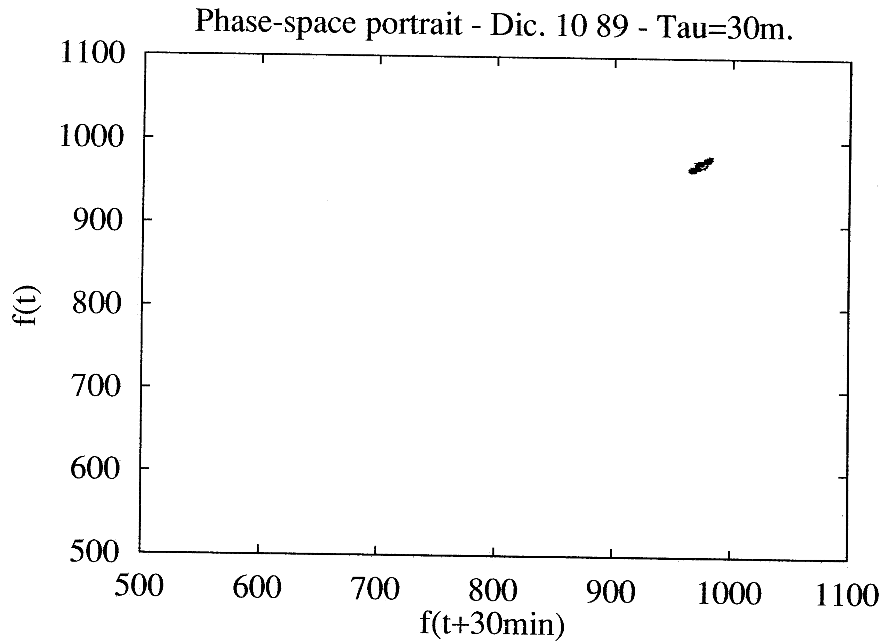

Fig. 2. Two-dimensional phase-space portrait for the *H* component of the geomagnetic field on 10th December 1989 (quiet day) as recorded at L'Aquila Observatory.

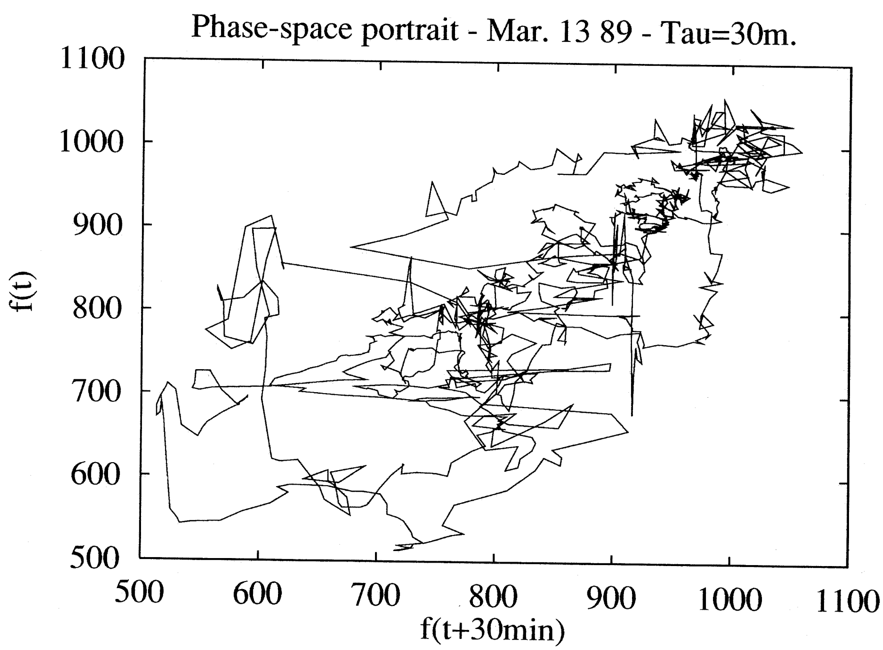

Fig. 3. As fig. 2 but for 13th March 1989 (very perturbed day).

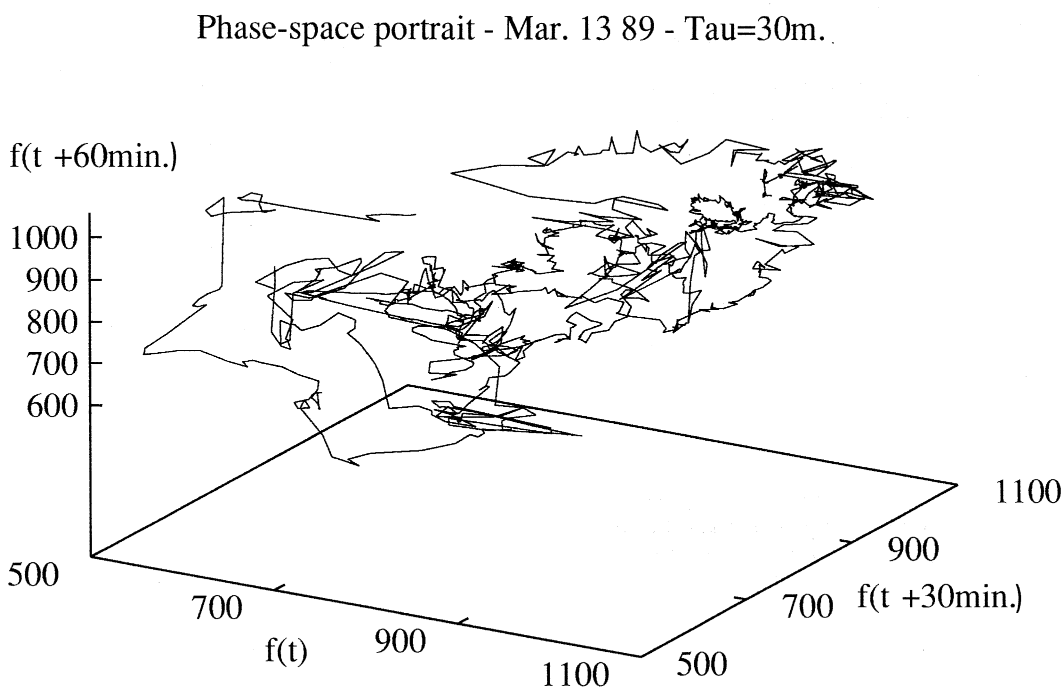

Fig. 4. Three dimensional phase-space portrait for 13th March 1989.

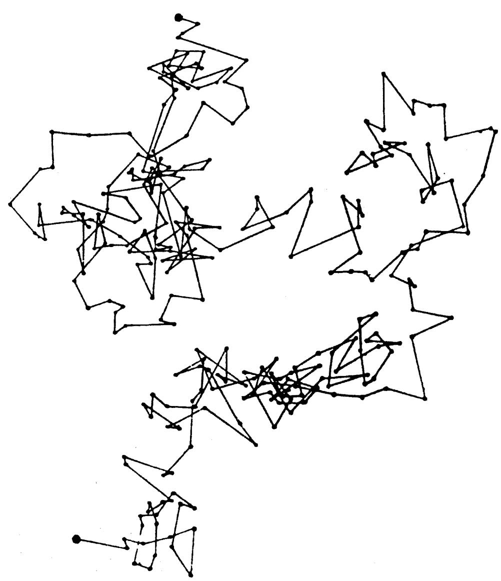

Fig. 5. The fractal Brownian motion is a curve with topological dimension 1. In practice, with its plane filling it shows a fractal dimension 2.

different phenomena in size and components, are roughly the same. This fact could be simply interpreted affirming that the same differential (non-linear) equations that regulate the *life* of the system in the conventional space (the particle system in a glass of water; size of few centimeters), are still valid for the other system in the reconstructed phase space (magnetospheric system; size of thousands of kilometers).

Finally we can conclude that Nature shows the same aspects at different scales; its apparent complexity sometimes disappears when we look at different phenomena in different spaces, as this characteristic contains a universal property of unicity. Chaotic and fractal analyses can definitely help in studying geophysical signals *(e.g.* Korvin, 1992), even when their fractal and/or chaotic aspects are not so obvious. However, if these features are found they force researchers to look for answers to specific questions of deep and large (say, philosophical) content.

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