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Closure properties of function spaces

Ljubiša D.R. Kočinac*

Dedicated to Professor S. Naimpally on the occasion of his 70^{th} birthday.

Abstract. In this paper we investigate some closure properties of the space $\mathsf{C}_k(X)$ of continuous real-valued functions on a Tychonoff space X endowed with the compact-open topology.

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1. Introduction.

All spaces in this paper are assumed to be Tychonoff. Our notation and terminology are standard, mainly the same as in [2]. Some notions will be defined when we need them. By $\mathsf{C}_k(X)$ we denote the space of continuous real-valued functions on a space X in the compact-open topology. Basic open sets of $\mathsf{C}_k(X)$ are of the form

$$W(K_1,\dots,K_n;V_1,\dots,V_n) := \{f \in C(X) : f(K_i) \subset V_i, i = 1,\dots,n\},$$

where $K_1,\dots,K_n$ are compact subsets of X and $V_1\dots,V_n$ are open in $\mathbb{R}$. For a function $f \in \mathsf{C}_k(X)$, a compact set $K \subset X$ and a positive real number ε we let

$$W(f;K;\varepsilon) := \{g \in \mathsf{C}_k(X) : |g(x) - f(x)| < \varepsilon,\ \forall x \in K\}.$$

The standard local base of a point $f \in \mathsf{C}_k(X)$ consists of the sets $W(f;K;\varepsilon)$, where K is a compact subset of X and ε is a positive real number.

The symbol $\underline{0}$ denotes the constantly zero function in $\mathsf{C}_k(X)$. The space $\mathsf{C}_k(X)$ is homogeneous so that we may consider the point $\underline{0}$ when studying local properties of $\mathsf{C}_k(X)$.

Many results in the literature show that for a Tychonoff space X closure properties of the function space $\mathsf{C}_p(X)$ of continuous real-valued functions on

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X endowed with the topology of pointwise convergence can be characterized by covering properties of X. We list here some properties which are expressed in this manner.

(A) [**Arhangel'skii–Pytkeev**] $\mathsf{C}_p(X)$ has countable tightness if and only if all finite powers of X have the Lindelöf property ([1]).

(B) [**Arhangel'skii**] $\mathsf{C}_p(X)$ has countable fan tightness if and only if all finite powers of X have the Menger property ([1]).

(C) [**Sakai**] $\mathsf{C}_p(X)$ has countable strong fan tightness if and only if all finite powers of X have the Rothberger property ([13]).

(D) [**Kočinac–Scheepers**] $\mathsf{C}_p(X)$ has countable fan tightness and the Reznichenko property if and only if all finite powers of X have the Hurewicz property ([7]).

In this paper we consider these properties in the context of spaces $\mathsf{C}_k(X)$. To get analogues for the compact-open topology one need modify the role of covers, preferably ω-covers from C_p-theory should be replaced by k-covers in C_k-theory.

Recall that an open cover $\mathcal{U}$ of X is called a *k-cover* if for each compact set $K \subset X$ there is a $U \in \mathcal{U}$ such that $K \subset U$. The symbol $\mathcal{K}$ denotes the collection of all k-covers of a space.

Let us mention one result of this sort which should be compared with the Arhangel'skii-Pytkeev theorem (A) above.

In [9] it was remarked (without proof) that the following holds:

Theorem 1.1. *A space $\mathsf{C}_k(X)$ has countable tightness if and only if for each k-cover $\mathcal{U}$ of X there is a countable set $\mathcal{V} \subset \mathcal{U}$ which is a k-cover of X.*

The proof of this result can be found in Theorem 3.13 of [11].

1.1. **Selection principles and games.** In this paper we shall need selection principles of the following two sorts: Let S be an infinite set and let $\mathcal{A}$ and $\mathcal{B}$ both be sets whose members are families of subsets of S.

Then $\mathsf{S}_1(\mathcal{A}, \mathcal{B})$ denotes the selection principle:

> For each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(b_n : n \in \mathbb{N})$ such that for each n $b_n \in A_n$, and $\{b_n : n \in \mathbb{N}\}$ is an element of $\mathcal{B}$.

$\mathsf{S}_{fin}(\mathcal{A}, \mathcal{B})$ denotes the selection principle:

> For each sequence $(A_n : n \in \mathbb{N})$ of elements of $\mathcal{A}$ there is a sequence $(B_n : n \in \mathbb{N})$ of finite sets such that for each n $B_n \subset A_n$, and $\bigcup_{n\in\mathbb{N}} B_n$ is an element of $\mathcal{B}$.

For a topological space X, let $\mathcal{O}$ denote the collection of open covers of X. Then the property $\mathsf{S}_1(\mathcal{O}, \mathcal{O})$ is called the *Rothberger property* [12],[5], and the property $\mathsf{S}_{fin}(\mathcal{O}, \mathcal{O})$ is known as the *Menger property* [10], [3],[5].

There is a natural game for two players, ONE and TWO, denoted $\mathsf{G}_{fin}(\mathcal{A}, \mathcal{B})$, associated with $\mathsf{S}_{fin}(\mathcal{A}, \mathcal{B})$. This game is played as follows: There is a round per positive integer. In the n-th round ONE chooses an $A_n \in \mathcal{A}$, and TWO responds with a finite set $B_n \subset A_n$. A play $A_1, B_1; \cdots ; A_n, B_n; \cdots$ is won by TWO if $\bigcup_{n\in\mathbb{N}} B_n$ is an element of $\mathcal{B}$; otherwise, ONE wins.

Similarly, one defines the game $\mathsf{G}_1(\mathcal{A}, \mathcal{B})$, associated with $\mathsf{S}_1(\mathcal{A}, \mathcal{B})$.

2. Countable (strong) fan tightness of $\mathsf{C}_k(X)$.

For X a space and a point $x \in X$ the symbol Ω_x denotes the set $\{A \subset X \setminus \{x\} : x \in \overline{A}\}$.

A space X has *countable fan tightness* [1] if for each $x \in X$ and each sequence $(A_n : n \in \mathbb{N})$ of elements of Ω_x there is a sequence $(B_n : n \in \mathbb{N})$ of finite sets such that for each n $B_n \subset A_n$ and $x \in \overline{\bigcup_{n\in\mathbb{N}} B_n}$, i.e. if $\mathsf{S}_{fin}(\Omega_x, \Omega_x)$ holds for each $x \in X$. A space X has *countable strong fan tightness* [13] if for each $x \in X$ the selection principle $\mathsf{S}_1(\Omega_x, \Omega_x)$ holds.

In [8], it was shown (compare with the corresponding theorem (B) of Arhangel'-skii for $\mathsf{C}_p(X)$):

Theorem 2.1. *The space $\mathsf{C}_k(X)$ has countable fan tightness if and only if X has property $\mathsf{S}_{fin}(\mathcal{K}, \mathcal{K})$.*

We show

Theorem 2.2. *For a space X the following are equivalent:*

(*a*) *$\mathsf{C}_k(X)$ has countable strong fan tightness;*

(*b*) *X has property $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$.*

Proof. $(a) \Rightarrow (b)$: Let $(\mathcal{U}_n : n \in \mathbb{N})$ be a sequence of k-covers of X. For a fixed $n \in \mathbb{N}$ and a compact subset K of X let $\mathcal{U}_{n,K} := \{U \in \mathcal{U}_n : K \subset U\}$. For each $U \in \mathcal{U}_{n,K}$ let $f_{K,U}$ be a continuous function from X into [0,1] such that $f_{K,U}(K) = \{0\}$ and $f_{K,U}(X \setminus U) = \{1\}$. Let for each n,

$$A_n = \{f_{K,U} : K \text{ compact in } X, U \in \mathcal{U}_{n,K}\}.$$

Then, as it is easily verified, $\underline{0}$ is in the closure of A_n for each $n \in \mathbb{N}$.

Since $\mathsf{C}_k(X)$ has countable strong fan tightness there is a sequence $(f_{K_n,U_n} : n \in \mathbb{N})$ such that for each n, $f_{K_n,U_n} \in A_n$ and $\underline{0} \in \overline{\{f_{K_n,U_n} : n \in \mathbb{N}\}}$. Consider the sets U_n, $n \in \mathbb{N}$. We claim that the sequence $(U_n : n \in \mathbb{N})$ witnesses that X has property $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$.

Let C be a compact subset of X. From $\underline{0} \in \overline{\{f_{K_n,U_n} : n \in \mathbb{N}\}}$ it follows that there is $i \in \mathbb{N}$ such that $W = W(\underline{0}; C; 1)$ contains the function f_{K_i,U_i}. Then $C \subset U_i$. Otherwise, for some $x \in C$ one has $x \notin U_i$ so that $f_{K_i,U_i}(x) = 1$, which contradicts the fact $f_{K_i,U_i} \in W$.

$(b) \Rightarrow (a)$: Let $(A_n : n \in \mathbb{N})$ be a sequence of subsets of $\mathsf{C}_k(X)$ the closures of which contain $\underline{0}$. Fix n. For every compact set $K \subset X$ the neighborhood $W = W(\underline{0}; K; 1/n)$ of $\underline{0}$ intersects A_n so that there exists a function $f_{K,n} \in A_n$ such that $|f_{K,n}(x)| < 1/n$ for each $x \in K$. Since $f_{K,n}$ is a continuous function there are neighborhoods O_x, $x \in K$, such that for $U_{K,n} = \bigcup_{x\in K} O_x \supset K$ we have $f_{K,n}(U_{K,n}) \subset (-1/n, 1/n)$. Let $\mathcal{U}_n = \{U_{K,n} : K \text{ a compact subset of } X\}$. For each $n \in \mathbb{N}$, $\mathcal{U}_n$ is a k-cover of X. Apply that X is an $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$-set: for each

$m \in \mathbb{N}$ there exists a sequence $(U_{K,n} : n \geq m)$ such that for each n, $U_{K,n} \in \mathcal{U}_n$ and $\{U_{K,n} : n \geq m\}$ is a k-cover for X. Look at the corresponding functions $f_{K,n}$ in A_n.

Let us show $\underline{0} \in \overline{\{f_{K,n} : n \in \mathbb{N}\}}$. Let $W = W(\underline{0}; C; \varepsilon)$ be a neighborhood of $\underline{0}$ in $\mathsf{C}_k(X)$ and let m be a natural number such that $1/m < \varepsilon$. Since C is a compact subset of X and X is an $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$-set, there is $j \in \mathbb{N}$, $j \geq m$ such that one can find a $U_{K,j}$ with $C \subset U_{K,j}$. We have

$$f_{K,j}(C) \subset f_{K,j}(U_{K,j}) \subset (-1/j, 1/j) \subset (-1/m, 1/m) \subset (-\varepsilon, \varepsilon),$$

i.e. $f_{K,j} \in W$. □

3. Countable T-tightness of $\mathsf{C}_k(X)$.

The notion of T-tightness was introduced by I. Juhász at the IV International Conference "Topology and its Applications", Dubrovnik, September 30 – October 5, 1985 (see [4]). A space X has *countable T-tightness* if for each uncountable regular cardinal ρ and each increasing sequence $(A_\alpha : \alpha < \rho)$ of closed subsets of X the set $\cup\{A_\alpha : \alpha < \rho\}$ is closed. In [14], the T-tightness of $\mathsf{C}_p(X)$ was characterized by a covering property of X. The next theorem is an analogue of this result in the $\mathsf{C}_k(X)$ context.

Theorem 3.1. *For a space X the following are equivalent:*

(*a*) $\mathsf{C}_k(X)$ *has countable T-tightness;*

(*b*) *for each regular cardinal ρ and each increasing sequence $(\mathcal{U}_\alpha : \alpha < \rho)$ of families of open subsets of X such that $\bigcup_{\alpha<\rho} \mathcal{U}_\alpha$ is a k-cover of X there is a $\beta < \rho$ so that $\mathcal{U}_\beta$ is a k-cover of X.*

Proof. $(a) \Rightarrow (b)$: Let ρ be a regular uncountable cardinal and let $(\mathcal{U}_\alpha : \alpha < \rho)$ be an increasing sequence of families of open subsets of X such that $\bigcup_{\alpha<\rho} \mathcal{U}_\alpha$ is a k-cover for X. For each $\alpha < \rho$ and each compact set $K \subset X$ let $\mathcal{U}_{\alpha,K} := \{U \in \mathcal{U}_\alpha : K \subset U\}$. For each $U \in \mathcal{U}_{\alpha,K}$ let $f_{K,U}$ be a continuous function from X into [0,1] such that $f_{K,U}(K) = \{0\}$ and $f_{K,U}(X \setminus U) = \{1\}$. For each $\alpha < \rho$ put

$$A_\alpha = \{f_{K,U} : U \in \mathcal{U}_{\alpha,K}\}.$$

Since the T-tightness of $\mathsf{C}_k(X)$ is countable, the set $A = \bigcup_{\alpha<\rho} \overline{A_\alpha}$ is closed in $\mathsf{C}_k(X)$.

Let $W(\underline{0}; K; \varepsilon)$ be a standard basic neighborhood of $\underline{0}$. There is an $\alpha < \rho$ such that for some $U \in \mathcal{U}_\alpha$, $K \subset U$. Then $U \in \mathcal{U}_{\alpha,K}$ and thus there is $f \in A_\alpha$, hence $f \in A_\alpha \cap W(\underline{0}; K; \varepsilon)$. Therefore, each neighborhood of $\underline{0}$ intersects some of the sets A_α, $\alpha < \rho$, which means that $\underline{0}$ belongs to the closure of the set $\bigcup_{\alpha<\rho} A_\alpha$. Since this set is actually the set A it follows there exists a $\beta < \rho$ with $\underline{0} \in \overline{A_\beta}$. We claim that the corresponding family $\mathcal{U}_\beta$ is a k-cover of X.

Let C be a compact subset of X. Then the neighborhood $W(\underline{0}; C; 1)$ of $\underline{0}$ intersects A_β; let $f_{K,U} \in A_\beta \cap W(\underline{0}; C; 1)$. By the definition of A_β, then from $f_{K,U}(X \setminus U) = 1$ it follows $C \subset U \in \mathcal{U}_\beta$.

(*b*) ⇒ (*a*): Let $(A_\alpha : \alpha < \rho)$ be an increasing sequence of closed subsets of $\mathsf{C}_k(X)$, with ρ a regular uncountable cardinal. We shall prove that the set $A := \bigcup_{\alpha<\rho} A_\alpha$ is closed. Let $f \in \overline{A}$. For each $n \in \mathbb{N}$ and each compact set $K \subset X$ the neighborhood $W(f; K; 1/n)$ of f intersects A. Put

$$\mathcal{U}_{n,\alpha} = \{g^{\leftarrow}(-1/n, 1/n) : g \in A_\alpha\}$$

and

$$\mathcal{U}_n = \bigcup_{\alpha<\rho} \mathcal{U}_{n,\alpha}.$$

Let us check that for each $n \in \mathbb{N}$, $\mathcal{U}_n$ is a k-cover of X. Let K be a compact subset of X. The neighborhood $W := W(f; K; 1/n)$ of f intersects A, i.e. there is $g \in A$ such that $|f(x) - g(x)| < 1/n$ for all $x \in K$; this means $K \subset g^{\leftarrow}(-1/n, 1/n) \in \mathcal{U}_n$.

By (b) there is $\mathcal{U}_{n,\beta_n} \subset \mathcal{U}_n$ which is a k-cover of X. Put $\beta_0 = \sup\{\beta_n : n \in \mathbb{N}\}$. Since ρ is a regular cardinal, $\beta_0 < \rho$. It is easy to verify that for each n the set $\mathcal{U}_{n,\beta_0}$ is a k-cover of X. Let us show that $f \in A_{\beta_0}$. Take a neighborhood $W(f; C; \varepsilon)$ of f and let m be a positive integer such that $1/m < \varepsilon$. Since $\mathcal{U}_{m,\beta_0}$ is a k-cover of X one can find $g \in A_{\beta_0}$ such that $C \subset g^{\leftarrow}(-1/m, 1/m)$. Then $g \in W(f; C; 1/m) \cap A_{\beta_0} \subset W(f; C; \varepsilon) \cap A_{\beta_0}$, i.e. $f \in \overline{A_{\beta_0}} = A_{\beta_0}$ and thus $f \in A$. So, A is closed. □

4. The Reznichenko property of $\mathsf{C}_k(X)$.

In this section we shall need the notion of groupability (see [7]).

1. A k-cover $\mathcal{U}$ of a space X is *groupable* if there is a partition $(\mathcal{U}_n : n \in \mathbb{N})$ of $\mathcal{U}$ into pairwise disjoint finite sets such that: For each compact subset K of X, for all but finitely many n, there is a $U \in \mathcal{U}_n$ such that $K \subset U$.
2. An element A of Ω_x is *groupable* if there is a partition $(A_n : n \in \mathbb{N})$ of A into pairwise disjoint finite sets such that each neighborhood of x has nonempty intersection with all but finitely many of the A_n.

We use the following notation:

- $\mathcal{K}^{gp}$ – the collection of all groupable k-covers of a space;
- Ω_x^{gp} – the family of all groupable elements of Ω_x.

In 1996 Reznichenko introduced (in a seminar at Moscow State University) the following property: Each countable element of Ω_x is a member of Ω_x^{gp}. This property was further studied in [6] and [7] (see Introduction). In [6] it was called the *Reznichenko property at x*. When X has the Reznichenko property at each of its points, then X is said to have the *Reznichenko property*.

We study now the Reznichenko property in spaces $\mathsf{C}_k(X)$.

Theorem 4.1. *Let X be a Tychonoff space. If* ONE *has no winning strategy in the game $\mathsf{G}_1(\mathcal{K}, \mathcal{K}^{gp})$, then $\mathsf{C}_k(X)$ has property $\mathsf{S}_1(\Omega_{\underline{0}}, \Omega_{\underline{0}}^{gp})$ (i.e. $\mathsf{C}_k(X)$ has countable strong fan tightness and the Reznichenko property).*

Proof. Evidently, from the fact that ONE has no winning strategy in the game $\mathsf{G}_1(\mathcal{K}, \mathcal{K}^{gp})$, it follows that X satisfies $\mathsf{S}_1(\mathcal{K}, \mathcal{K}^{gr})$ and consequently X is in the class $\mathsf{S}_1(\mathcal{K}, \mathcal{K})$. By Theorem 2.2 $\mathsf{C}_k(X)$ has countable strong fan tightness, and thus countable tightness. Therefore, it remains to prove that each countable subset of $\Omega_{\underline{0}}$ is groupable (i.e. that $\mathsf{C}_k(X)$ has the Reznichenko property).

Let A be a *countable* subset of $\mathsf{C}_k(X)$ such that $\underline{0} \in \overline{A}$. We define the following strategy σ for ONE in $\mathsf{G}_1(\mathcal{K}, \mathcal{K}^{gp})$. For a compact set $K \subset X$ the neighborhood $W = W(\underline{0}; K; 1)$ of $\underline{0}$ intersects A. Let $f_K \in A \cap W$. As f_K is continuous, for every $x \in K$ there is a neighborhood O_x of x such that $f_K(O_x) \subset (-1, 1)$. From the open cover $\{O_x : x \in K\}$ of K choose a finite subcover $\{O_{x_1}, \ldots, O_{x_m}\}$ and let $U_K = O_{x_1} \cup \cdots \cup O_{x_m}$. Then $f_K(U_K) \subset (-1, 1)$ and the set $\mathcal{U}_1 = \{U_K : K \text{ a compact subset of } X\}$ is a k-cover of X. ONE's first move, $\sigma(\varnothing)$, will be $\mathcal{U}_1$. Let TWO's response be an element $U_{K_1} \in \mathcal{U}_1$. ONE considers now the corresponding function $f_{K_1} \in A$ (satisfying $f_{K_1}(U_{K_1}) \subset (-1, 1)$) and looks at the set $A_1 = A \setminus \{f_{K_1}\}$ which obviously satisfies $\underline{0} \in \overline{A_1}$. For every compact subset K of X ONE chooses a function $f_K \in A \cap W(\underline{0}; K; 1/2)$ and a neighborhood U_K of K such that $f_K(U_K) \subset (-1/2, 1/2)$. The set $\mathcal{U}_2 = \{U_K : K \text{ a compact subset of } X\} \setminus \{U_{K_1}\}$ is a k-cover of X. ONE plays $\sigma(U_{K_1}) = \mathcal{U}_2$. Suppose that $U_{K_2} \in \mathcal{U}_2$ is TWO's response. ONE first considers the function $f_{K_2} \in A_1$ with $f_{K_2}(U_{K_2}) \subset (-1/2, 1/2)$ and then looks at the set $A_2 = A_1 \setminus \{f_{K_2}\}$ the closure of which contains $\underline{0}$, and so on.

The strategy σ, by definition, gives sequences $(\mathcal{U}_n : n \in \mathbb{N})$, $(U_n : n \in \mathbb{N})$ and $(f_n : n \in \mathbb{N})$ having the following properties:

(i) $(\mathcal{U}_n : n \in \mathbb{N})$ is a sequence of k-covers of X and for each n $\mathcal{U}_n = \sigma(U_1, \ldots, U_{n-1})$;
(ii) For each n, $U_n \in \mathcal{U}_n$ and $U_n \notin \{U_1, \ldots, U_{n-1})\}$;
(iii) For each n, f_n is a member of $A \setminus \{f_1, \ldots, f_{n-1})$;
(iv) For each n, $f_n(U_n) \subset (-1/n, 1/n)$.

Since σ is not a winning strategy for ONE, the play $\mathcal{U}_1, U_1; \ldots; \mathcal{U}_n, U_n; \ldots$ is lost by ONE so that $\mathcal{V} := \{U_n : n \in \mathbb{N}\}$ is a groupable k-cover of X. Therefore there is an increasing infinite sequence $n_1 < n_2 < \cdots < n_k < \ldots$ such that the sets $\mathcal{H}_k := \{U_i : n_k \leq i < n_{k+1}\}$, $k = 1, 2, \ldots$, are pairwise disjoint and for every compact set $K \subset X$ there is k_0 such that for each $k > k_0$ there is $H \in \mathcal{H}_k$ with $K \subset H$. Define also $M_k := \{f_i : n_k \leq i < n_{k+1}\}$. Then the sets M_k are finite pairwise disjoint subsets of A. One can also suppose that $A = \bigcup_{k \in \mathbb{N}} M_k$; otherwise we distribute countably many elements of $A \setminus \bigcup_{k \in \mathbb{N}} M_k$ among M_k's so that after distribution new sets are still finite and pairwise disjoint. We claim that the sequence $(M_k : k \in \mathbb{N})$ witnesses that A is groupable (i.e. that $\mathsf{C}_k(X)$ has the Reznichenko property).

Let $W(\underline{0}; K; \varepsilon)$ be a neighborhood of $\underline{0}$ and let m be the smallest natural number such that $1/m < \varepsilon$. There is n_0 such that for each $n > n_0$ one can choose an element $H_n \in \mathcal{H}_n$ with $K \subset H_n$; choose also a corresponding function $f_n \in M_n$ satisfying $f_n(H_n) \subset (-1/n, 1/n)$. So for each $n > \max\{n_0, m\}$ we have $f_n \in M_n \cap W(\underline{0}; K; \varepsilon)$, i.e for all but finitely many n $W(\underline{0}; K; \varepsilon) \cap M_n \neq \varnothing$. The theorem is shown. □

In a similar way one can prove

Theorem 4.2. *For a space X the statement (a) below implies the statement (b):*

(a) ONE *has no winning strategy in the game $\mathsf{G}_{fin}(\mathcal{K}, \mathcal{K}^{gp})$;*
(b) *$\mathsf{C}_k(X)$ has countable fan tightness and the Reznichenko property (i.e. $\mathsf{C}_k(X)$ has property $\mathsf{S}_{fin}(\Omega_{\underline{0}}, \Omega_{\underline{0}}^{gp})$).*

Problem 4.3. *Is the converse in Theorem 4.1 and in Theorem 4.2 true?*

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Ljubiša D.R. Kočinac
Faculty of Sciences, University of Niš, 18000 Niš, Serbia

E-mail address: lkocinac@ptt.yu