Neutrosophic Hedge Algebras

Florentin Smarandache
University of New Mexico
705 Gurley Ave., Gallup, New Mexico 87301, USA
Phone: +1 505-277-0111
fsmarandache@gmail.com

Abstract

We introduce now for the first time the neutrosophic hedge algebras as an extension of classical hedge algebras, together with an application of neutrosophic hedge algebras.

1. Introduction

The classical hedge algebras deal with linguistic variables. In neutrosophic environment we have introduced the neutrosophic linguistic variables. We have defined neutrosophic partial relationships between single-valued neutrosophic numbers. Neutrosophic operations are used in order to aggregate the neutrosophic linguistic values.

2. Materials and Methods

We introduce now, for the first time, the Neutrosophic Hedge Algebras, as extension of classical Hedge Algebras.

Let's consider a Linguistic Variable:

with $Dom(x)$ as the word domain of x, whose each element is a word (label), or string of words.

Let $\mathcal{A}$ be an attribute that describes the value of each element $x \in Dom(x)$, as follows:

$\mathcal{A}: Dom(x) \rightarrow [0,1]^3$. (1)

$\mathcal{A}(x)$ is the neutrosophic value of x with respect to this attribute:

$A(x) = \langle t_x, i_x, f_x \rangle$, (2)

where $t_x, i_x, f_x \in [0,1]$, such that

- t_x means the degree of value of x;
- i_x means the indeterminate degree of value of x;
- f_x means the degree of non-value of x.

We may also use the notation: $x\langle t_x, i_x, f_x \rangle$.

A neutrosophic partial relationship $\leq_N$ on $Dom(x)$, defined as follows:

$x\langle t_x, i_x, f_x \rangle \leq_N y\langle t_y, i_y, f_y \rangle$, (3)

if and only if $t_x \leq t_y$, and $i_x \geq i_y$, $f_x \geq f_y$.

Therefore, $(Dom(x), \leq_N)$ becomes a neutros-ophic partial order set (or neutrosophic poset), and $\leq_N$ is called a neutrosophic inequality.

Let $C = \{0, w, 1\}$ be a set of constants, $C \subset Dom(x)$, where:

- 0 = the least element, or $0_{\langle 0,1,1 \rangle}$;
- w = the neutral (middle) element, or $w_{\langle 0.5,0.5,0.5 \rangle}$;
- and 1 = the greatest element, or $1_{\langle 1,0,0 \rangle}$.

Let G be a word-set of two *neutrosophic generators*, $G \subset Dom(x)$, qualitatively a negative primary neutrosophic term (denoted g^-), and the other one that is qualitatively a positive primary neutrosophic term (denoted g^+), such that:

$0 \leq_N g^- \leq_N w \leq_N g^+ \leq_N 1,$ (4)

or transcribed using the neutrosophic com-ponents:

$0_{\langle 0,1,1\rangle} \leq_N {g^-}_{\langle t_{g^-}, i_{g^-}, f_{g^-}\rangle} \leq_N w_{\langle 0.5,0.5,0.5\rangle}$

$\leq_N {g^+}_{\langle t_{g^+}, i_{g^+}, f_{g^+}\rangle} \leq_N 1_{\langle 1,0,0\rangle},$

where

- $0 \leq t_{g^-} \leq 0.5 \leq t_{g^+} \leq 1$ (here there are classical inequalities)
- $1 \geq i_{g^-} \geq 0.5 \geq i_{g^+} \geq 0$, and
- $1 \geq f_{g^-} \geq 0.5 \geq f_{g^+} \geq 0$.

Let $H \subset Dom(x)$ be the set of *neutrosophic hedges*, regarded as unary operations. Each hedge h∈H is a functor, or comparative particle for adjectives and adverbs as in the natural language (English).

h:Dom(x)→Dom(x)

x→h(x). (5)

Instead of h(x) one easily writes hx to be closer to the natural language.

By associating the neutrosophic components, one has:

h_⟨t_h,i_h,f_h ⟩ x_⟨t_x,i_x,f_x ⟩ .

A hedge applied to x may increase, decrease, or approximate the neutrosophic value of the element x.

There also exists a neutrosophic identity I∈Dom(x), denoted I_⟨0,0,0⟩ that does not hange on the elements:

I_⟨0,0,0⟩ x_⟨t_x,i_x,f_x ⟩ .

In most cases, if a hedge increases / decreases the neutrosophic value of an element x situated above the neutral element w, the same hedge does the opposite, decreases / increases the neutrosophic value of an element y situated below the neutral element w.

And reciprocally.

If a hedge approximates the neutrosophic value, by diminishing it, of an element x situated above the neutral element w, then it approximates the neutrosophic value, by enlarging it, of an element y situated below the neutral element w.

Let's refer the hedges with respect to the upper part (⊔), above the neutral element, since for the lower part (L) it will automatically be the opposite effect.

We split de set of hedges into three disjoint subsets:

H_⊔^+ = the hedges that increase the neutrosophic value of the upper elements;

H_⊔^- = the hedges that decrease the neutrosophic value of the upper elements;

H_⊔^∼ = the hedges that approximate the neutrosophic value of the upper elements.

Notations: Let $x = x_\sqcup \cup w \cup x_L$, where $x_\sqcup$ cons-titutes the upper element set, while x_L the lower element subset, w the neutral element. $x_\sqcup$ and x_L are disjoint two by two.

3. Operations on Neutrosophic Components

Let $\langle t_1, i_1, f_1\rangle$, $\langle t_2, i_2, f_2\rangle$ neutrosophic numbers.

Then:

$$t_1 + t_2 = \begin{cases} t_1 + t_2, \text{if } t_1 + t_2 \leq 1; \\ 1, \text{if } t_1 + t_2 > 1; \end{cases} \quad (6)$$

and

$$t_1 - t_2 = \begin{cases} 0, \text{if } t_1 - t_2 < 0; \\ t_1 - t_2, \text{if } t_1 - t_2 \geq 0. \end{cases} \tag{7}$$

Similarly for i_1 and f_1:

$$i_1 + i_2 = \begin{cases} i_1 + i_2, \text{if } i_1 + i_2 \leq 1; \\ 1, \text{if } i_1 + i_2 > 1; \end{cases} \tag{8}$$

$$i_1 - i_2 = \begin{cases} 0, \text{if } i_1 - i_2 < 0; \\ i_1 - i_2, \text{if } i_1 - i_2 \geq 0. \end{cases} \tag{9}$$

and

$$f_1 + f_2 = \begin{cases} f_1 + f_2, \text{if } f_1 + f_2 \leq 1; \\ 1, \text{if } f_1 + f_2 > 1; \end{cases} \tag{10}$$

$$f_1 - f_2 = \begin{cases} 0, \text{if } f_1 - f_2 < 0; \\ f_1 - f_2, \text{if } f_1 - f_2 \geq 0. \end{cases} \tag{11}$$

4. Neutrosophic Hedge-Element Operators

We define the following operators:

4.1. Neutrosophic Increment

Hedge $\boxed{\uparrow}$ Element = $\langle t_1, i_1, f_1 \rangle \boxed{\uparrow} \langle t_2, i_2, f_2 \rangle = \langle t_2 + t_1, i_2 - i_1, f_2 - f_1 \rangle$, (12)

meaning that the first triplet increases the second.

4.2. Neutrosophic Decrement

Hedge $\boxed{\downarrow}$ Element = $\langle t_1, i_1, f_1 \rangle \boxed{\downarrow} \langle t_2, i_2, f_2 \rangle = \langle t_2 - t_1, i_2 + i_1, f_2 + f_1 \rangle$, (13)

meaning that the first triplet decreases the second.

4.3. Theorem 1

The neutrosophic increment and decrement operators are non-commutattive.

5. Neutrosophic Hedge-Hedge Operators

Hedge $\boxed{\uparrow}$ Hedge = $\langle t_1, i_1, f_1 \rangle \boxed{\uparrow} \langle t_2, i_2, f_2 \rangle = \langle t_1 + t_2, i_1 + i_2, f_1 + f_2 \rangle$ (14)

Hedge $\boxed{\downarrow}$ Hedge = $\langle t_1, i_1, f_1 \rangle \boxed{\downarrow} \langle t_2, i_2, f_2 \rangle = \langle t_1 - t_2, i_1 - i_2, f_1 - f_2 \rangle$ (15)

6. Neutrosophic Hedge Operators

Let $x_\sqcup \langle t_{x_\sqcup}, i_{x_\sqcup}, f_{x_\sqcup} \rangle \in Dom(x)$ i.e. $x_\sqcup$ is an upper element of $Dom(x)$, and

– $h_\sqcup^+ \langle t_{h_\sqcup^+}, i_{h_\sqcup^+}, f_{h_\sqcup^+} \rangle \in H_\sqcup^+$,

– $h_\sqcup^- \langle t_{h_\sqcup^-}, i_{h_\sqcup^-}, f_{h_\sqcup^-} \rangle \in H_\sqcup^-$,

– $h_\sqcup^{\sim} \langle t_{h_\sqcup^{\sim}}, i_{h_\sqcup^{\sim}}, f_{h_\sqcup^{\sim}} \rangle \in H_\sqcup^{\sim}$,

then $h_\sqcup^+$ applied to $x_\sqcup$ gives

$(h_\sqcup^+ x_\sqcup) \langle t_{x_\sqcup}, i_{x_\sqcup}, f_{x_\sqcup} \rangle \boxed{\uparrow} \langle t_{h_\sqcup^+}, i_{h_\sqcup^+}, f_{h_\sqcup^+} \rangle$,

and $h_\sqcup^-$ applied to $x_\sqcup$ gives

$(h_\sqcup^- x_\sqcup) \langle t_{x_\sqcup}, i_{x_\sqcup}, f_{x_\sqcup} \rangle \boxed{\downarrow} \langle t_{h_\sqcup^-}, i_{h_\sqcup^-}, f_{h_\sqcup^-} \rangle$,

and $h_\sqcup^{\sim}$ applied to $x_\sqcup$ gives

$(h_\sqcup^{\sim} x_\sqcup) \langle t_{x_\sqcup}, i_{x_\sqcup}, f_{x_\sqcup} \rangle \boxed{\downarrow} \langle t_{h_\sqcup^{\sim}}, i_{h_\sqcup^{\sim}}, f_{h_\sqcup^{\sim}} \rangle$.

Now, let $x_L\langle t_{x_L}, i_{x_L}, f_{x_L}\rangle \in Dom(x_L)$, i.e. x_L is a lower element of $Dom(x)$. Then, $h_{\sqcup}^{+}$ applied to x_L gives:

$h_{\sqcup}^{+}x_L\langle t_{x_L}, i_{x_L}, f_{x_L}\rangle \boxed{\downarrow} \langle t_{h_{\sqcup}^{+}}, i_{h_{\sqcup}^{+}}, f_{h_{\sqcup}^{+}}\rangle$,

and $h_{\sqcup}^{-}$ applied to x_L gives:

$h_{\sqcup}^{-}x_L\langle t_{x_L}, i_{x_L}, f_{x_L}\rangle \boxed{\uparrow} \langle t_{h_{\sqcup}^{-}}, i_{h_{\sqcup}^{-}}, f_{h_{\sqcup}^{-}}\rangle$,

and $h_{\sqcup}^{\frown}$ applied to x_L gives:

$h_{\sqcup}^{\frown}x_L\langle t_{x_L}, i_{x_L}, f_{x_L}\rangle \boxed{\uparrow} \langle t_{h_{\sqcup}^{\frown}}, i_{h_{\sqcup}^{\frown}}, f_{h_{\sqcup}^{\frown}}\rangle$.

In the same way, we may apply many increasing, decreasing, approximate or other type of hedges to the same upper or lower element

$h_{\sqcup_n}^{+}h_{\sqcup_{n-1}}^{-}h_{\sqcup}^{v} \dots h_{\sqcup_1}^{+}x$,

generating new elements in $Dom(x)$.

The hedges may be applied to the constants as well.

6.1. Theorem 2

A hedge applied to another hedge wekeans or stengthens or approximates it.

6.2. Theorem 3

If $h_{\sqcup}^{+} \in H_{\sqcup}^{+}$ and $x_{\sqcup} \in Dom(x_{\sqcup})$, then $h_{\sqcup}^{+}x_{\sqcup} \geq x_{\sqcup}$.
If $h_{\sqcup}^{-} \in H_{\sqcup}^{-}$ and $x_{\sqcup} \in Dom(x_{\sqcup})$, then $h_{\sqcup}^{-}x_{\sqcup} \geq x_{\sqcup}$.
If $h_{\sqcup}^{+} \in H_{\sqcup}^{+}$ and $x_L \in Dom(x_L)$, then $h_{\sqcup}^{+}x_L \leq_N x_L$.
If $h_{\sqcup}^{-} \in H_{\sqcup}^{-}$ and $x_L \in Dom(x_L)$, then $h_{\sqcup}^{-}x_L \geq_N x_L$.

6.3. Converse Hedges

Two hedges h_1 and $h_2 \in H$ are converse to each other, if $\forall x \in Dom(x)$, $h_1x \leq_N x$ is equivalent to $h_2x \geq_N x$.

6.4. Compatible Hedges

Two hedges h_1 and $h_2 \in H$ are compatible, if $\forall x \in Dom(x)$, $h_1x \leq_N x$ is equivalent to $h_2x \leq_N x$.

6.5. Commutative Hedges

Two hedges h_1 and $h_2 \in H$ are commutative, if $\forall x \in Dom(x)$, $h_1h_2x = h_2h_1x$. Otherwise they are called non-commutative.

6.6. Cumulative Hedges

If $h_{1_{\sqcup}}^{+}$ and $h_{2_{\sqcup}}^{+} \in H^{+}$, then two neutrosophic edges can be cumulated into one:

$h_{1_{\sqcup}}^{+}\langle t_{h_{1_{\sqcup}}^{+}}, i_{h_{1_{\sqcup}}^{+}}, f_{h_{1_{\sqcup}}^{+}}\rangle h_{2_{\sqcup}}^{+}\langle t_{h_{2_{\sqcup}}^{+}}, i_{h_{2_{\sqcup}}^{+}}, f_{h_{2_{\sqcup}}^{+}}\rangle = h_{12_{\sqcup}}^{+}\langle t_{h_{1_{\sqcup}}^{+}}, i_{h_{1_{\sqcup}}^{+}}, f_{h_{1_{\sqcup}}^{+}}\rangle \boxed{\uparrow} \langle t_{h_{2_{\sqcup}}^{+}}, i_{h_{2_{\sqcup}}^{+}}, f_{h_{2_{\sqcup}}^{+}}\rangle$. (16)

Similarly, if $h_{1_{\sqcup}}^{-}$ and $h_{2_{\sqcup}}^{-} \in H^{-}$, then we can cumulate them into one:

$h_{1_{\sqcup}}^{-}\langle t_{h_{1_{\sqcup}}^{-}}, i_{h_{1_{\sqcup}}^{-}}, f_{h_{1_{\sqcup}}^{-}}\rangle h_{2_{\sqcup}}^{-}\langle t_{h_{2_{\sqcup}}^{-}}, i_{h_{2_{\sqcup}}^{-}}, f_{h_{2_{\sqcup}}^{-}}\rangle = h_{12_{\sqcup}}^{-}\langle t_{h_{1_{\sqcup}}^{-}}, i_{h_{1_{\sqcup}}^{-}}, f_{h_{1_{\sqcup}}^{-}}\rangle \boxed{\uparrow} \langle t_{h_{2_{\sqcup}}^{-}}, i_{h_{2_{\sqcup}}^{-}}, f_{h_{2_{\sqcup}}^{-}}\rangle$. (17)

Now, if the two hedges are converse, $h^+_{1_\sqcup}$ and $h^-_{1_\sqcup}$, but the neutrosophic components of the first (which is actually a neutrosophic number) are greater than the second, we cumulate them into one as follows:

$$h^+_{3_\sqcup} = \left(h^+_{1_\sqcup} h^-_{2_\sqcup}\right) \langle t_{h^+_{1_\sqcup}}, i_{h^+_{1_\sqcup}}, f_{h^+_{1_\sqcup}} \rangle \boxed{\downarrow} \langle t_{h^-_{2_\sqcup}}, i_{h^-_{2_\sqcup}}, f_{h^-_{2_\sqcup}} \rangle. \qquad (18)$$

But, if the neutrosophic components of the second are greater, and the hedges are commutative, we cumulate them into one as follows:

$$h^+_{3_\sqcup} = \left(h^+_{1_\sqcup} h^-_{2_\sqcup}\right) \langle t_{h^-_{2_\sqcup}}, i_{h^-_{2_\sqcup}}, f_{h^-_{2_\sqcup}} \rangle \boxed{\downarrow} \langle t_{h^+_{1_\sqcup}}, i_{h^+_{1_\sqcup}}, f_{h^+_{1_\sqcup}} \rangle \qquad (19)$$

7. Neutrosophic Hedge Algebra

$NHA = (x, G, C, H \cup I, \leq_N)$ constitutes an abstract algebra, called Neutrosophic Hedge Algebra.

7.1. Example of a Neutrosophic Hedge Algebra τ

Let $G = \{Small, Big\}$ the set of generators, repres-ented as neutrosophic generators as follows:

$Small_{\langle 0.3,0.6,0.7\rangle}, Big_{\langle 0.7,0.2,0.3\rangle}$.

Let $H = \{Very, Less\}$ the set of hedges, repres-ented as neutrosophic hedges as follows:

$Very_{\langle 0.1,0.1,0.1\rangle}, Less_{\langle 0.1,0.2,0.3\rangle}$,

where $Very \in H^+_\sqcup$ and $Less \in H^-_\sqcup$.

x is a neutrosophic linguistic variable whose domain is G at the beginning, but extended by generators.

The neutrosophic constants are

$C = \left\{0_{\langle 0,1,1\rangle}, Medium_{\langle 0.5,0.5,0.5\rangle}, 1_{\langle 1,0,0\rangle}\right\}$.

The neutrosophic identity is $I_{\langle 0,0,0\rangle}$.

We use the neutrosophic inequality $\leq_N$, and the neutrosophic increment / decrement operators previously defined.

Let's apply the neutrosophic hedges in order to generate new neutrosophic elements of the neutrosophic linguistic variable x.

$Very$ applied to Big [upper element] has a positive effect:

$Very_{\langle 0.1,0.1,0.1\rangle} Big_{\langle 0.7,0.2,0.3\rangle} = (Very\ Big)_{\langle 0.7+0.1,0.2-0.1,0.3-0.1\rangle} = (Very\ Big)_{\langle 0.8,0.1,0.2\rangle}$.

Then:

$Very_{\langle 0.1,0.1,0.1\rangle}(Very\ Big)_{\langle 0.9,0.1,0.2\rangle} = (Very\ Very\ Big)_{\langle 0.9,0,0.1\rangle}$.

$Very$ applied to $Small$ [lower element] has a negative effect:

$Very_{\langle 0.1,0.1,0.1\rangle} Small_{\langle 0.3,0.6,0.7\rangle} = (Very\ Small)_{\langle 0.3-0.1,0.6+0.1,0.7+0.1\rangle} = (Very\ Small)_{\langle 0.2,0.7,0.8\rangle}$.

If we compute $(Very\ Very)$ first, which is a neutrosophic hedge-hedge operator:

$Very_{\langle 0.1,0.1,0.1\rangle} Very_{\langle 0.1,0.1,0.1\rangle} = (Very\ Very)_{\langle 0.1+0.1,0.1+0.1,0.1+0.1\rangle} = (Very\ Very)_{\langle 0.2,0.2,0.2\rangle}$,

and we apply it to Big, we get:

$$\begin{aligned}(Very\ Very)_{\langle 0.2,0.2,0.2\rangle} Big_{\langle 0.7,0.2,0.3\rangle} &= (Very\ Very\ Big)_{\langle 0.7+0.2,0.2-0.2,0.3-0.2\rangle}\\ &= (Very\ Very\ Big)_{\langle 0.9,0,0.1\rangle},\end{aligned}$$

so, we get the same result.

$Less$ applied to Big has a negative effect:

$Less_{\langle 0.1,0.2,0.3\rangle}Big_{\langle 0.7,0.2,0.3\rangle} = (Less\ Big)_{\langle 0.7-0.1,0.2+0.2,0.3\rangle} = (Less\ Big)_{\langle 0.6,0.4,0.6\rangle}$.

$Less$ applied to $Small$ has a positive effect:

$Less_{\langle 0.1,0.2,0.3\rangle}Small_{\langle 0.3,0.6,0.7\rangle} = (Less\ Small)_{\langle 0.1+0.3,0.6-0.2,0.7-0.3\rangle} = (Less\ Small)_{\langle 0.4,0.4,0.4\rangle}$.

The set of neutrosophic hedges H is enriched through the generation of new neutrosophic hedges by combining a hedge with another one using the neutrosophic hedge-hedge operators.

Further, the newly generated neutrosophic hedges are applied to the elements of the linguistic variable, and more new elements are generated.

Let's compute more neutrosophic elements:

$$VLB = Very_{\langle 0.1,0.1,0.1\rangle}Less_{\langle 0.1,0.2,0.3\rangle}Big_{\langle 0.7,0.2,0.3\rangle}$$
$$= (Very\ Less\ Big)_{\left[\langle 0.1,0.1,0.1\rangle \underset{h}{\boxed{\uparrow}} \langle 0.1,0.2,0.3\rangle\right]\boxed{\downarrow}\langle 0.7,0.2,0.3\rangle}$$
$$= (Very\ Less\ Big)_{\langle 0.1+0.1,0.1+0.2,0.1+0.3\rangle\boxed{\downarrow}\langle 0.7,0.2,0.3\rangle}$$
$$= (Very\ Less\ Big)_{\langle 0.7-0.2,0.2-0.3,0.3-0.4\rangle} = (Very\ Less\ Big)_{\langle 0.5,0,0\rangle}$$

$$VM = Very_{\langle 0.1,0.1,0.1\rangle}Medium_{\langle 0.5,0.5,0.5\rangle} = (Very\ Medium)_{\langle 0.1,0.1,0.1\rangle\boxed{\uparrow}\langle 0.5,0.5,0.5\rangle}$$
$$= (Very\ Medium)_{\langle 0.6,0.4,0.4\rangle}$$

$$LM = Less_{\langle 0.1,0.2,0.3\rangle}Medium_{\langle 0.5,0.5,0.5\rangle} = (Less\ Medium)_{\langle 0.1,0.2,0.3\rangle\boxed{\downarrow}\langle 0.5,0.5,0.5\rangle}$$
$$= (Less\ Medium)_{\langle 0.4,0.7,0.8\rangle}$$

$$VVS = Very_{\langle 0.1,0.1,0.1\rangle}Very_{\langle 0.1,0.1,0.1\rangle}Small_{\langle 0.3,0.6,0.7\rangle} = (Very\ Very)_{\langle 0.2,0.2,0.2\rangle}Small_{\langle 0.3,0.6,0.7\rangle}$$
$$= (Very\ Very\ Small)_{\langle 0.1,0.8,0.9\rangle}$$

$$VLS = Very_{\langle 0.1,0.1,0.1\rangle}Less_{\langle 0.1,0.2,0.3\rangle}Small_{\langle 0.3,0.6,0.7\rangle} = Very_{\langle 0.1,0.1,0.1\rangle}(Less\ Small)_{\langle 0.4,0.4,0.4\rangle}$$
$$= (Very\ Less\ Small)_{\langle 0.5,0.3,0.3\rangle}$$

$$LAMax = Less_{\langle 0.1,0.2,0.3\rangle}Absolute\ Maximum_{\langle 1,0,0\rangle}$$
$$= (Less\ Absolute\ Maximum)_{\langle 0.1,0.2,0.3\rangle\boxed{\downarrow}\langle 1,0,0\rangle}$$
$$= (Less\ Absolute\ Maximum)_{\langle 0.9,0.2,0.3\rangle}$$

$$LAMin = Less_{\langle 0.1,0.2,0.3\rangle}Absolute\ Minimum_{\langle 0,1,1\rangle} = (Less\ Absolute\ Minimum)_{\langle 0.1,0.2,0.3\rangle\boxed{\uparrow}\langle 0,1,1\rangle}$$
$$= (Less\ Absolute\ Maximum)_{\langle 0.1,0.8,0.7\rangle}$$

7.2. Theorem 4

Any increasing hedge $h_{\langle t,i,f\rangle}$ applied to the absolute maximum cannot overpass the absolute maximum.

Proof:

$$h_{\langle t,i,f\rangle}\ \boxed{\uparrow}\ 1_{\langle 1,0,0\rangle} = (h1)_{\langle 1+t,0-i,0-f\rangle}$$
$$= (h1)_{\langle 1,0,0\rangle} = 1_{\langle 1,0,0\rangle}.$$

7.3. Theorem 5

Any decreasing hedge $h_{\langle t,i,f\rangle}$ applied to the absolute minimum cannot pass below the absolute minimum.

Proof:

$$h_{\langle t,i,f\rangle}\ \boxed{\downarrow}\ 0_{\langle 0,1,1\rangle} = (ho)_{\langle 0-t,1+i,1+f\rangle}$$
$$= (ho)_{\langle 0,1,1\rangle} = 0_{\langle 0,1,1\rangle}.$$

8. Diagram of the Neutrosophic Hedge Algebra τ

$1_{\langle 1,0,0\rangle}$ ABSOLUTE MAXIMUM

$VVB_{\langle 0.9,0,0.1\rangle}$ Very Very Big

$LAM_{\langle 0.9,0.2,0.3\rangle}$ Less Absolute Maximum
$VB_{\langle 0.8,0.1,0.2\rangle}$ Very Big

$Big_{\langle 0.7,0.2,0.3\rangle}$
$VM_{\langle 0.6,0.4,0.4\rangle}$ Very Medium
$LV_{\langle 0.5,0.4,0.6\rangle}$ Less Big

$VLB_{\langle 0.5,0,0\rangle}$ Very Less Big
$VLS_{\langle 0.5,0.3,0.3\rangle}$ Very Less Small
$M_{\langle 0.5,0.5,0.5\rangle}$ MEDIUM
$LM_{\langle 0.4,0.7,0.8\rangle}$ Less Medium

$LS_{\langle 0.4,0.4,0.4\rangle}$ Less Small
$Small_{\langle 0.3,0.6,0.7\rangle}$
$VS_{\langle 0.2,0.7,0.8\rangle}$ Very Small
$LAMin_{\langle 0.1,0.8,0.7\rangle}$ Less Absolute Minimum
$VVS_{\langle 0.1,0.8,0.9\rangle}$ Very Very Small

$0_{\langle 0,1,1\rangle}$ ABSOLUTE MINIMUM

9. Conclusions

In this paper, the classical hedge algebras have been extended for the first time to neutrosophic hedge algebras. With respect to an attribute, we have inserted the neutrosophic degrees of membership / indeterminacy / nonmembership of each generator, hedge, and constant. More than in the classical hedge algebras, we have introduced several numerical hedge operators: for hedge applied to element, and for hedge combined with hedge. An extensive example of a neutrosophic hedge algebra is given, and important properties related to it are presented.

References

Cat Ho, N.; Wechler, W. *Hedge Algebras: An algebraic Approach to Structure of Sets of Linguistic Truth Values*. Fuzzy Sets and Systems 1990, 281-293.

Lakoff, G. *Hedges, a study in meaning criteria and the logic of fuzzy concepts*. 8th Regional Meeting of the Chicago Linguistic Society, 1972.

Zadeh, L.A. *A fuzzy-set theoretic interpretation of linguistic hedges*. Journal of Cybernetics 1972, Volume 2, 04-34.