



# Estimation of Power System and Electrical Transient Status Based on Particle Filter

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In order to improve the accuracy and convergence speed of the transient estimation of the power system, taking into account the nonlinearity of the system, the Kalman filter (EKF) algorithm which is mainly used in the current status estimation of the power system, has the disadvantages of slow convergence and poor robustness, using particle filter (PF) algorithm in this work. In order to solve the problem of computationally occupied space and large amount of computation and sample degradation, a particle filter algorithm with sequence importance resampling is introduced on the basis of basic PF algorithm, which is closer to the approximate expression of true distribution of status. Compared with the EKF algorithm, the power system can converge to the real value quickly after the disturbance of the power system, and it has higher estimation precision and stability than the extended Kalman filter algorithm, achieving the requirement of accurate estimation.

## 1. Introduction

Transient status estimation is a branch of status estimation studies. The actual power system is a complex, nonlinear, transient system and the transient status estimation is more in line with the nature of the power system than the static status estimation. Transient estimation has the function of forecasting, can provide real-time operation of power grid, is an important part of energy management system (EMS), attracting attention of academic circles (Chiang et al., 1994; Hauer et al., 1990; Karami, 2011; Esmaeil and Kamwa, 2011; Dhaouadi et al., 1991; Amjady, 2001; Ariff et al., 2015).

Based on the basic PF algorithm, this work proposes an algorithm based on sequential importance resampling (SIR). In order to verify the superiority of the algorithm proposed, we study the traditional EKF (extended Kalman filter) algorithm for solving the nonlinear status estimation problem, theoretically, two algorithms are compared and analyzed. The results show that the estimation result of the particle filter algorithm based on SIR is highly correlated with the actual result, and the RMS error of the real value is smaller than that of the EKF, which effectively reduces the influence of the error (Meloni and Palma, 2017).

## 2. Transient system

### 2.1 The status space model of transient system

In the status space model, the time can be regarded as an independent variable in the model, so the model is a time domain model, which describes the operating characteristics of the transient system in the time domain. The status space model can be divided into several categories from different perspectives: determinism and uncertainty, discrete and continuous, linear and nonlinear. Regardless of the type of status space model, it is a system model that describes the status of the moment and describes the observed model representation of the relationship between the output and the status (Plett, 2004)

$$\begin{cases} x_{k+1} = f(x_k, u_k, w_k) \\ y_k = h(x_k, v_k) \end{cases} \quad (1)$$

Where  $u_k$  - System input;  $n$ -System status dimensions;  $m$ -Measure dimensions;  $w_k$  -System noise;  $v_k$  - Observation noise

$w_k$  and  $v_k$  are independent of each other and are independent of status,  $x_k \in R^n$  is status quantity,  $y_k \in R^m$  is observation quantity.

From the statistical point of view, the status transition probability density of the system model is represented by  $p(x_k|x_{k-1})$  in the status space model, that is, at time  $k-1$  status variable  $x_{k-1}$ , at time  $k$  status variable  $x_k$  probability, and in the observation model  $p(y_k|x_k)$ , The likelihood probability of the status value, that is, the status variable at the time  $k$  is  $x_k$ , the observed equation observes the probability of the status value  $y_k$ . It should be noted that the distribution of the status variable  $x_k$  of the system obeys the first-order Markov process in probability statistics, independent of  $y_k$ .

## 2.2 Typical algorithm for transient status estimation

According to the Bayesian estimation principle, the status estimation of the system can be expressed as expected. The estimation process can be described as follows: First, the prior probability distribution of the system status and the likelihood function between the contact status and the system quantity measurement are obtained, and then the system status is inferred to obtain the posterior probability distributed. This recursive process is called recursive Bayesian filtering, and since the equation contains high-dimensional integrals, it is difficult to find explicit solutions. Therefore, we can only obtain the suboptimal estimation result by the approximate method. The difference between the various estimation methods lies in the idea of its approximate processing and the means of realization (Esmaeil and Kamwa, 2011; Huang et al., 2007, Dhaouadi et al., 1991).

The assumption of the extended Kalman filter technique is that the input probability of the system and the prior probability of the status obey the Gaussian distribution. The conditional distribution of the status is completely characterized by its mean and covariance matrices, and then in the vicinity of the estimated and predicted values of the status, the status equation and the measurement equation are Taylor developed. i.e.

$$\begin{cases} x_k = f_k(\hat{x}_{k-1|k-1}) + A_k(x_{k-1} - \hat{x}_{k-1|k-1}) + \Delta_1(x_{k-1} - \hat{x}_{k-1|k-1}) + w_k \\ y_k = h_k(\hat{x}_{k|k-1}) + C_k(x_k - \hat{x}_{k|k-1}) + \Delta_2(x_k - \hat{x}_{k|k-1}) + v_k \end{cases} \quad (2)$$

Where

$$A_k = \frac{\partial f_k(x_{k-1})}{\partial x_{k-1}} \Big|_{x_{k-1} = \hat{x}_{k-1} | k-1} \quad (3)$$

$$C_k = \frac{\partial h_k(x_k)}{\partial x_k} \Big|_{x_k = \hat{x}_k | k-1}$$

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$$\begin{aligned} &\Delta_1(x_{k-1} - \hat{x}_{k-1|k-1}) \\ &\Delta_2(x_k - \hat{x}_{k|k-1}) \end{aligned} \quad (4)$$

Represents higher order terms for second order and above.

The above formula is simply derived, the following relationship:

$$\begin{cases} x_k = A_k x_{k-1} + c_k + \tilde{w}_k \\ y_k = C_k x_k + d_k + \tilde{v}_k \end{cases} \quad (5)$$

Where

$$\begin{aligned} c_k &= f_k(\hat{x}_{k-1|k-1}) - A_k \hat{x}_{k-1|k-1} \\ d_k &= h_k(\hat{x}_{k-1|k-1}) - C_k \hat{x}_{k-1|k-1} \end{aligned} \quad (6)$$

For the given amount of  $\hat{x}_{k-1|k-1}$  and  $\hat{x}_{k|k-1}$ , and

$$\begin{aligned}\tilde{w}_k &= \Delta_1(x_{k-1} - \hat{x}_{k-1|k-1}) + w_k \\ \tilde{v}_k &= \Delta_2(x_k - \hat{x}_{k|k-1}) + v_k\end{aligned}\quad (7)$$

represents random input noise, which contains high-order items, also known as virtual process noise and virtual measurement noise.

If the high-order items are ignored

$$\tilde{w} = w_k \quad \tilde{v} = v_k$$

From the above, we can approximate the Kalman filter formula recursive solution, which is the current power system transient status estimation commonly used filtering algorithm.

### 3. Particle filter algorithm

#### 3.1 The basic principle of particle filter algorithm

After determining the status space model of the transient system, we need to find a theory as the basis of this kind of estimation problem. Bayesian estimation theory is an effective method, it is based on the historical information to construct the corresponding probability density estimation, the probability is combined with the actual observed data of the system to determine the probability density function of the unknown system:  $x_{0:k} = \{x_i; i=0,1,\dots,k\}$  represents the moment of time 0 to the moment k, the unknown status variable vector;  $y_{1:k} = \{y_i; i=0,1,\dots,k\}$  represents the moment of time 1 to the moment k, the observation vector of the status variable.

If the status variable observations of  $y_{1:k}$  are given, then the conditional probability density of the unknown status variable  $x_{0:k}$ , that is, the posterior probability density  $p(x_{0:k} | y_{1:k})$  of the system is:

$$p(x_{0:k} | y_{1:k}) = \frac{p(y_{1:k} | x_{0:k})p(x_{0:k})}{\int p(y_{1:k} | x_{0:k})p(x_{0:k})dx_{0:k}}\quad (8)$$

Where,  $p(x_{0:k})$  is the prior probability density, and  $p(y_{1:k} | x_{0:k})$  is the status observed likelihood probability corresponding to the observation  $y_{1:k}$ , this is the content of Bayesian estimation theory. From the formula, we can see that the role of likelihood probability density is to use the observed data to correct the prior information. Bayesian theory describes the process by using the previous time of the prior probability density, combined with the current time to obtain the observation of new data, to modify the process. The prior probability density expresses the calendar, the probability of the status variable is expressed in the case of the known historical observation data, and the probability of the unknown variable is obtained, which reflects the relationship between the unknown status variable value and the actual observed value. This method makes the posterior probability density closer to the true value, and at the same time obtains the prior probability density next moment. This process of solving is a detailed solution to the whole Bayesian estimation problem.

However, the filtering problem is solved by finding the posterior filter probability density of the status variable  $p(x_k | y_{k+1})$ , we can make a solution by using the boundary density function of posterior density probability  $p(x_{0:k} | y_{1:k})$ , namely:

$$p(x_k | y_{1:k}) = \int \int \dots \int p(x_{0:k} | y_{1:k}) dx_0 dx_1 \dots dx_{k-1}\quad (9)$$

Using the above formula, we can solve the various filtering values of the status variables in the dynamic system. For example, the mean value of the status can be used as its estimated value.

#### 3.2 Particle filtering algorithm based on sequential importance sampling

In the actual status of the dynamic system to estimate, our purpose is not through the appeal method to obtain the status variable after the probability density of the random variable in a function satisfying this probability density is obtained by this probability density. The posterior probability density is only one of the intermediate steps. The following is the basic Principle introduced.

$g(x_{0:k})$  is set to represent any status representation of the status variable from 0 to k, it is easy to get the mathematical expectation expression for this function:

$$E(g(x_{0:k})) = \int g(x_{0:k})p(x_{0:k} | y_{1:k})dx_{0:k}\quad (10)$$

According to the monte carlo theory, It is possible to randomly extract N sets of samples independent of each other and satisfy the same probability distribution from  $p(x_{0:k}|y_{1:k})$  to satisfy the following formula:

$$E(g(x_{0:k})) \approx \frac{1}{N} \sum_{i=1}^N g(x_{0:k}(i)) \quad (11)$$

When N to a certain extent, the above formula converges to  $E(g(x_{0:k}))$ .

$$\begin{aligned} E(g(x_{0:k})) &= \int g(x_{0:k}) \frac{p(x_{0:k} | y_{1:k})}{q(x_{0:k} | y_{1:k})} q(x_{0:k} | y_{1:k}) dx_{0:k} = \int g(x_{0:k}) \frac{p(y_{1:k} | x_{0:k}) p(x_{0:k})}{p(y_{1:k}) q(x_{0:k} | y_{1:k})} q(x_{0:k} | y_{1:k}) dx_{0:k} \\ &= \int g(x_{0:k}) \frac{w_k^*}{p(y_{1:k})} q(x_{0:k} | y_{1:k}) dx_{0:k} \end{aligned} \quad (12)$$

Where,  $w_k^*(x_{0:k})$  is weight, since the above formula is an integral operation,  $y_{1:k}$  can be used as a constant for integral operations, further finishing:

$$E(g(x_{0:k})) = \frac{\int g(x_{0:k}) w_k^*(x_{0:k}) q(x_{0:k} | y_{1:k}) dx_{0:k}}{\int w_k^*(x_{0:k}) q(x_{0:k} | y_{1:k}) dx_{0:k}} \quad (13)$$

According to the monte carlo theory, It is easy to randomly extract N sets of samples independent of each other and satisfy the same probability distribution from  $p(x_{0:k}|y_{1:k})$ , the formula can be approximated as:

$$E(g(x_{0:k})) \approx \frac{\frac{1}{N} \sum_{i=1}^N g(x_{0:k}(i)) w_k^*(x_{0:k}(i))}{\frac{1}{N} \sum_{i=1}^N w_k^*(x_{0:k}(i))} = \sum_{i=1}^N g(x_{0:k}(i)) w_k(x_{0:k}(i)) \quad (14)$$

Where,  $w_k(w_{0:k}(i))$  is the normalized weight.

In order to solve the problem of large memory occupancy in the calculation process, the above formula is written in the form of sequence, and its weight is also recursively updated, even if the probability density estimation of the status variable is realized by recursive way. The recursive formula of calculating the weight iteration of can be collated as:

$$w_k^*(x_{0:k}) = w_{k-1}^*(x_{0:k-1}) \frac{p(y_k | x_k) p(x_k | x_{k-1})}{q(x_k | x_{0:k-1}, y_{1:k})} \quad (15)$$

The probability density of the status can be approximated as:

$$p(x_{0:k} | y_{1:k}) \approx \frac{1}{N} \sum_{i=1}^N w_k(x_{0:k}(i)) \delta(x_{0:k} - x_{0:k}(i)) \quad (16)$$

According to the above derivation, the filter probability density of  $x_k$  is approximately expressed as follows:

$$p(x_k | y_{1:k}) \approx \frac{1}{N} \sum_{i=1}^N w_k(i) \delta(x_k - x_k(i)) \quad (17)$$

Where,  $w_k(i) = w_k(x_{0:k}(i))$ .

The minimum mean square error estimate for  $x_k$  is:

$$\hat{x}_k = \sum_{i=1}^N w_k(i) x_k(i) \quad (18)$$

The most important feature of the sequential importance sampling method described above is the use of recursive iterations to update the data, this method avoids the computational complexity that is useless in each calculation. This method is the basis of the particle filter algorithm (Zhou et al., 2013; Konstandopoulos et al., 2000). Regardless of how to improve the particle filter algorithm, the essence is based on the sequential importance sampling algorithm. In the nonlinear power system, PF and EKF have the obvious ability to filter out the noise when the system transient changes, PF is obviously better than EKF. The simulation condition is

shown in figure 1: when the system is 2s, the three-phase short circuit occurs between the bus 3 and the bus 101, the fault is cut off at 2.1s, the transient status estimation step is estimated to be 0.1s. Taking bus 3 as an example, Figure 2 and 3 show the two algorithmic estimates of the voltage amplitude and phase angle on bus 3, respectively.

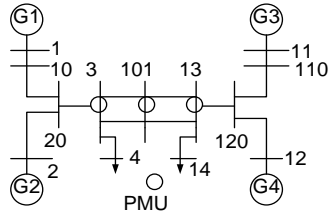


Figure 1: Simulation model

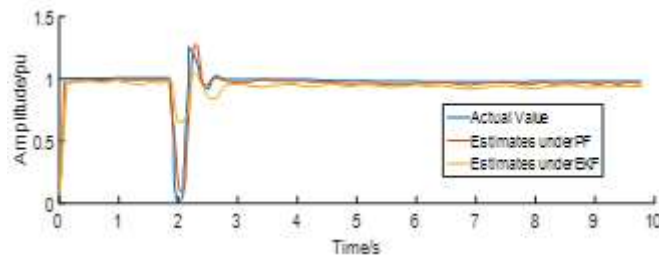


Figure 2: Comparison of amplitude estimation

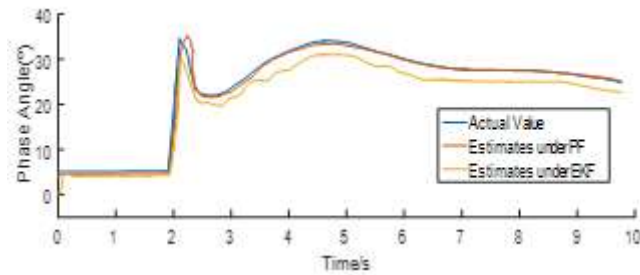


Figure 3: Comparison of phase angle estimation

This work also uses the root mean square error (root mean square error, RMSE) to evaluate the error

$$y = \sqrt{\frac{1}{N} \sum_{k=0}^N (v_k - \hat{v}_k)^2} \quad (19)$$

Where,  $v_k$  represents the true status of the system,  $\hat{v}_k$  represents the system status estimate,  $N$  represents the total number of simulated samples. The error of the two statuses of the power system transient and the phase angle is evaluated as follows:

The distributions of the error estimates of the results of the two estimation algorithms and the actual values are analyzed statistically. It can be seen from figure 4 and 5 that the error density of the result is estimated by the PF algorithm, and the spike is about 0, which means that the result of the PF method is closer to the true value.

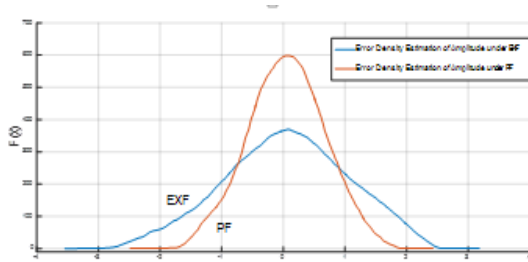


Figure 4: Error Density Estimation of Amplitude under Two Algorithms

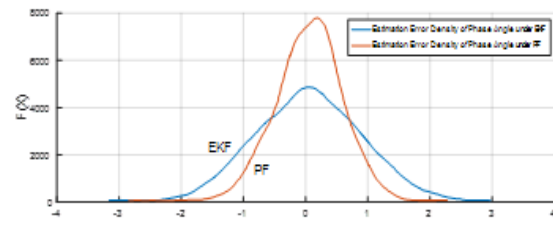


Figure 5: Estimation Error Density of Phase Angle under Two Algorithms

#### 4. Conclusion

Based on the transient equation of power system, a transient power model of nonlinear power system is established. The power system amplitude and phase angle are used as status variables. The nonlinear algorithm of PF is applied to the transient status estimation, the PF algorithm combining the sequence importance resampling method not only improves the computational speed of the algorithm, but also improves the accuracy of the estimation result effectively. The estimation accuracy is evaluated by the mean square error evaluation index. Comparing the results of PF and EKF, the simulation results show that the proposed method can effectively filter the errors in the observations of status variables and is better than the EKF algorithm, which can better meet the practical application.

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