POLARISATION OF THE *S* - PHASE OF SEISMOGRAMS (*)

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It has been observed by F. Neumann [1], P. Byerly [2] and I. Lehmann [3] that in seismograms of earthquakes recorded at moderate distances, the *S* phase appears first as *SH*, followed some 10-14 seconds later by *SV*. Miss Lehmann attributes this phenomenon to the small angle that the *S* — oscillations make with the horizontal, while Prof. Byerly suggests that some kind of double refraction may be the cause. Both agree that the interval does not seem to depend on epicentral distance.

Evidently more observational evidence is needed, but a preliminary examination can be made of the possibility that the phenomenon arises because the outer part of the Earth's mantle is not elastically isotropic. The simplest hypothesis we can make to conform to this assumption is perhaps that all directions at right angles to the vertical are equivalent; this type of solid is termed « transversely isotropic ». The assumed type of anisotropy is sufficiently general to give an indication of the order of magnitude of the anomaly to be expected. Further, it is unlikely that the outer layers are very markedly anisotropic, else this departure from isotropy would probably have been noted long ago. It should suffice then, to take as a standard of comparison a transversely isotropic material whose elastic constants are known: such a crystal is beryl, the constants of which were determined by Voigt [4].

The seismological implications of a transversely isotropie medium have been investigated, and one important fact is that if all directions perpendicular to the vertical are equivalent then (neglecting the curvature of the Earth) the velocities of *SH* and *SV* are different, and they depend on the angle of incidence of the wave. The difference in the time of propagation through a surface layer of thickness h can easily be calculated. Let suffixes H, V refer to *SH*, *SV* respectively; let c be the wave-velocity in any medium and i the angle of incidence

(*) La traduzione in lingua italiana della presente Nota è riportata a pag. 113.

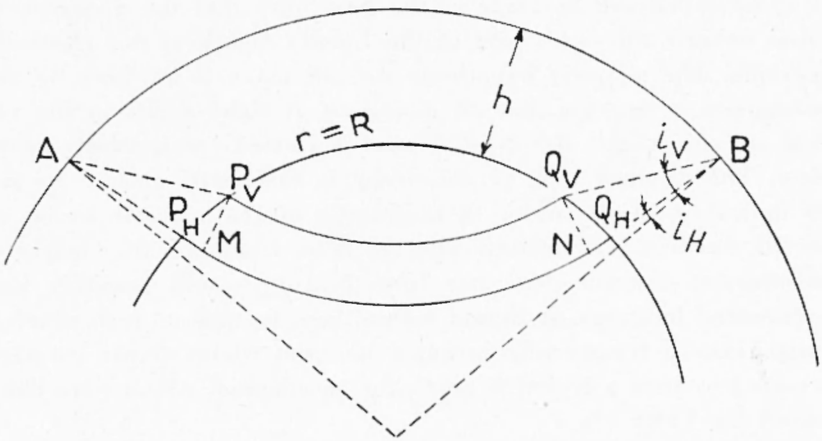
in that medium. If r is the radial distance from the centre of the Earth, then for any ray, in the standard notation,

$$p = \frac{r \sin i}{c} = \frac{dT}{d\Delta}$$

Suppose that a disturbance originating at A generates SH and SV waves arriving at B along paths $PQ_H B$ and $PQ_V B$. Let $P_V M$ and $Q_V N$ be perpendicular to $P_H\, M$ and $Q_H\, N$ respectively, where refraction is supposed to occur at the base of a surface layer.

Then from the diagram $P_H\, M = NQ_H = h\ (\tan i_V - \tan i_H) \sin i$, so that the difference of the time of transmission of S along the paths $P_V\, Q_V$ and $P_H\, Q_H$ in the lower material, supposed isotropic, is $2(h/c)$ $\sin i\ (\tan i_V - \tan i_H) = 2\, dT_1$ say.

Alternatively, if O is the centre of the Earth and R the radius at the discontinuity, while the angle $Q_V\, OQ_H$ is $d\,\Delta$, then $Q_V Q_H = R.\ d\Delta$ $= R(d\Delta/dT_1)\ dT_1 = c\ dT_1/\sin i$, giving the same result as before.



Thus, in all, the difference in the times of transmission of SH and SV

$$T_H - T_V = \frac{2h}{c_H} \sec i_H - \frac{2h}{c_V} \sec i_V + \frac{2h}{c} \sin i\, (\tan i_V - \tan i_H)$$

where c, i refer to the medium immediately below the junction.

To see the type of variation of c_V and c_H with inclination 0° to the vertical, it will suffice to work with the density and elastic constants of beryl, and to scale down the results so as to correspond to a velocity 3.40 km/sec. in the layer for SV at $\vartheta = 0^\circ$ and $\vartheta = 90^\circ$. In the underlying material we take $c=4.38$ km/sec.; thus the model corresponds to a granitic continent resting on ultrabasic material.

The requisite formulae for c_{H} and c_{V} are found in the paper already cited ([6]). For a transversely isotropic body in which the strain-energy function W is given by

$$2W=A(e^2_{xx}+e^2_{yy})+Ce^2_{zz}+2F(e_{xx}+e_{yy})e_{zz}+$$
$$+2(A-2N)e_{xx}e_{yy}+L(e^2_{yz}+e^2_{zx})+Ne^2_{xy}$$

the velocity c of a wave corresponding to HS is given by

$$\rho c^2_{H}=Nl^2+Ln^2$$

where $n=\cos\vartheta$; $l=\sin\vartheta$. For waves of SV type

$$2\rho c^2_{V}=Al^2+Cn^2-\left|\left\{(A-L)l^2-(C-L)n^2\right\}^2+4J^2l^2n^2\right|^{\frac{1}{2}}$$

in which $J=F+L$.

Voigt's values, in dynes/cm^2, are

$A=2.694\times10^{12}$; $C=2.363\times10^{12}$; $A-2N=0.961\times10^{12}$;
$F=0.661\times10^{12}$; $L=0.653\times10^{12}$; $N=0.866\times10^{12}$.

From the foregoing formulae, the values of c_{H} and c_{V}, scaled so that they correspond to material in which c^2_{H} and c^2_{V} for $\vartheta=0°$ are 11.56×10^{10}, are given in the following table:

TABLE I

$\vartheta°$	$c_{H}\times10^{-5}$	$c_{V}\times10^{-5}$
0	3.40	3.40
10	3.41	3.48
20	3.46	3.68
30	3.54	3.905
40	3.62	4.04
50	3.71	4.02
60	3.79	3.855
70	3.86	3.64
80	3.90	3.47
90	3.92	3.40

The differing trends of c_{H} and c_{V} with ϑ are evident from this table.

In what follows the layer will be treated as thin, so that for propagation in this layer the Earth may be considered as plane. Then i_{H} and i_{V} are given by

$$\frac{\sin i}{c} = \frac{\sin i_{\text{H}}}{c_{\text{H}}} = \frac{\sin i_{\text{V}}}{c_{\text{V}}}$$

where $c = 4.38$ km/sec. and $(\sin i)/c = (dT/d\Delta^{\circ})/110.7$
Since $dT/d\Delta$ is known from seismological tables, the angle i can be computed for various epicentral distances Δ.

To complete the calculations we first consider SH, and put $\vartheta = i_{\text{H}}$ in Table I. Thus $(\sin i_{\text{H}})/c$ can be computed as a function of i, and from the foregoing equation i_{H} is found as a function of Δ. The calculations for SV are much longer, but the general principle is the same. We thus obtain Table II. It will be seen that the differences between i_{H} and i_{V} are not large. The value of $(T_{\text{H}} - T_{\text{V}})/2h$ can now be computed directly.

TABLE II

Δ degrees	dT/Δ sec/deg	i_{H} degrees	i_{V} degrees	i degrees
10	24.4	56.3	58.7	74.5
20	19.4 Sd	39.3	45.0	50.0
20	18.7 Sr	38.0	43.0	47.7
30	15.8	30.0	34.8	38.8
40	15.0	28.5	32.3	36.4
50	13.9	26	29.4	33.5
60	12.8	23.5	26.4	30.5
70	11.8	21.5	23.0	27.8
80	10.5	19.0	20.7	24.5
90	9.0	16.2	17.2	20.9
100	9.4	15.0	16.0	19.2

Since the object of this investigation is to ascertain only the order of magnitude of the effect of departure from isotropy, it suffices to consider three values of $(T_{\text{H}} - T_{\text{V}})/2h$. For $\Delta = 20^{\circ}(Sr)$ it is 0.039, for $\Delta = 50^{\circ}$ it is 0.035, and for $\Delta = 80^{\circ}$ it is 0.025. Thus, the variation with epicentral distance is not large. However, the time difference is quite small, e.g. for $h = 33$ km., about the maximum permissible thickness of the granitic layer, $T_{\text{H}} - T_{\text{V}}$ amounts only to about 1.6 sec. Thus the explanation in terms of departure from isotropy in the outer layers seems to fail, for with a typical crystalline medium (« adjusted » to fit the velocity of Sg), it would need a thickness of anisotropic rock of the order of 300 km to account for a difference of 14 seconds in the times of arrival of *SH* and *SV* at an epicentral distance of 80°. While one cannot entirely rule out the possibility of a preferred orientation in the outer crystalline part of the mantle (roughly the part above the 20° discontinuity) it is to be remembered that the model chosen is one in which the axes of symmetry of *all* the constituent crystals are radial. Accordingly, unless decisive evidence of anisotropy in the Earth's crust is forthcoming from other sources, this will not be an adequate explanation of the observed time difference $T_H - T_V$. Provisionally, then, Miss Lehmann's explanation seems the more likely.

Comunicazione presentata al Convegno per la costituzione della F.S.E., tenutosi a Verona dal 12 al 16 settembre 1950 (v. « Annali di Geofisica », vol. III, pag. 579).

SUMMARY

In earthquakes recorded at moderate distances it has been observed that S phase appears first as SH, followed some 10 to 14 seconds later by SV. The object of this paper is to try to decide whether double refraction is likely to be the explanation of this phenomenon.

A simple model to consider would be a « transversely isotropic » material, symmetrical about the radial direction. Formulae for the velocities of SH and SV waves are available; these velocities depend on the angle that the ray makes with the normal. It is unlikely that the Earth could be as markedly anisotropic as the mineral beryl, which is transversely isotropic; accordingly, this material, of which the five elastic constants are known is taken as an extreme example, and the velocities of SH and SV for different angles of incidence are « scaled down » so as to match the velocity of distortional waves in granite. It is then possible to calculate the difference in the time taken by waves from one point of the surface of the Earth to another point on the surface according as the S wave in the surface layer is of SH or SV type.

It is found that, even in this extreme case, a layer of anisotropic rock some 30 km thick would account only for a time difference of

about 1½ seconds. Thus, if the Earth were as strongly anisotropic as beryl (which is unlikely) one would need the layer of preferred orientation to extend to a depth of about 300 km. This is difficult to believe, and thus no great credence can be attached to an explanation in terms of double refraction.

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(4) W. Voigt, *Lehrbuck der Kristallphysik* (1928).

(5) R. Stoneley, *Geophys. Supp.* 5, 343, 1949, e.g. K. E. Bullen, Theoretical Seismology, p. 109.

(6) R. Stoneley, *Loc. cit.* equations (11) o (11 a).