CHEMICAL ENGINEERING TRANSACTIONS

VOL. 61, 2017

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Petar S Varbanov, Rongxin Su, Hon Loong Lam, Xia Liu, Jiří J Klemeš

ISBN 978-88-95608-51-8; **ISSN** 2283-9216

DOI: 10.3303/CET1761112

Robust Optimization of Refinery Hydrogen Networks using Worst-Case Conditional Value-at-Risk Concept

Lili Wei, Zuwei Liao*, Binbo Jiang, Jingdai Wang, Yongrong Yang

State Key Laboratory of Chemical Engineering, College of Chemical and Biological Engineering, Zhejiang University, Hangzhou, Zhejiang 310027, P.R. China
liaozw@zju.edu.cn

The hydrogen supply in many refineries is becoming a critical issue because of a trend of heavier crude oils and increasingly rigorous legislation. One of the significant problems is that the concentration fluctuation of hydrogen affects product quality of refineries and causes economic losses. This article investigates the disturbance resistance ability of hydrogen network. The Worst-Case Conditional Value-at-Risk (WCVaR) concept which indicates possible minimum hydrogen content in hydrogen networks is introduced to handle this problem. The disturbance resistance ability is optimized in maximization of WCVaR associated with uncertainty distribution. The article can obtain the hydrogen network whose WCVaR is not less than limit hydrogen content. The resistance ability of the obtained network structure is verified by Monte Carlo simulation. The literature example illustrate that the hydrogen network optimized by the WCVaR model performs robustly.

1. Introduction

VaR (Value-at-Risk), defined as the maximum anticipated loss in portfolio value because of market fluctuations has become the standard risk measure adopted by financial institutions in risk management since 1995 (Artzner and Delbaen et al., 1999). However, Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev, 2002), defined as the mean of the tail distribution exceeding VaR, has drawn much attention in recent years. CVaR performs some better properties than VaR in the measure of risk. The CVaR minimization formulation (Rockafellar and Uryasev, 2002) can result in convex programs, and even linear programs. Recently, a few researchers express more concern about the study of robustness (Goldfarb and Iyengar, 2003). Zhu and Fukushima (2009) introduced Worst-Case Conditional to robust portfolio optimization.
The approaches mentioned above have been applied in many other risk management fields. One of the significant problems of the hydrogen supply is that the concentration fluctuation of hydrogen affects product quality of refineries and results in economic losses in many refineries (Wang et al., 2012). Lou et al. (2014) employed the robust optimization approach to optimize hydrogen networks. Later, they presented a thermodynamic irreversibility based method for the design of hydrogen networks with multiple impurities (2015). Zhang et al. (2016) proposed a MILP model based on relative concentration analysis to optimize refinery multi-impurity hydrogen networks. This article investigates the disturbance resistance ability of hydrogen network. The WCVaR concept which indicates possible minimum hydrogen content in hydrogen networks is presented. The hydrogen network whose WCVaR is not less than limit hydrogen content can be obtained. The disturbance resistance ability of the obtained network structure is validated by Monte Carlo simulation. The example is investigated to illustrate the effectiveness of the method.

2. Problem statement

The robust optimization problem of refinery hydrogen system is stated as follows. Given a set of source and sink streams with certain concentration and flowrates, given the flowrate disturbance distribution of the source streams, it is desired to maximize the disturbance resistance ability of the source – sink allocation network. The disturbance resistance ability of the network structure be verified by the parameter P obtained in Monte Carlo simulation.

Please cite this article as: Wei L., Liao Z., Jiang B., Wang J., Yang Y., 2017, Robust optimization of refinery hydrogen networks using worst-case conditional value-at-risk concept, Chemical Engineering Transactions, 61, 685-690 DOI:10.3303/CET1761112

3. Methodology

3.1 Definition of the WCVaR in the hydrogen network

VaR (Artzner and Delbaen et al., 1999) and CVaR (Rockafellar and Uryasev, 2002) represent the maximum anticipated loss in portfolio value due to adverse market movements and the mean of the tail distribution exceeding VaR with a certain confidence interval respectively in financial risk management. WCVaR is the maximum Conditional Value-at-Risk value in the worst case (Zhu and Fukushima, 2009).
Given a confidence level β, the VaR $\alpha_\beta(x)$ is defined as (Rockafellar and Uryasev, 2002):

$$\varphi(x,\alpha) = \int_{f(x,y)\le\alpha} p(y)dy \tag{1}$$

$$\alpha_\beta(x) = \min\{\alpha \in R; \varphi(x,\alpha) \ge \beta\} \tag{2}$$

where f(x,y) denote the loss associated with decision vector $x \in X \subseteq R^n$ and random vector $y \in R^m$, p(y) is the density function of y.
The corresponding CVaR is defined as (Rockafellar and Uryasev, 2002):

$$\phi_\beta(x) = E\left[f(x,y) \mid f(x,y) \ge VaR(x)\right] = \frac{1}{1-\beta}\int_{f(x,y)\ge\alpha_\beta(x)} f(x,y)p(y)dy \tag{3}$$

Rockafellar and Uryasev （2002） demonstrated that the calculation of CVaR can be achieved by minimizing the following auxiliary function and present its approximate function:

$$F_\beta(x,\alpha) = \alpha + \frac{1}{1-\beta}\int_{y\in R^m}\left[f(x,y)-\alpha\right]^+ p(y)dy \tag{4}$$

$$\tilde{F}_\beta(x,\alpha) = \alpha + \frac{1}{S(1-\beta)}\sum_{j=1}^{S}\left[f(x,y_j)-\alpha\right]^+ \tag{5}$$

where $[t]^+=\max\{t,0\}$.
Then Zhu and Fukushima (2009) obtain the formulas:

$$WCVaR_\beta(x) = \min_{\alpha\in R}\max_{i\in L} F_\beta^{\,i}(x,\alpha) \tag{6}$$

$$\min_{x\in X} WCVaR_\beta(x) = \min_{(x,\alpha)\in X\times R}\max_{i\in L} F_\beta^{\,i}(x,\alpha) = \min_{(x,\alpha)\in X\times R} F_\beta^{\,L}(x,\alpha) \tag{7}$$

The problem can be formulated as the equation presented by Zhu and Fukushima (2009):

$$\begin{aligned} &\min_{(x,\alpha,\theta)\in X\times R\times R} \theta \\ &s.t.\quad \alpha + \frac{1}{S^i(1-\beta)}\sum_{j=1}^{S^i}\left[f(x,y_k^{\,i})-\alpha\right]^+ \le \theta, i = 1,\cdots l, \end{aligned} \tag{8}$$

where y_k^i is the k^{th} sample with respect to the i^{th} likelihood distribution $p^i(\cdot)$, and S^i denotes the number of corresponding samples.

Then, by introducing an auxiliary vector $u = (u^1;u^2;\cdots;u^l) \in R^m$, where $m = \sum_{i=1}^{l} S^i$, the optimization problem can be reformulated as the following tractable minimization problem with variables $(x,u,\alpha,\theta) \in R^n \times R^m \times R \times R$ (Zhu and Fukushima, 2009):

$$\begin{aligned} &\min\ \theta \\ &s.t.\quad x \in X, \\ &\qquad \alpha + \frac{1}{1-\beta}(\pi^i)^T u^i \le \theta, \quad i = 1,\cdots,l, \\ &\qquad u_k^i \ge f(x,y_k^{\,i}) - \alpha, \quad k=1,\cdots,S^i, \ \ i = 1,\cdots,l, \\ &\qquad u_k^i \ge 0, \quad k=1,\cdots,S^i, \ \ i = 1,\cdots,l. \end{aligned} \tag{9}$$

where π_k^i denotes the probability according to the k^{th} sample with respect to the i^{th} likelihood distribution $p^i(\cdot)$.

In the hydrogen system, VaR and CVaR represent the minimum anticipated hydrogen content due to flowrate fluctuation of source streams and the mean of the tail distribution less than VaR with a certain confidence interval accordingly. WCVaR is the minimum CVaR value in the worst case. The optimization problem can be formulated as:

$$\begin{aligned}
&\max\ \theta \\
&s.t.\quad \alpha + \frac{1}{1-\beta}(\pi^n)^T u^n \geq \theta, \quad n = 1, \cdots, N. \\
&\qquad u_k^n \leq f(C, FC2_k^n) - \alpha, \quad k=1, \cdots, S^n, \ \ n = 1, \cdots, N. \\
&\qquad u_k^n \leq 0, \quad k=1, \cdots, S^n, \ \ n = 1, \cdots, N.
\end{aligned} \tag{10}$$

where π^n denotes the probability according to the kth sample with respect to the nth likelihood distribution $p^i(\cdot)$; $FC2_k^n$ is the flowrate of the kth sample with respect to the n$_{th}$ likelihood distribution $p^i(\cdot)$; S^n denotes the number of samples of the nth likelihood distribution $p^i(\cdot)$; C represents the concentration of hydrogen of source.

3.2 Optimization model of WCVaR in the hydrogen network

Objective function:

$$\text{Max}(\theta) \tag{11}$$

Hydrogen sink flowrate constraint:

$$F_d = \sum_{i=1}^{NI} FC_{i,d} \tag{12}$$

where F_d denote the required flowrate of the sink; $FC_{i,d}$ is the flowrate of hydrogen source i to hydrogen sink d; ND is the number of hydrogen sink; NI is the number of hydrogen source.
Constraint on hydrogen load:

$$F_d \times C_{d,m} \leq \sum_{i=1}^{NI} (FC_{i,m} \times C_{i,m}) \ \ (m=1) \tag{13}$$

Impurity load constraint:

$$F_d \times C_{d,m} \geq \sum_{i=1}^{NI} (FC_{i,m} \times C_{i,m}) \ \ (m>1) \tag{14}$$

where $C_{d,m}$ is the demanded concentration of component m of hydrogen sink d; $C_{i,m}$ is the concentration of component m of hydrogen source i; m=1 represents hydrogen, m>1 represents other impurities.
Hydrogen source flowrate constraint:

$$F_i \geq \sum_{d=1}^{ND} FC_{i,d} \tag{15}$$

where F_i is the flowrate of hydrogen source i.

$$FC2_{k,i,d}{}^n = \frac{F2_{k,i}{}^n \times FC_{i,d}}{F_i} \tag{16}$$

where $FC2_{k,i,d}{}^n$ is the flowrate of hydrogen source i to hydrogen sink d of the k-th sample with respect to the n-th like-lihood distribution $p^i(\cdot)$; $F2_{k,i}{}^n$ represents flowrate of hydrogen source i of the k-th sample with respect to the n-th likelihood distribution $p^i(\cdot)$
Hydrogen utility flowrate constraint:

$$\sum_{d=1}^{ND} FC_{i,d} \leq U \ \ (i=NF) \tag{17}$$

where U is the flowrate of the hydrogen utility to ensure the WCVaR of the network meet the requirement; NF denotes the hydrogen utility.

$$U_k^{\ n} \le \sum_{d=1}^{ND}\sum_{i=1}^{NI}(FC2_{k,i,d}^{\ \ n} \times C_i) - \alpha \tag{18}$$

where C_i represents the concentration of hydrogen of source i.

$$U_k^{\ n} \le 0 \tag{19}$$

$$\alpha + \frac{1}{S^n \times (1-\beta)}\sum_{k=1}^{S^n} U_k^{\ n} \ge \theta \tag{20}$$

4. Monte Carlo verification method

Monte Carlo simulation is introduced in this section to verify the effectiveness of the above presented WCVaR based optimizing approach. Monte Carlo simulation is a numerical procedure for predicting statistical properties such as sample mean and standard deviation of the system (Moore and Weatherford, 2001). Monte Carlo simulation was performed using the Crystal Ball software.The parameter P which denotes the probability of satisfying hydrogen content requirement can be obtained from the simulation.

5. Case study

This example is taken from Zhang and Feng et al. (2013) with the data shown in Table 1. The hydrogen network structure of literature and of this work is shown in Figure 1 and Figure 2. Monte Carlo simulations are performed with the two distributions for the literature result and this work result. The simulating results are given by Figure 3 and Figure 4.

Table 1: Data for the source and sink streams of example 1

		Impurity	Concentration	(mol %)	
Stream	Flowrate (mol/s)	A	B	C	total(mol %)
FH	unlimited	0.01	0	0	0.01
SR2	50	2.5	5	3	10.5
SR3	75	9	3.5	6	18.5
SR4	20	10	7.5	5	22.5
SK1	40	0.5	0	0.1	0.6
SK2	60	3	1.75	4.5	9.25
SK3	50	5	6	4	15

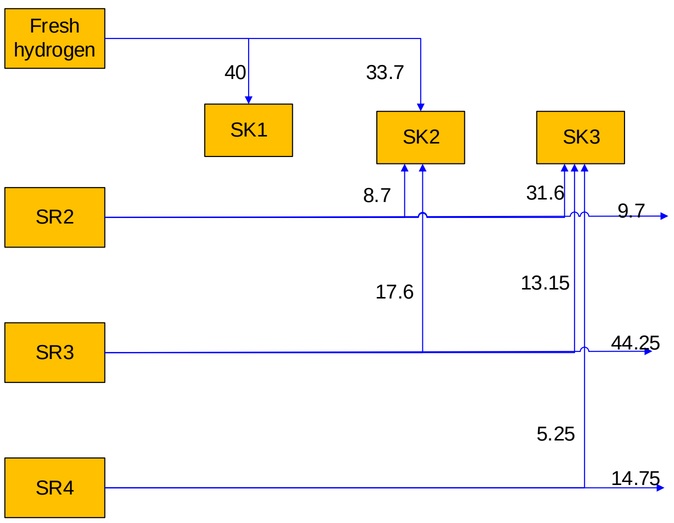

Figure 1: Literature network for example

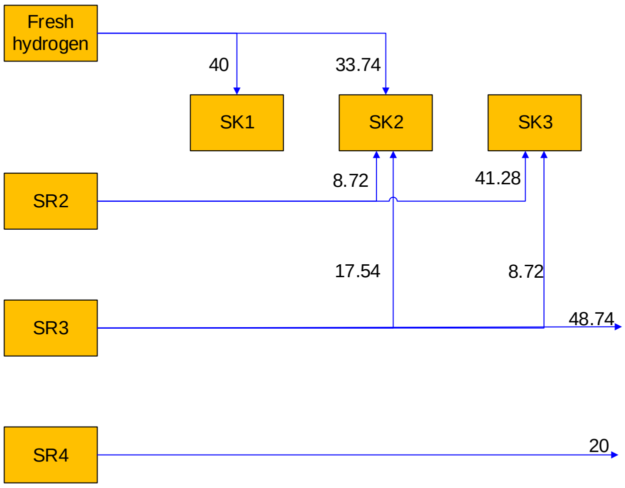

Figure 2: The achieved network for example

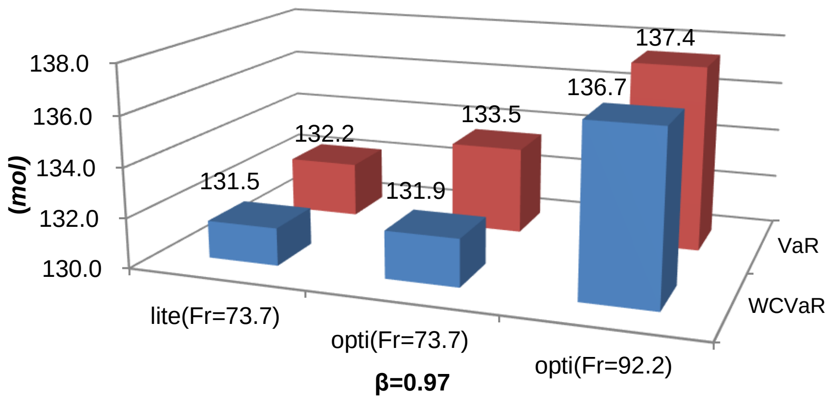

Figure 3: The comparison of optimized results for example

The ordinate of Figure 3 stands for the value of the VaR and WCVaR, while the abscissa denotes the literature result and optimized results with the different hydrogen utility. The lite and opti illustrate the literature result and optimized result respectively. The series axis represents the VaR and WCVaR.

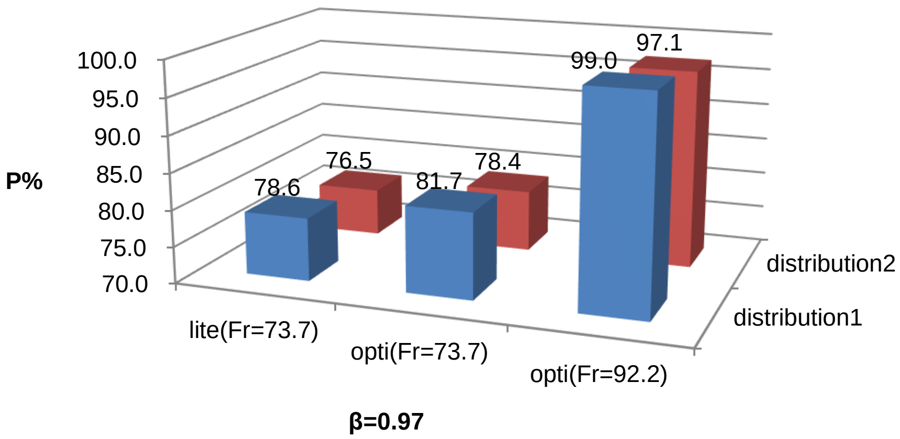

Figure 4: The comparison of Monte Carlo simulations results for example

The ordinate of Figure 4 stands for the probability of satisfying hydrogen content requirement, while the abscissa denotes the literature result and optimized results with the different hydrogen utility. The lite and opti illustrate the literature result and optimized result respectively. The series axis represents the two distributions. The Figure 3 and Figure 4 show that optimized result with the same hydrogen utility as literature performs better than the literature result. 92.2 mol·s^{-1} is the minimum flowrate of the hydrogen utility to make the WCVaR meets the hydrogen content requirement which is 136.7 mol. The probability of satisfying hydrogen content requirement in this situation improves greatly and approaches 100 % in the two distributions. Therefore, it attests the definition of the WCVaR. The p values do not fully satisfy the requirement because of the limit of the sample numbers.

6. Conclusions

Robust optimization of hydrogen network operating parameters is important for its product quality. The WCVaR concept which indicates possible minimum hydrogen content in hydrogen networks is applied to deal with this problem. The disturbance resistance ability is optimized in maximization of WCVaR associated with uncertainty distribution. The article can obtain the hydrogen network whose WCVaR is not less than limit hydrogen content. In comparison with the literature result, Monte Carlo simulation results in the two fluctuation distributions show that the hydrogen network optimized by the WCVaR model performs robustly.

Acknowledgments

The financial support provided by the Project of National Natural Science Foundation of China (91434205), the National Science Fund for Distinguished Young (21525627), the National Natural Science Foundation of China (Grant 61590925), and the International S&T Cooperation Projects of China (2015DFA40660) are gratefully acknowledged.

References

Artzner P., Delbaen F., Eber J.M., Heath D., 1999, Coherent Measures of Risk, Mathematical Finance, 9, 203–228.

Goldfarb D., Iyengar G., 2003, Robust Portfolio Selection Problems, Mathematics of Operations Research, 28, 1-38.

Lou J.Y., Liao Z.W., Jiang B.B., Wang J.D., Yang Y.R., 2014, Robust Optimization of Hydrogen Network, International Journal of Hydrogen Energy, 39 (3), 1210-1219.

Lou J.Y., Liao Z.W., Jiang B.B., Wang J.D., Yang Y.R., 2015. A Thermodynamic Irreversibility based Design Method for Multi-Contaminant Hydrogen Networks. International Journal of Hydrogen Energy, 40 (1), 435-443.

Moore J.H., Weatherford L.R., 2001, Decision modeling with Microsoft Excel, Prentice Hall PTR, New Jersey, USA, ISBN: 013017789X.

Rockafellar R.T., Uryasev S., 2002, Conditional Value-at-Risk for General Loss Distributions, Journal of Banking and Finance, 26, 1443-1471.

Wang B., Feng X., Chu K.H., 2012, A Novel Graphical Procedure Based on Ternary Diagram for Minimizing Refinery Consumption of Fresh Hydrogen, Journal of Cleaner Production, 37, 202-10.

Zhang Q., Feng X., Chu K.H., 2013, Evolutionary Graphical Approach for Simultaneous Targeting and Design of Resource Conservation Networks with Multiple Contaminants, Industrial & Engineering Chemistry Research, 52(3), 1309-1321.

Zhang Q., Song H.C., Liu G.L., Feng X., 2016, Relative Concentration-based Mathematical Optimization for the Fluctuant Analysis of Multi-impurity Hydrogen Networks, Industrial & Engineering Chemistry Research, 55(39), 10344-10354.

Zhu S.S., Fukushima M., 2009, Worst-Case Conditional Value-at-Risk with Application to Robust Portfolio Management, Operations Research, 57, 1155-1168.