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# P-graph Model of Economic Networks with Partial Substitution

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P-graph models of economic networks based on the Leontief input-output framework have been demonstrated in previous work. However, such models are subject to the limiting assumption of fixed technical coefficients that reflect the average state of technology currently in use. This limitation restricts the use of such models to incremental changes; more radical shifts in economic structure cannot be adequately represented. In this paper, a modified P-graph model of economic networks that allows partial substitution of inputs is developed. This modified formulation can represent major structural shifts, such as deep decarbonization or increased circularity, that are needed for improving the sustainability of modern economies. The new model is illustrated using a hypothetical case study. Results show that economic systems which allow for partial substitution are more resilient when disruptions occur in certain sectors of the economy.

## 1. Introduction

Input-output (I-O) modelling was developed by Leontief (1936) as an approach to represent economic networks using systems of linear equations. The I-O model is now widely used as part of standard economic statistics and records in most countries. It provides a powerful framework for understanding economic interdependencies that result from the aggregation of supply chain linkages among sectors. In the I-O model, the flow of economic goods is normally reported in monetary units, although in principle it is also possible to use physical units (Wachs and Singh, 2018) or a combination of both (Merciai, 2019). A comprehensive treatment of the I-O model can be found in a reference book by Miller and Blair (2009). Environmentally extended I-O have also been used to address different sustainability issues. They provide quantitative basis in the areas of Industrial Ecology (Duchin, 1992) and Circular Economy (Aguilar-Hernandez et al., 2018). Multi-region I-O (MRIO) models can be used to trace the virtual flows of environmental footprints that are embedded in trade (Liu et al., 2017). This approach has recently been applied to trade in the European Union and China (Wang et al., 2020) and the Asia-Pacific region (Yang et al., 2020). Optimization models based on an I-O formulation have been developed to explore decarbonization options in countries such as the Philippines (Cayamanda et al., 2017) and for determining the effect of economic structural changes on carbon emission in China (Su et al., 2021), or to determine maximum economic welfare within planetary boundaries (Heijungs et al., 2014).

The I-O model has also been combined in previous work with Process Integration (PI) and Process Systems Engineering (PSE) tools. For example, Mathematical Programming (MP) based on I-O models is well established, and is described in books (Miller and Blair, 2009) and encyclopaedias (Tan et al., 2017). Other than MP, Tan et al. (2018) developed a hybrid Carbon Emissions Pinch Analysis (CEPA) and I-O model for country-level carbon management problems. Aviso et al. (2015) implemented economic I-O networks into the P-graph framework and applied it to the problem of mitigating economic losses following a climate-change induced perturbation. The latter model is the basis for the development reported in this paper. However, one key limitation of the conventional I-O model is that it relies on the assumption of a fixed set of inputs for any given sector. This assumption prevents it from accurately reflecting the flexibility inherent in real economic sectors and limits the capability of identifying alternative system structures. In practice, some degree of substitution is usually possible among the inputs into a given economic sector. This flexibility can be described by a production function that describes the rate of substitution. The inability to capture the effects of partial substitution introduces errors into projections made using the standard form of the I-O model.

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This work extends the previous work of Aviso et al. (2015) by allowing for partial substitution of inputs into economic sectors in the hybrid I-O/P-graph model. The rate of substitution is defined by a production function and is implemented using the approach recently developed by Éles et al. (2021). The rest of this paper is organized as follows. Section 2 gives the formal problem statement. Section 3 discusses the P-graph implementation of the model. In Section 4, an illustrative three-sector case is solved, using both fixed and flexible input scenarios. Section 5 then states the conclusions and prospects for future research.

## 2. Problem Statement

The optimization problem may be formally stated as follows:

- Given an economic system with m number of sectors;
- Given that the technical coefficient matrix allows for partial substitution in the inputs of some sectors;
- Given that a disruption occurs in at least one of the sectors;

The problem is to determine the optimal transaction matrix in the economic system which maximizes total GDP given a disruption.

## 3. P-graph Model Development

The P-graph framework was originally developed as a graph theoretic approach for solving Process Network Synthesis (PNS) problems in plant design. In the P-graph framework, a set of axioms (Friedler et al., 1992a) are the basis for the rigorous development of combinatorial algorithms for PNS (Friedler et al., 1992b). The component algorithms of P-graph are Maximal Structure Generation (MSG), Solution Structure Generation (SSG), and Accelerated Branch-and-Bound (ABB). The MSG algorithm (Friedler et al., 1993) is used to rigorously assemble an error-free, non-redundant superstructure given a set of individually specified component process units. The SSG algorithm is used to enumerate combinatorially or structurally feasible networks to allow for a comprehensive evaluation by the decision-maker (Friedler et al., 1992b). These two algorithms require only structural information as inputs. Once economic and flow data are given, the ABB algorithm can be used to determine optimal and near-optimal designs (Friedler et al., 1996). These algorithms can be implemented in different software, such as P-graph Studio (P-graph, 2021) or Visual Basic for Applications (Lao et al., 2020). Applications of P-graph for the analysis and planning of sustainable systems are reviewed in the paper of Cabezas et al. (2018). A prospective paper by Friedler et al. (2019) discusses important directions for future research. The potential of P-graph methodology for addressing applications that are analogous to conventional PNS problems is the basis for possible diversification of its use.

Aviso et al. (2015) showed that economic I-O models can be implemented in P-graph. In this approach, each economic sector is analogous to a process unit. The fundamental differences are that the I-O ratios in an economic sector are high level (i.e., low resolution), are given in monetary units, and are derived empirically from economic statistics. By comparison, the I-O ratios in conventional PNS problems are process-level (i.e., high resolution), are given in physical units, and are based on engineering knowledge. Despite these differences, the same computational principles apply to both types of systems. In that work, P-graph was used to determine the optimal allocation of production capacity that minimizes economic losses in an economic network following a perturbation. Note that the problem is analogous to the process engineering problem of optimizing plant operations under abnormal conditions (Tan et al., 2014). The main limitation of these previous works are the assumption of fixed I-O ratios for the economic sectors or process units.

Éles et al. (2021) recently extended the standard P-graph model by relaxing the assumption of fixed I-O ratios for the process units. Their approach is used here to allow flexible substitution of inputs into economic sectors. Such flexibility exists in many real systems. For example, if a country or region that has excess installed power generation capacity, the actual mix of fuel inputs needed per unit of electricity generated by the sector can vary, depending on the output of the different power plants at any given time. In the P-graph framework, an economic sector with fixed I-O ratios is represented as shown in Figure 1a where the blue bars represent the economic sectors, the green nodes represent the output of each economic sector and the black arcs represent the flow of transactions between sectors. For simplicity, each sector's transaction with itself is not shown. A sector with flexible I-O ratios allowing partial substitution for Sector 3 (S3) is represented as shown in Figure 1b. The red rectangular bars are added to allow flexibility in the input streams of S3 through the node F03. By representing it in this way, the amount of Y1 or Y2 enters S3 in any of two pathways: 1) directly as represented by the flow from the green nodes to S3 and 2) via F03 through the red rectangular bars. This indicates that the input streams from Y1, Y2 and Y3 are in fixed ratios; partial substitution and input flexibility is introduced by the input from F03 which can be a mix of Y1, Y2 and Y3 in varying proportions.

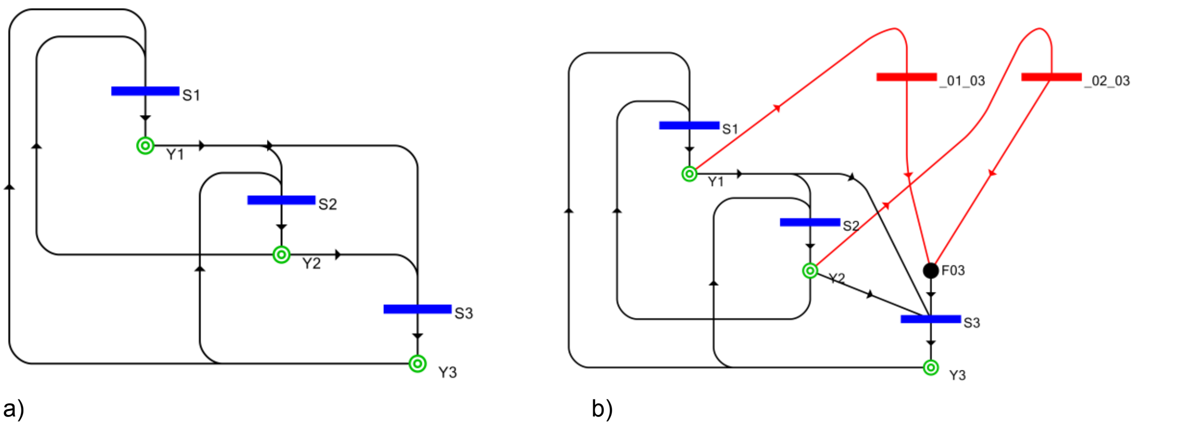



*Figure 1: P-graph representation of an economic system with (a) fixed I-O ratios and (b) flexible I-O ratios for partial substitution*

## 4. Case Study

This case study uses the hypothetical transaction matrix shown in Table 1 where the entries, $Z_{ij}$, represent the transaction that occurs between two sectors interpreted as the number of units needed from sector i by sector j, x represents the total size of the economic sector and y represents the net output of each sector. The second row shows that the total output of Sector 1 (S1) is $x_1$ = 1,000 units, this is allocated to other sectors where 100 units is used as input to S1, 500 units are inputs for S2, 100 units are inputs to S3, leaving $y_1$ = 300 units as its net output. The same interpretation can be done for S2 and S3. Using Eq(1), Table 1 is translated to the technical coefficient matrix, with elements $a_{ij}$, representing the fixed input-output ratios between sectors as shown in Table 2. However, if partial substitution is possible then some degree of flexibility in the coefficients is expected. In this case study it is assumed that partial substitution between the outputs of S1 and S2 can be considered as input to S3. This flexibility is reflected in the technical coefficient matrix shown in Table 3. Here it is assumed that S3 will need a total of 0.4 units of input to manufacture 1.0 units of output, like the requirements in Table 2. However, instead of having completely fixed ratios between the contributions of S1 and S2, it is assumed that a minimum input of 0.1 units is obtained from both S1 and S2 while the remaining 0.2 units can be supplied from a mixture of S1 and S2 at any proportion. The equivalent P-graph representation for the fixed and flexible ratios are shown in Figure 2. It is important to note that when internal consumption occurs within a sector, the net output should be used when generating its P-graph representation. Sector 1 (S1) for example uses 0.1 units of its own output to generate 1.0 units of $x_1$ (see Table 2). The net output of S1 is therefore 0.9 if we account for this internal consumption (1.0 – 0.1 = 0.9). Similarly, the net output of S2 is 0.6 (1.0 – 0.4 = 0.6). This is reflected by the streams coming out of S1 and S2 as shown in Figure 2.

*Table 1: Transactions matrix for case study* ($Z_{ij}$)

| | S1 | S2 | S3 | x | y |
|---|---|---|---|---|---|
| S1 | 100 | 500 | 100 | 1,000 | 300 |
| S2 | 200 | 400 | 100 | 1,000 | 300 |
| S3 | 0 | 100 | 0 | 500 | 400 |

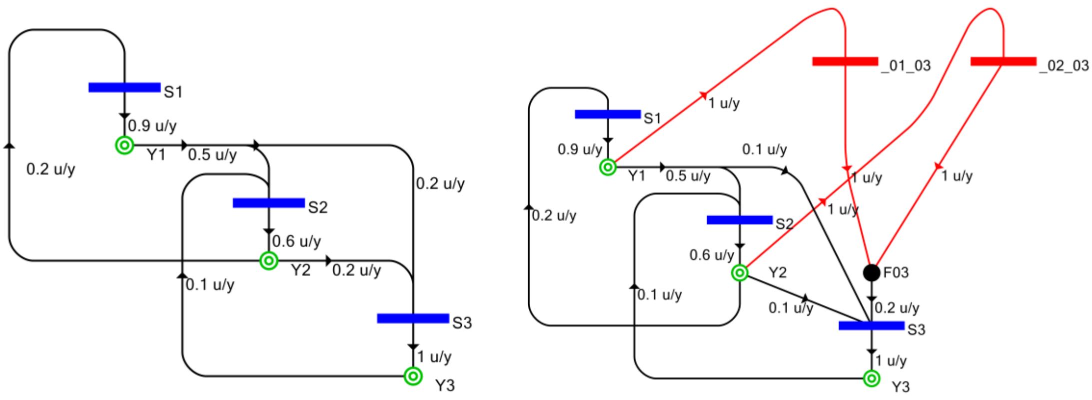
$$a_{ij} = \frac{Z_{ij}}{x_j} \qquad (1)$$

*Table 2: Technical coefficient matrix for case study*

| | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0.1 | 0.5 | 0.2 |
| S2 | 0.2 | 0.4 | 0.2 |
| S3 | 0 | 0.1 | 0 |

*Table 3: Technical coefficient matrix with partial substitution*

| | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0.1 | 0.5 | 0.1–0.3 |
| S2 | 0.2 | 0.4 | 0.1–0.3 |
| S3 | 0 | 0.1 | 0 |




a) b)

*Figure 2: P-graph representation of case study (a) fixed I-O ratios and (b) flexible I-O ratios*

Scenario analysis is then conducted by introducing a reduction of the capacity of S1 and determining the optimal allocation of goods which would maximize total GDP as indicated in Eq(2) given the disruption. The performance of the different sectors at different disruption levels in GDP units are shown in Figure 3.

$$max \sum_{i} y_i \tag{2}$$

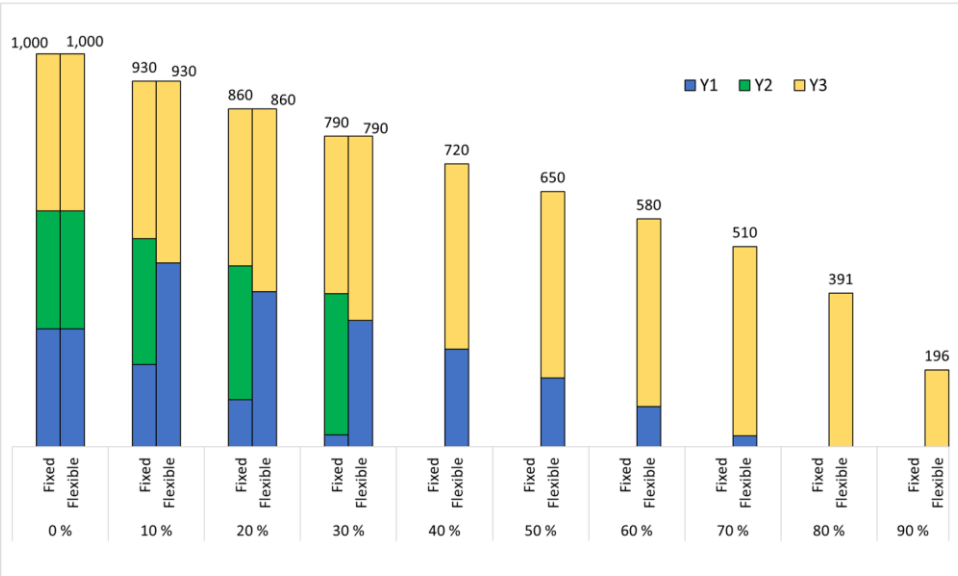



*Figure 3: Optimal performance of the economic system at varying levels of disruption to S1*

Based on the scenario analysis, the two economic systems were able to achieve the same GDP level (but with different distribution across sectors) when the lost output from S1 due to disruption is up to 30 %. The economic system with fixed coefficients can only tolerate up to 30 % disruption in the capacity of S1, beyond which no feasible solution is possible. Because of the fixed structure, the contribution of the different sectors to the total

GDP remains proportional to each other. Excessive loss of output from S1 without option for substitution leaves the economic system incapable of producing the final outputs. On the other hand, the economic system which allows for partial substitution and therefore has flexible coefficients can withstand up to a disruption level of up to 90 % output loss from S1. Since the minimum net output of the individual sectors is zero, the optimal solution of the flexible system only consisted of the outputs from S1 and S3, while products generated from S2 were used internally by the system. The flexible economic system is more robust since it was able to tolerate higher levels of disruption because S3 can use outputs from S2 to substitute for reduced supply of goods from S1. When the disruption level for S1 reached 80 %, the contribution of S1 and S2 per unit output of S3 was 0.1 and 0.3, respectively. These proportions represent the maximum degree of feasible substitution. The optimal network for this system is shown in Figure 4.

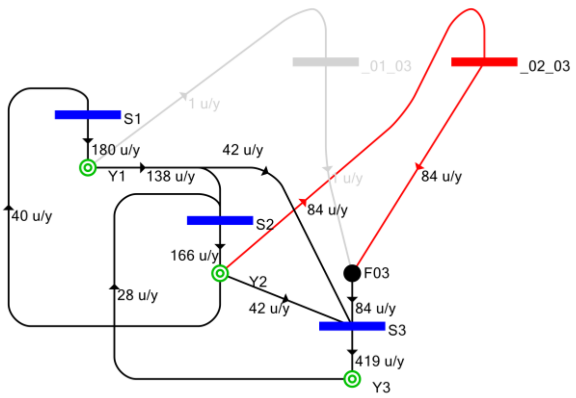



*Figure 4: Optimal performance of the economic system at 80 % disruption to S1*

These hypothetical results have significant practical implications for real economic systems. The restrictive assumptions of the classical I-O models prevent them from adequately representing an important aspect of production systems. The COVID-19 pandemic illustrates that capacity for reallocation contributes to system resilience in industrial supply chain networks (Queiroz et al., 2020).

## 5. Conclusions

An improved P-graph approach to modelling economic input-output systems has been developed in this work. This extension allows for partial substitution of inputs into each sector represented in the model based on a predefined objective function. A case study was solved to illustrate the modelling approach. Comparing the results of the model with and without partial substitution of inputs, it was demonstrated that the former model underestimates the resilience of an economic system to perturbations. The improved model can provide better representation of the real behaviour of economic input-output systems. Partial substitution of inputs is normally possible as described by a production function. The results of this work are relevant in the development of sustainable strategies such as identifying optimal economic structures which remain resilient despite pressure from sustainability targets. The P-graph approach automatically generates optimal and near optimal solution structures which can be further examined for other features and characteristics. Insights from these designs can be used for developing economic policies for sustainable trade. Future work should explore the application of this modelling approach to real-world economic data and applications. This method can also be applied to other types of input-output systems, such as models of organizations or ecosystems.

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