Lamb and Love wave propagation in an infinite micropolar elastic plate

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Summary. — This paper is concerned with monochromatic wave propagation in an infinite homogeneous micropolar elastic plate bounded by two parallel free planes. Two kinds of propagation are discussed: Lamb and Love waves. We find that a displacement field $(u_1, u_2, 0)$ and a microrotation field $(0, 0, \varphi_3)$ leads to Lamb's waves, while a displacement field $(0, 0, u_3)$ and a microrotation field $(\varphi_1, \varphi_2, 0)$ leads to Love's waves.

Riassunto. — In questo lavoro si tratta la propagazione di onde monocromatiche in un piatto elastico, micropolare, omogeneo ed infinito, limitato da due piani paralleli e liberi. Vengono discussi due tipi di propagazione, quello di Lamb e quello di Love. Si trova che il campo di spostamenti $(u_1, u_2, 0)$ ed il campo di microrotazioni $(0, 0, \varphi_3)$ porta ad onde di Lamb, mentre un campo di spostamenti $(0, 0, u_3)$ ed un campo di microrotazioni $(\varphi_1, \varphi_2, 0)$ porta ad onde di Love.

Introduction

Oscillations of an elastic plate, with stress-free surfaces, have been investigated by Rayleigh [13], Lamb [11] and others, and more recently by Prescott [12], Gogoladze [9], Satô [14] and Ewing, Jardetzky and Press [4].

In this paper we want to deal with the same problem in the framework of the theory of micropolar elasticity.

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The linear theory of micropolar elasticity has been introduced by Eringen ([3]). Iesan ([10]) has derived the fundamental equations using invariance conditions under superposed rigid body motions.

Micropolar elasticity may give interesting results when applied to geophysical problems. We have already applied it to derive an explicit expression of the body force and the body couple equivalents for seismic dislocations ([1,2]).

Basic equations

Throughout this paper we employ the usual indicial notations. All regularity hypotheses on the considered functions will be omitted.

We consider a rectangular Cartesian frame $0x_k$ $(k = 1, 2, 3)$. The basic equations in the linear theory of homogeneous and anisotropic elastic solids are ([10]):

— the kinematic relations:

$$e_{ij} = u_{j,i} - \varepsilon_{ijk}\,\varphi_k \qquad [1.a]$$

$$\varkappa_{ij} = \varphi_{j,i} \qquad [1.b]$$

— the equations of motion:

$$t_{ji,j} + F_i = \varrho\,\ddot{u}_i \qquad [2.a]$$

$$m_{ji,j} + \varepsilon_{ijk}\,t_{jk} + M_i = I_{ij}\,\ddot{\varphi}_j \qquad [2.b]$$

— the constitutive laws:

$$t_{ij} = A_{ijkl}\,e_{kl} + B_{ijkl}\,\varkappa_{kl} \qquad [3.a]$$

$$m_{ij} = B_{klij}\,e_{kl} + C_{ijkl}\,\varkappa_{kl} \qquad [3.b]$$

In the above equations we have used the following notations: u_i, the components of the displacement vector; φ_i, the components of the microrotation vector; e_{ij} and $\varkappa_{ij}$, the kinematic characteristics of the strain; t_{ji}, the components of the stress tensor; m_{ji}, the components of the couple stress tensor; F_i, the components of the body force; M_i, the components of the body couple; ϱ, the mass density; A_{ijkl},

B_{ijkl}, C_{ijkl}, I_{ij}, the characteristic constants of the material; ε_{ijk}, the alternating tensor; a comma denotes partial derivation with respect to space variables, and a superposed dot partial derivation with respect to the time t. Furthermore, we have:

$$\left.\begin{aligned} A_{ijkl} &= A_{klij} \\ C_{ijkl} &= C_{klij} \\ I_{ij} &= I_{ji} \end{aligned}\right\} \qquad [4]$$

If the body is homogeneous, isotropic and centrosymmetric, we have:

$$\begin{aligned} A_{ijkl} &= \lambda\, \delta_{ij}\, \delta_{kl} + (\mu - \varkappa)\, \delta_{ik}\, \delta_{jl} + \mu\, \delta_{il}\, \delta_{jk} \\ B_{ijkl} &= 0 \\ C_{ijkl} &= \alpha\, \delta_{ij}\, \delta_{kl} + \gamma\, \delta_{ik}\, \delta_{jl} + \beta\, \delta_{il}\, \delta_{jk} \end{aligned}$$

and equations [3] are altered to:

$$\left.\begin{aligned} t_{ij} &= \lambda\, e_{kk}\, \delta_{ij} + (\mu + \varkappa)\, e_{ij} + \mu\, e_{ji} \\ m_{ij} &= \alpha\, \varkappa_{kk}\, \delta_{ij} + \gamma\, \varkappa_{ij} + \beta\, \varkappa_{ji} \end{aligned}\right\} \qquad [5]$$

where λ, μ, $\varkappa$, α, γ and β are material constants satisfying the following inequalities:

$$\left.\begin{aligned} & 3\,\lambda + 2\,\mu + \varkappa \geqslant 0 \\ & 2\,\mu + \varkappa \geqslant 0 \\ & \varkappa \geqslant 0 \\ & 3\,\alpha + \beta + \gamma \geqslant 0 \\ & -\gamma \leqslant \beta \leqslant \gamma \\ & \gamma \geqslant 0 \end{aligned}\right\} \qquad [6]$$

which are the necessary and sufficient conditions for the internal energy to be non-negative. Furthermore we can also write:

$$I_{ij} = I\, \delta_{ij} \qquad [7]$$

where I is another material constant.

If equations [5] and [7] are substituted into equations [1], we get:

$$(\lambda + \mu)\, u_{j,ji} + (\mu + \varkappa)\, u_{i,jj} + \varkappa\, \varepsilon_{ijk}\, \varphi_{k,j} + F_i = \varrho \ddot{u}_i, \qquad [8.a]$$

$$(\alpha + \beta)\, \varphi_{j,ji} + \gamma\, \varphi_{i,jj} + \varkappa \varepsilon_{ijk}\, u_{k,j} - 2\, \varkappa\, \varphi_i + M_i = I\, \ddot{\varphi}_i \qquad [8.b]$$

Let us now specialize our analysis to the case in which:

$$F_i = M_i = 0$$

$$u_i = u_i\,(x_1, x_2, t);\ \varphi_i = \varphi_i\,(x_1, x_2, t)$$

Equations [8] lead to:

$$\left.\begin{array}{l} (\mu + \varkappa)\, \nabla^2 u_1 + (\lambda + \mu)\, e_{,1} + \varkappa\, \varphi_{3,2} = \varrho\, \ddot{u}_1 \\ (\mu + \varkappa)\, \nabla^2 u_2 + (\lambda + \mu)\, e_{,2} - \varkappa\, \varphi_{3,1} = \varrho\, \ddot{u}_2 \\ \gamma\, \nabla^2 \varphi_3 + \varkappa\, (u_{2,1} - u_{1,2}) - 2\, \varkappa\, \varphi_3 = I\, \ddot{\varphi}_3 \end{array}\right\} \qquad [9.a]$$

$$\left.\begin{array}{l} \gamma\, \nabla^2 \varphi_1 + (\alpha + \beta)\, \Theta_{,1} - 2\, \varkappa\, \varphi_1 + \varkappa\, u_{3,2} = I\, \ddot{\varphi}_1 \\ \gamma\, \nabla^2 \varphi_2 + (\alpha + \beta)\, \Theta_{,2} - 2\, \varkappa\, \varphi_2 - \varkappa\, u_{3,1} = I\, \ddot{\varphi}_2 \\ (\mu + \varkappa)\, \nabla^2 u_3 + \varkappa\, (\varphi_{2,1} - \varphi_{1,2}) = \varrho\, \ddot{u}_3 \end{array}\right\} \qquad [9.b]$$

where we have introduced the following notations:

$$e = u_{1,1} + u_{2,2}$$

$$\Theta = \varphi_{1,1} + \varphi_{2,2}$$

$$\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$$

The two systems of equations [9.a] and [9.b] are independent. The system [9.a] describes the displacement field $(u_1, u_2, 0)$ and the microrotation field $(0, 0, \varphi_3)$. The system [9.b] describes the displacement field $(0, 0, u_3)$ and the microrotation field $(\varphi_1, \varphi_2, 0)$. Our purpose is to show that the system [9.a] leads to waves of Lamb kind, while the system [9.b] leads to waves of Love kind.

LAMB WAVES.

The components of the stress and couple stress tensor determined by the displacement field $(u_1, u_2, 0)$ and by the microrotation field $(0, 0, \varphi_3)$ are:

$$\left.\begin{aligned}
t_{11} &= (2\,\mu + \varkappa)\,u_{1,1} + \lambda\,e \\
t_{22} &= (2\,\mu + \varkappa)\,u_{2,2} + \lambda\,e \\
t_{33} &= \lambda\,e \\
t_{12} &= \mu\,(u_{2,1} + u_{1,2}) + \varkappa\,u_{2,1} - \varkappa\,\varphi_3 \\
t_{21} &= \mu\,(u_{1,2} + u_{2,1}) + \varkappa\,u_{1,2} + \varkappa\,\varphi_3 \\
t_{13} &= t_{31} = t_{23} = t_{32} = 0 \\
m_{13} &= \gamma\,\varphi_{3,1} \\
m_{31} &= \beta\,\varphi_{3,1} \\
m_{23} &= \gamma\,\varphi_{3,2} \\
m_{32} &= \beta\,\varphi_{3,2} \\
m_{11} &= m_{22} = m_{33} = m_{12} = m_{21} = 0
\end{aligned}\right\} \qquad [10]$$

Let us now assume that a monochromatic wave propagates, along the x_2 – direction, in an infinite micropolar elastic plate. Let $2H$ be the thickness of the considered plate. Moreover we assume that the following conditions:

$$t_{11} = t_{12} = m_{13} = 0, \quad \text{at} \quad x_1 = \pm H \qquad [11]$$

should be satisfied. In other words, the surfaces of the plate are supposed free of stresses and couple stresses.

Equations [9.a] are satisfied by:

$$\begin{aligned}
u_1 &= \Phi_{,1} - \Psi_{,2} \\
u_2 &= \Phi_{,2} + \Psi_{,1}
\end{aligned}$$

provided

$$\left.\begin{aligned}
&(\lambda + 2\,\mu + \varkappa)\,\nabla^2\,\Phi - \varrho\,\ddot{\Phi} = 0 \\
&(\mu + \varkappa)\,\nabla^2\,\Psi - \varrho\,\ddot{\Psi} - \varkappa\,\varphi_3 = 0 \\
&[\gamma\,\nabla^2 - 2\,\varkappa - I\,\partial^2/\partial t^2]\,\varphi_3 + \varkappa\,\nabla^2\,\Psi = 0
\end{aligned}\right\} \qquad [12]$$

The first equation [12] describes the propagation of longitudinal displacement waves with velocity:

$$v_1^2 = \frac{\lambda + 2\,\mu + \varkappa}{\varrho} \qquad [13]$$

The last two equations [12] can be reduced to the following form:

$$\left\{[\gamma\,\nabla^2 - 2\,\varkappa - I\,\partial^2/\partial t^2]\,[(\mu + \varkappa)\,\nabla^2 - \varrho\,\partial^2/\partial t^2] + \varkappa^2\,\nabla^2\right\}\,\Psi = 0 \qquad [14.a]$$

$$\left\{[\gamma\,\nabla^2 - 2\,\varkappa - I\,\partial^2/\partial t^2]\,[(\mu + \varkappa)\,\nabla^2 - \varrho\,\partial^2/\partial t^2] + \varkappa^2\,\nabla^2\right\}\,\varphi_3 = 0 \qquad [14.b]$$

Let us assume solutions of the form:

$$\left.\begin{aligned} \Phi\ (x_1, x_2, t) &= \Phi^*\ (x_1)\, e^{i\,(kx_2 - \omega t)} \\ \Psi\ (x_1, x_2, t) &= \Psi^*\ (x_1)\, e^{i\,(kx_2 - \omega t)} \\ \varphi_3\ (x_1, x_2, t) &= \varphi_3^*\ (x_1)\, e^{i\,(kx_2 - \omega t)} \end{aligned}\right\} \quad [15]$$

The first equation [12] and equations [14] become:

$$\Phi^*_{,11} - \left(k^2 - \frac{\omega^2}{v_1^2}\right) \Phi^* = 0 \quad [16]$$

$$\left\{\left(\frac{d^2}{dx_1^2} - k^2 + a^2\right)\left(\frac{d^2}{dx_1^2} - k^2 + d^2 - b^2\right) + \sigma^2 \left(\frac{d^2}{dx_1^2} - k^2\right)\right\} \Psi^* = 0 \quad [17.a]$$

$$\left\{\left(\frac{d^2}{dx_1^2} - k^2 + a^2\right)\left(\frac{d^2}{dx_1^2} - k^2 + d^2 - b^2\right) + \sigma^2 \left(\frac{d^2}{dx_1^2} - k^2\right)\right\} \varphi_3^* = 0, \quad [17.b]$$

where the following notations have been introduced:

$$\left.\begin{aligned} a^2 &= \omega^2/(c_2^2 + c_3^2) \\ d^2 &= \omega^2/c_4^2 \\ b^2 &= 2\,\varkappa/\gamma \\ \sigma^2 &= \varkappa^2/[\gamma\,(\mu + \varkappa)] \\ c_2^2 &= \mu/\varrho \\ c_3^2 &= \varkappa/\varrho \\ c_4^2 &= \gamma/I \end{aligned}\right\} \quad [18]$$

Equations [17] describe the modified transverse waves. We search solutions of equations [16] and [17] of the form:

$$\Phi^* = A \sinh(\eta\, x_1) + B \cosh(\eta\, x_1) \quad [19]$$
$$\Psi^* = C \sinh(k_1\, x_1) + D \cosh(k_1\, x_1) + E \sinh(k_2\, x_1) + F \cosh(k_2\, x_1)$$
$$\varphi_3^* = G \sinh(k_1\, x_1) + L \cosh(k_1\, x_1) + P \sinh(k_2\, x_1) + Q \cosh(k_2\, x_1)$$

where

$$k_{1,2}{}^2 = k^2 + \frac{1}{2}\left\{ b^2 - a^2 - \sigma^2 - d^2 \pm \left[\left(a^2 + \sigma^2 + d^2 - b^2 \right) - \right.\right.$$
$$\left.\left. - 4\, a^2 \left(d^2 - b^2 \right) \right) \right]^{1/2} \right\}$$
$$\eta = \left(k^2 - \frac{\omega^2}{v_{12}} \right)$$

Let us now specialize our study to the case of symmetric vibrations. Then the component u_2 of the displacement vector, the components t_{11} and t_{22} of the stress tensor and the component m_{13} of the couple stress tensor must be symmetric with respect to the plane $x_1 = 0$. This leads to:

$$A = D = F = L = Q = 0$$

and the expressions [19] are altered to:

$$\left.\begin{aligned} \Phi^* &= B \cosh (\eta\, x_1) \\ \Psi^* &= C \sinh (k_1\, x_1) + E \sinh (k_2\, x_1) \\ \varphi_3{}^* &= G \sinh (k_1\, x_1) + P \sinh (k_2\, x_1) \end{aligned}\right\} \qquad [20]$$

Furthermore introducing these expressions into equations [12], we get:

$$G = \tau_1\, C$$
$$P = \tau_2\, E$$

where

$$\tau_n = \frac{\mu + \varkappa}{\varkappa}\,(a^2 + k^2 - k_n{}^2), \;\; n = 1,2.$$

Therefore $\varphi_3{}^*$ can be written as:

$$\varphi_3{}^* = \tau_1\, C \sinh (k_1\, x_1) + \tau_2\, E \sinh (k_2\, x_1). \qquad [21]$$

The boundary conditions [11] are expressed as:

$$B\,[\eta^2\,(2\,\mu + \lambda + \varkappa) - \lambda\, k^2] \cosh\, (\eta\, H) - C\, i\, k\, k_1\, (2\,\mu + \varkappa) \cosh\, (k_1\, H)$$
$$- E\, i\, k\, k_2\, (2\,\mu + \varkappa) \cosh\, (k_2\, H) = 0,$$
$$B\, i\, k\, \eta\, (2\,\mu + \varkappa) \sinh\, (\eta\, H) + C\, \sigma_1 \sinh\, (k_1\, H) + E\, \sigma_2 \sinh\, (k_2\, H) = 0,$$
$$C\, k_1\, \tau_1\, \cosh(k_1\, H) + E\, k_2\, \tau_2\, \cosh(k_2\, H) = 0, \qquad [22]$$

where

$$\sigma_n = (\mu + \varkappa)\, k_n^2 + \mu\, k^2 - \varkappa\, \tau_n, \qquad n = 1,2.$$

The characteristic equation of the system [22] is:

$$\frac{\operatorname{tgh}(\eta H)}{\operatorname{tgh}(k_1 H)} = \frac{(2\mu + \varkappa + \lambda)\,\eta^2 - k^2 \lambda}{(2\mu + \varkappa)^2\, \eta\, k^2\, k_1\,(\tau_2 - \tau_1)} \cdot$$
$$\cdot \left| \sigma_1 \tau_2 - \sigma_2 \tau_1 \frac{k_1}{k_2} \frac{\operatorname{tgh}(k_2 H)}{\operatorname{tgh}(k_1 H)} \right| \qquad [23]$$

In the limit of the usual elasticity, equation [23] reduces to the period equation for Lamb waves (5,11). The discussion of the trascendental equation [23] presents great difficulties and, therefore, only the asymptotic limits for long and short waves are considered.

For waves long compared with the thickness $2H$, the products ηH, $k_1 H$, $k_2 H$ are so small that the hyperbolic functions are replaced by their arguments and equation [23] takes the form:

$$(2\mu + \varkappa)^2\, \eta^2\, k^2\,(\tau_2 - \tau_1) = [(2\mu + \varkappa + \lambda)\,\eta^2 - k^2 \lambda] \cdot (\sigma_1 \tau_2 - \sigma_2 \tau_1) \qquad [24]$$

For very short waves, the quantities ηH, $k_1 H$, $k_2 H$ are large and the two ratios of hyperbolic tangents, which appear in equation [23] become unity, giving:

$$[(2\,\mu + \varkappa + \lambda)\,\eta^2 - k^2 \lambda]\,(\sigma_1\,\tau_2\,k_2 - \sigma_2\,\tau_1\,k_1) =$$
$$= (2\,\mu + \varkappa)^2\,\eta\,k^2\,k_1\,k_2\,(\tau_2 - \tau_1) \qquad [25]$$

In the classical limit both the equations [24] and [25] reduce to the corresponding relations found in the framework of the usual elasticity (6). In particular, equation [25] can be recognized as the characteristic equation for Rayleigh waves in a micropolar elastic half space.

So far we have discussed the case of symmetric vibrations. Let us now specialize our study to the case of antisymmetric vibrations. The component u_2 of the displacement vector, the components t_{11} and t_{22} of the stress tensor and the component m_{13} of the couple stress tensor are now antisymmetric with respect to the plane $x_1 = 0$. These conditions lead to:

$$B = C = E = G = P = 0$$

and the expressions [19] become:

$$\left.\begin{aligned} \Phi^* &= A \sinh(\eta\, x_1) \\ \Psi^* &= D \cosh(k_1 x_1) + F \cosh(k_2 x_1) \\ \varphi_3^* &= L \cosh(k_1 x_1) + Q \cosh(k_2 x_1) \end{aligned}\right\} \qquad [26]$$

As in a previously considered case, by introducing these expressions into equations [12], we get:

$$\begin{aligned} L &= \tau_1 D \\ Q &= \tau_2 F \end{aligned}$$

where τ_1 and τ_2 have been already defined. Therefore φ_3^* can now be written as:

$$\varphi_3^* = \tau_1 D \cosh(k_1 x_1) + \tau_2 F \cosh(k_2 x_1) \qquad [27]$$

The boundary conditions [1] are now expressed as:

$$\begin{aligned} &A\,[\eta^2 (2\,\mu + \varkappa + \lambda) - \lambda\, k^2] \sinh(\eta H) - D\, i\, k\, k_1 (2\,\mu + \varkappa) \sinh(k_1 H) - \\ &\qquad - F\, i\, k\, k_2\, (2\,\mu + \varkappa) \sinh(k_2 H) = 0, \\ &A\, i\, k\, \eta\, (2\,\mu + \varkappa) \cosh(\eta H) + D \sigma_1 \cosh(k_1 H) + F \sigma_2 \cosh(k_2 H) = 0, \\ &\qquad D\, \tau_1\, k_1 \sinh(k_1 H) + F\, \tau_2\, k_2 \sinh(k_2 H) = 0. \end{aligned} \qquad [28]$$

The characteristic equation of the system [28] is:

$$\begin{aligned} &\left(\frac{\sigma_1 \tau_2 k_2}{\operatorname{tgh}(k_1 H)} - \frac{\sigma_2 \tau_1 k_1}{\operatorname{tgh}(k_2 H)}\right) \operatorname{tgh}(\eta H) = \\ &\qquad = \frac{(2\,\mu + \varkappa)^2 k^2 \eta^2 k_1 k_2 (\tau_2 - \tau_1)}{(2\,\mu + \varkappa + \lambda)\,\eta^2 - k^2 \lambda} \end{aligned} \qquad [29]$$

In the classical limit equation [29] reduces to the corresponding relation found in the framework of the usual elasticity (7).

For waves long compared with the thickness $2H$, retaining up to the third terms of the power expansion of the hyperbolic functions, equation [29] becomes:

$$(3 - \eta H^2) \left| \frac{\sigma_1 \tau_2}{k_1^2 (3 - k_2^2 H^2)} - \frac{\sigma_2 \tau_1}{k_2^2 (3 - k_1^2 H^2)} \right| = \\ = \frac{(2\mu + \varkappa)^2 (\tau_2 - \tau_1)}{(2\mu + \varkappa + \lambda)\eta^2 - k^2 \lambda} \tag{30}$$

This is the period equation for long flexural waves. Dispersion occurs for these waves, with phase velocity decreasing to zero with increasing wave length.

In the classical limit, equation [30] reduces to the corresponding relation found in the framework of the usual elasticity (8).

For wave lengths small compared to the thickness $2H$, equation [30] reduces to equation [25].

Love waves

The components of the stress and couple stress tensors determined by the displacement field $(0, 0, u_3)$ and by the microrotation field $(\varphi_1, \varphi_2, 0)$ are:

$$\left.\begin{aligned}
t_{11} &= t_{22} = t_{33} = t_{21} - t_{12} = 0 \\
t_{13} &= (\mu + \varkappa) u_{3,1} + \varkappa \varphi_2 \\
t_{31} &= \mu u_{3,1} - \varkappa \varphi_2 \\
t_{23} &= (\mu + \varkappa) u_{3,2} - \varkappa \varphi_1 \\
t_{32} &= \mu u_{3,2} + \varkappa \varphi_1 \\
m_{11} &= (\beta + \gamma) \varphi_{1,1} + \alpha \Theta \\
m_{22} &= (\beta + \gamma) \varphi_{2,2} + \alpha \Theta \\
m_{33} &= \alpha \Theta \\
m_{12} &= \gamma \varphi_{2,1} + \beta \varphi_{1,2} \\
m_{21} &= \gamma \varphi_{1,2} + \beta \varphi_{2,1} \\
m_{13} &= m_{31} = m_{23} = m_{32} = 0
\end{aligned}\right| \tag{31}$$

Let us now assume that a monochromatic wave propagates, along the x_2 – direction, in an infinite micropolar elastic plate, whose thickness is $2H$. Moreover we assume that the surfaces of the plate are free of stresses and couple stresses. This leads to the following boundary conditions:

$$t_{13} = m_{11} = m_{12} = 0, \quad \text{at} \quad x_1 = \pm H \tag{32}$$

Equations [9.b] are satisfied by:

$$\varphi_1 = \Lambda_{,1} - \Gamma_{,2}$$
$$\varphi_2 = \Lambda_{,2} + \Gamma_{,1}$$

provided:

$$\left.\begin{aligned} &[(\alpha + \beta + \gamma)\, \nabla^2 - 2\,\varkappa - I\, \partial^2/\partial t^2]\, \Lambda = 0 \\ &[\gamma\, \nabla^2 - 2\,\varkappa - I\, \partial^2/\partial t^2]\, \Gamma - \varkappa\, u_3 = 0 \\ &[(\mu + \varkappa)\, \nabla^2 - \varrho\, \partial^2/\partial t^2]\, u_3 + \varkappa\, \nabla^2\, \Gamma = 0 \end{aligned}\right\} \tag{33}$$

The last two equations [33], after simple manipulations, can be reduced to the following form:

$$\left\{[\gamma\, \nabla^2 - 2\,\varkappa - I\, \partial^2/\partial t^2]\,[(\mu + \varkappa)\, \nabla^2 - \varrho\, \partial^2/\partial t^2] + \varkappa\, \nabla^2\right\} \Gamma = 0 \tag{34.a}$$

$$\left\{[\gamma\, \nabla^2 - 2\,\varkappa - I\, \partial^2/\partial t^2]\,[(\mu + \varkappa)\, \nabla^2 - \varrho\, \partial^2/\partial t^2] + \varkappa\, \nabla^2\right\} u_3 = 0 \tag{34.b}$$

Let us assume solutions of the form:

$$\left.\begin{aligned} &\Lambda\ \ (x_1, x_2, t) = \Lambda^*\ \ (x_1)\, e^{i\,(k\, x_2 - \omega t)} \\ &\Gamma\ \ (x_1, x_2, t) = \Gamma^*\ \ (x_1)\, e^{i\,(k\, x_2 - \omega t)} \\ &u_3\ \ (x_1, x_2, t) = u_3^*\ \ (x_1)\, e^{i\,(k\, x_2 - \omega t)} \end{aligned}\right\} \tag{35}$$

Then the first equation [33] and equations [34] become:

$$\Lambda^*_{,11} - \left(k^2 + \frac{\omega^2}{v_2^2}\right) \Lambda^* = 0 \tag{36}$$

$$\left\{\left(\frac{d^2}{dx_1^2} - k^2 + a^2\right)\left(\frac{d^2}{dx_1^2} - k^2 + d^2 - b^2\right) + \sigma^2\left(\frac{d^2}{dx_1^2} - k^2\right)\right\} \Gamma^* = 0 \tag{37.a}$$

$$\left\{\left(\frac{d^2}{dx_1^2} - k^2 + a^2\right)\left(\frac{d^2}{dx_1^2} - k^2 + d^2 - b^2\right) + \sigma^2\left(\frac{d^2}{dx_1^2} - k^2\right)\right\} u_3^* = 0 \tag{37.b}$$

where

$$v_2^2 = \frac{\alpha + \beta + \gamma}{\frac{2\varkappa}{\omega^2} - I}$$

We search, as in the previous case, solutions of equations [36] and [37] of the form:

$$\left.\begin{aligned} A^* &= A \sinh(\zeta x_1) + B \cosh(\zeta x_1) \\ \Gamma^* &= C \sinh(k_1 x_1) + D \cosh(k_1 x_1) + \\ &\quad + E \sinh(k_2 x_1) + F \cosh(k_2 x_1) \\ u_2^* &= G \sinh(k_1 x_1) + L \cosh(k_1 x_1) + \\ &\quad + P \sinh(k_2 x_1) + Q \cosh(k_2 x_1) \end{aligned}\right\} \qquad [38]$$

where

$$\zeta^2 = k^2 - \frac{1}{v_2^2}$$

Let us now specialize our study to the case of symmetric vibrations. Then the component φ_2 of the microrotation vector, the components m_{11} and m_{22} of the couple stress tensor and the component t_{13} of the stress tensor must be symmetric with respect to the plane $x_1 = 0$. This leads to:

$$\begin{aligned} &A = D = F = L = Q = 0, \\ &G = \xi_1 C \\ &P = \xi_2 E \end{aligned}$$

where:

$$\xi_n = \frac{\varkappa (k^2 - k_n^2)}{(\mu + \varkappa)(k_n^2 - k) + \varrho \omega^2}, \quad n = 1,2.$$

From the assumption that the determinant of the system of the three homogeneous equations, obtained giving an explicit expression to the boundary conditions [32], must be zero, we get the following period equation:

$$\frac{\operatorname{tgh}(\zeta H)}{\operatorname{tgh}(k_1 H)} = \left\{ (\gamma + \beta) \varkappa k^2 \left(g_2 k_1 \frac{\operatorname{tgh}(k_2 H)}{\operatorname{tgh}(k_1 H)} - g_1 k_2 \right) + \right.$$
$$+ \left[(\alpha + \beta + \gamma) \zeta^2 - \alpha k^2 \right] \left(g_1 f_2 - g_2 f_1 \frac{\operatorname{tgh}(k_2 H)}{\operatorname{tgh}(k_1 H)} \right) \Big\} \cdot$$
$$\cdot \left\{ (\gamma + \beta)^2 k^2 (k_1 f_2 - k_2 f_1) \right\}^{-1} \qquad [39]$$

where:

$$\left. \begin{aligned} g_n &= \frac{\gamma + \beta}{2} (k_n^2 + k^2) + \frac{\gamma - \beta}{2} (k_n^2 - k^2) \\ f_n &= (\mu + \varkappa) k_n \xi_n + \varkappa k_n \end{aligned} \right\}, \quad n = 1,2.$$

For wave lengths small compared to the thickness $2H$, equation [39] reduces to:

$$2 (\gamma + \beta) k^2 \zeta (k_1 f_1 - k_2 f_2) = (g_2 f_1 - g_1 f_2) \cdot$$
$$\cdot [(\gamma + \beta) \varkappa k^2 - (\alpha + \beta + \gamma) \zeta^2 - \alpha^2 k^2] \qquad [40]$$

It is obvious that we have no counterpart of equations [39] and [40] in classical elasticity. However we may conclude that in a micropolar elastic medium we have waves of Love type, while, in the classical framework, this is possible only in a layered medium by imposing some conditions on the material constants.

Let us now specialize our study to the case of antisymmetric vibrations. The component φ_2 of the microrotation vector, the components m_{11} and m_{22} of the couple stress tensor and the component t_{13} of the stress tensor are antisymmetric with respect to the plane $x_1 = 0$. These conditions lead to:

$$B = C = E = G = P = 0$$
$$L = \xi_1 D$$
$$Q = \xi_2 F$$

and by the procedure, already employed in the previous cases, we get the following period equation:

$$\frac{\operatorname{tgh}(\zeta H)}{\operatorname{tgh}(k_1 H)} = \left| (\gamma + \beta)(k_1 f_2 - k_2 f_1) k^2 \zeta \right| \Bigg\{ \left| (\alpha + \beta + \gamma) \zeta^2 - \alpha k^2 \right| \cdot$$

$$\cdot \left[g_1 f_2 - g_2 f_1 \frac{\operatorname{tgh}(k_1 H)}{\operatorname{tgh}(k_2 H)} + \right.$$

$$\left. + \frac{1}{2} (\gamma + \beta) \varkappa^2 H^2 \left| k_1 g_2 \frac{\operatorname{tgh}(k_1 H)}{\operatorname{tgh}(k_2 H)} - k_2 g_1 \right| \right\}^{-1} \qquad [41]$$

For wave lengths small compared with the thickness $2H$, equation [41] reduces to equation [40].

Conclusions

We have studied the Lamb and Love wave propagation in an infinite homogeneous micropolar elastic plate with free boundary surfaces. We have found that a displacement field $(u_1, u_2, 0)$ and a microrotation field $(0, 0, \varphi_3)$ leads to waves which can be considered of Lamb's kind, while a displacement field $(0, 0, u_3)$ and a microrotation field $(\varphi_1, \varphi_2, 0)$ leads to waves which can be considered of Love's kind. Therefore it is possible to have, in micropolar media, Love waves without the conditions required in the framework of classical elasticity.

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References

(1) Boschi E., 1972. – *Body Force and Body Couple Equivalents for Seismic Dislocations in Micropolar Media*, to be published.

(2) Boschi E., 1972. – *Body Force, Body Couple and Heat Source Equivalents for Seismic Dislocations in Micropolar Media*, to be published.

(3) Eringen A. C., 1966. – *Linear Theory of Micropolar Elasticity*. "J. Math. and Mech.", **15**, 909.

(4) EWING W. M., JARDETZKY W. S., PRESS F., 1957. – *Elastic Waves in Layered Media*. New York.

(5) See, e.g., eq. [6-12], page 283, of Ref. 4.

(6) See, e.g., eq. [6-15] and [6-17], page 283, of Ref. 4.

(7) See, e.g., eq. [6-11], page 283, of Ref. 4.

(8) See, e.g., eq. [6-21], page 285, of Ref. 4.

(9) GOGOLADZE V. G., 1947. – *Dispersion of Rayleigh waves in a Layer*. Publ. Inst. Seism. Acad. Sci. U.R.S.S., **119**, 27.

(10) IESAN D., 1969. – *Sur la Théorie de la Thermoélasticité Micropolaire Couplée*. "C. Rend. Acad. Sc. Paris", **268**, 58.

(11) LAMB H., 1916. – *On Waves in an Elastic Plate*. "Proc. Roy. Soc. (London)", A, **93**, 114.

(12) PRESCOTT J., 1942. – *Elastic Waves and Vibrations of Thin Rods*. "Phil. Mog.", **33**, 703.

(13) RAYLEIGH, 1889. – *On the Free Vibrations of an Infinite Plate of Homogeneous Isotropic Elastic Matter*. "Proc. London Math. Soc.", **20**, 225.

(14) SATÔ Y., 1951. – *Study of Surface Waves II: Velocity of Surface Waves Propagated upon Elastic Plates*. "Bull. Earth. Res. Inst. (Tokyo)", **29**, 223.