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THE RESPIRATORY IMPEDANCE IN AN ASYMMETRIC MODEL OF THE LUNG STRUCTURE

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ABSTRACT

This paper presents a model of the respiratory tree as a recurrent, but asymmetric, structure. The intrinsic properties posed by such a system lead to a multi-fractal structure, i.e. a non-integer order model of the total impedance. The fractional order behavior of the asymmetric tree simulated as a dynamic system is assessed by means of Bode plots, on a wide range of frequencies. The results indicate than in a specific frequency range, both the symmetric and asymmetric representation of the respiratory tree lead to similar values in the impedance.

Keywords: *fractals, respiratory tree, non-integer order systems, impedance, self-organizing*

2000 *Mathematics Subject Classification*: 92B05, 93A30, 26A33.

1. Introduction

In his recent publication, Weibel discusses the reduction of diameter and length by a constant factor with respect to both blood vessels and airways [1]. He recognizes the theoretical contributions of Murray [2], i.e. that the dissipation of energy due to flow of blood or air in a branched tube system can be minimized if the diameter of the two daughter-branches are related to the diameter of the parent as in $d_{parent}^3 = d_1^3 + d_2^3$ [3]. In the context of fractal geometry, the reduction factor depends on the fractal dimension fD of the branching tree such that the correct formula is: $d_1 = d_{parent} \cdot 2^{-1/fD}$. For example, in the circulatory system, the Murray law gives $fD = 3$ because the tree is considered to be space-filling [4].

In this contribution, we investigate the frequency response of the respiratory system in terms of its equivalent electrical impedance, using an asymmetric and a symmetric tree structure. The comparison between the two responses will indicate whether or not the asymmetry is significant or not in modelling the respiratory impedance.

2.Proposal

In his investigations, Weibel found that the slope of the conducting airway diameters against the generations was given by $d(m) = d_0 \cdot 2^{-m/3}$, with d_0 the tracheal diameter and m the airway generation. He then concludes that the conducting airways of the human lung are designed as a self-similar and space-filling fractal tree, with a homothety factor of $2^{-1/3} = 0.79$ (similitude ratio). However, this averaged value has a significant variance in the first generations [5]. Hence, the average value changes in the diffusion zone (airways from 14^{th} generation onward). These observations and the fact that Weibel himself discusses that a small change in the homothety factor results in a dramatic increase in peripheral bronchiolar resistance, suggests that the lung must be capable to adjust itself to the optimality conditions. Indeed, a closer analysis reveals that the homothety factor is about 0.79 in the 6^{th} generation, but it increases slowly to about 0.9 in the 16^{th} generation, with an average of 0.85 for the small airways [1]. The physiological implications of this observation are [3]:

- the flow resistance decreases in the small airways and

- a small reduction in the homothety factor does not affect significantly the lung function.

In the context of the above observations, we explore the approach of modelling the respiratory system as a multi-fractal structure. A self-similar multi-fractal spatial distribution forms the basis for breaking the symmetry of bifurcation design within a tree. In his study, Bennet discusses the implications of self-affine scaling [3]. It turns out that the fractal dimension changes when viewed from different perspectives. Therefore, the slope determining the homothety factor changes when viewed at a fine or coarse grained diameter scale. This latter observation is of interest in the context of this work, since it supports the idea of a multi-fractal structure. For example, the average of the radius ratio changes from $2^{-0.1713} = 0.8881$ to 0.8923 when only the first 16 generations are taken into account, respectively to 0.8783 for the alveoli (generations 17-24) [5]. This implies that the homothety factor changes, depending on the spatial location within the tree. On the other hand, if we analyze the radius ratio from generations 1 to 24 in steps of 4, we obtain an average of 0.8535, whereas if we use steps of 2, we obtain an average homothety factor of 0.8623. These changes might not seem significant, but one should recall that they are originated by the symmetric geometry of the respiratory tree. However, when asymmetry is considered, one deals with several homothety factors, i.e. as schematically drawn in figure 1.

We propose to investigate the case of asymmetric branching in the human lungs, i.e. the Horsfield representation will be used [6]. In the asymmetric morphology, the respiratory tree consists of 35 dichotomous generations, whereas an airway of level m bifurcates into two daughters: one of order $m+1$ and one of order $m+1+\Delta$, with Δ the asymmetry index. Figure 3 shows the number of branches that are in one generation, for the symmetric and asymmetric lung structure. Notice the different slope which characterizes the space-filling distribution.

In the asymmetric case, the electrical network of figure 1 cannot be simplified [7], therefore one must calculate explicitly the impedance from level 36 to level 1. To avoid complex numerical formulations, the impedance along the longest path was calculated, as in [8]. From level 26 onward, the asymmetry index is zero, therefore symmetric bifurcation occurs. The effect of this change in the asymmetry index is visible in figure 3, i.e. a change in the slope. The initial values in the trachea are imposed similarly as in the symmetric case [9]. Figure 2 shows the total impedance by means of its Bode plot, for the

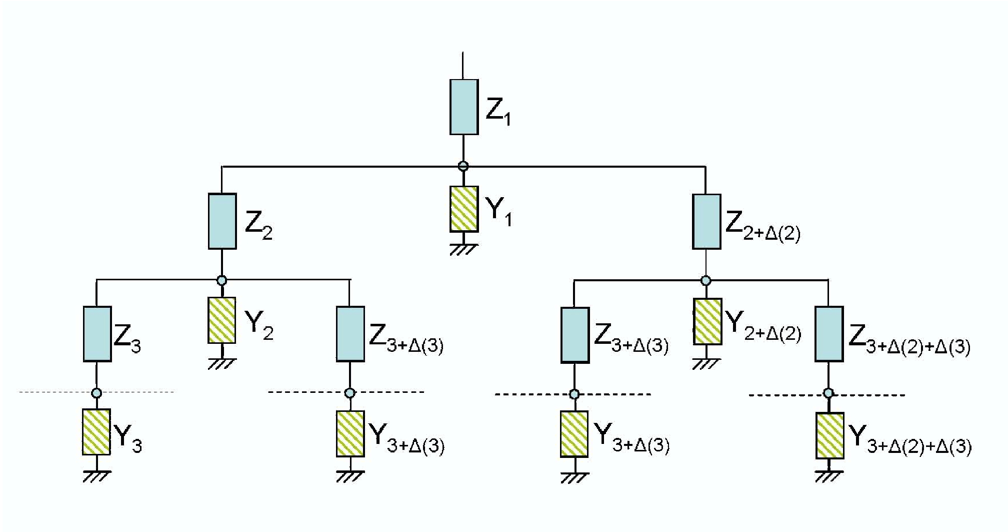

Figure 1: Asymmetric representation for the first four generations, in its electrical equivalent

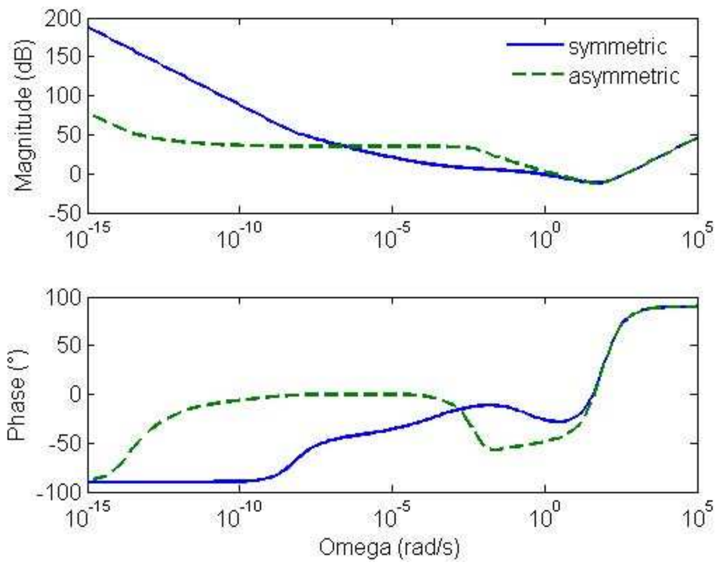

Figure 2: Impedance by means of Bode-plot representation, symmetric (continuous line) and the asymmetric (dashed line) tree.

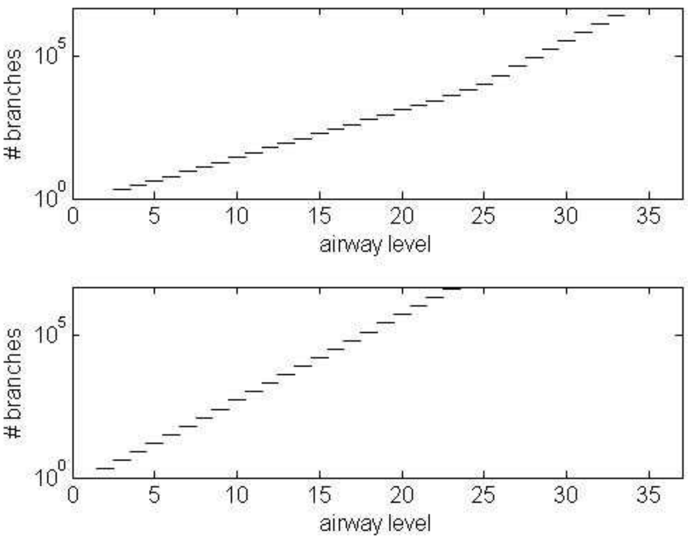

Figure 3: Number of branches for each generation, in the asymmetric (TOP) and symmetric (BOTTOM) generation. Notice that the Y-axis is logarithmic.

symmetric and the asymmetric tree, whereas the airway tubes are modelled by an $R-L-C$ element in both representations.

It is significant to observe that in the frequency interval of clinical interest, $\omega \in [25, 300]$ rad/s, the two impedances tend to behave similarly. For the asymmetric case, we have a decrease of about -10dB/dec and a phase of approximately -50^o, resulting in a fractional order of $n \cong 0.5$. This observation suggests that a combined effect of more than one fractal order is present in the lungs and that it leads naturally to values closer to measured data in the low frequency range. In other words, the symmetric tree representation does not suffice to obtain a good fit between the model and the measured impedance data. Another observation is that the constant-phase behavior is emphasized at frequencies below those evaluated standardly in clinical practice, i.e. below 5Hz. However, in the standard clinical range of frequencies for the forced oscillation technique, namely 4-48Hz, both symmetric and asymmetric tree models give similar results, as depicted in figure 4 and figure 5.

3. Conclusions

In this paper, it is observed that the asymmetric structure of the respiratory

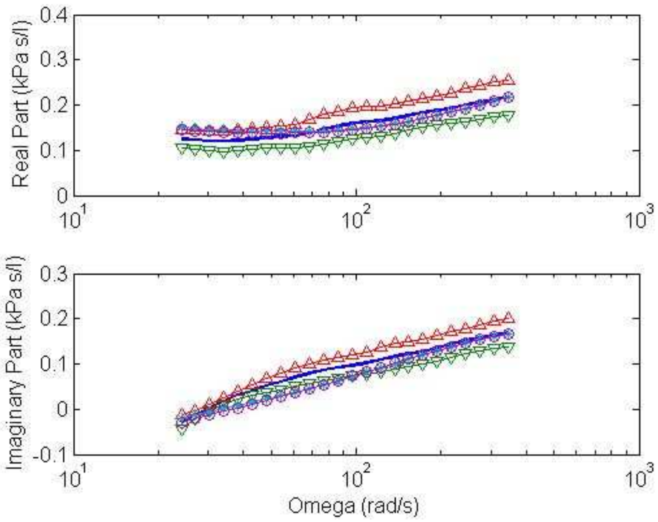

Figure 4: The estimated impedance within the measured frequency range for the symmetric (*) and the asymmetric (o) case against averaged data from healthy subjects

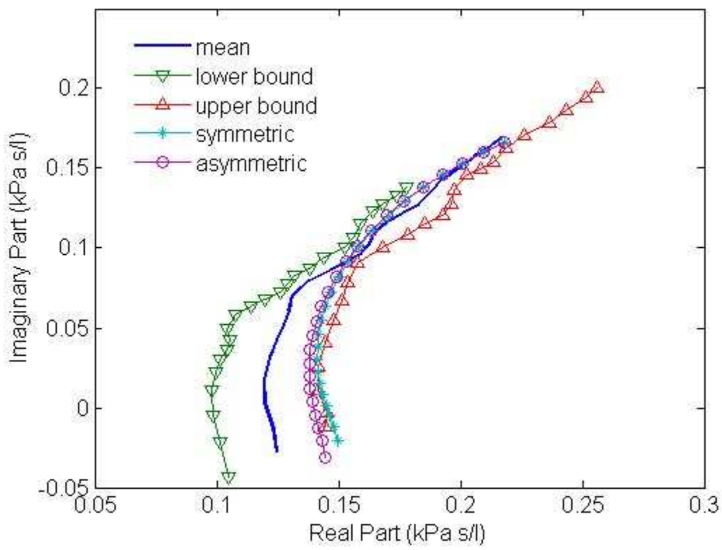

Figure 5: The estimated impedance within the measured frequency range for the symmetric (*) and the asymmetric (o) case, polar plot representation.

tree leads to an impedance exhibiting fractional-order behavior. The results indicate than in a specific frequency range, both the symmetric and asymmetric representation of the respiratory tree lead to similar values in the impedance.

The following remarks can be summarized: i) the fractional order behavior is still present, although the symmetric fractal structure is no more fulfilled; and ii) the fractional order value is changing if the degree of asymmetry is changed (this observation is significant in diseased lungs).

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