

Particle Swarm Algorithm with Adaptive Constraint Handling Technique for Heat Exchanger Network Synthesis

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The heat exchanger networks synthesis (HENS) still remains an open problem due to its non-linear characteristics but also due to a great number of local optima in its solution space. This paper deals with the development of effective techniques to generate optimal heat exchanger network(HEN) automatically aiming to simultaneously balance the energy recovery target and investment costs. The optimization formulations of such a problem turn out to be a non-convex NLP/MINLP problem with equality and inequality constraints. The stochastic or meta-heuristic optimization algorithms seem to have some special advantages for the synthesis of complex HENS. However, one of the major issues for stochastic algorithms is how to handle the constraints. In this paper, we develop a robust particle swarm algorithm coupled with a novel adaptive constraint-handling technique to search for the global optimum of these formulations. The problem can be transformed into with no equality constraints through analyzing and tearing equality constraints. Two Classic HENS problems are performed to prove the applicability and efficiency of the proposed algorithm.

1. Introduction

The optimal design for heat exchanger network (HEN) continues to be a major challenge throughout the world, due to its importance in energy-saving and environmental protection policies in process industry. The Synthesis/Retrofit of HEN is essentially a MINLP problem. To overcome nonlinear-problem solving difficulty in the MINLP model, effective initialization strategy and solution strategy are necessary (Koraviyotin and Siemanond, 2015). Many efficient optimization methods are developed for HEN synthesis (HENS), which can be grouped into three major categories: pinch analysis methods, mathematical programming methods and stochastic optimization methods (Gorji-Bandpy et al., 2011). Stochastic optimization methods do not suffer from the NP-hardness of the problem, and thus they are considered to be suitable for complex HENS problems (Zhang et al., 2016). Simulated annealing approach (SA) (Cardoso et al., 1997), genetic algorithm (GA) (Lewin, 1998), different evolution (DE) (Yerramsetty and Murty, 2008), Tabu search algorithm (Lin and Miller, 2004), randomized algorithm (Pariyani et al., 2006), and particle swarm algorithm (PSO) (Silva et al., 2006) are widely used for dealing with HENS problems. These algorithms do not depend on gradients and work on function evaluation alone. PSO is a population based stochastic optimization technique developed by Kennedy and Eberhart (1995), inspired by social behaviour of bird flocking, fish schooling and swarm theory. Comparing with other stochastic algorithms, PSO is simple in concept. It has few parameters to adjust and is easy to implement. However, since the plain PSO algorithm does not handle constraints on its own, an adaptive constraint-handling technique (CHT) need to be developed and integrated into the optimizer (Luo et al., 2007). A novel optimization strategy based on PSO algorithm is developed in this paper for solving HENS problem. For the solution of test previous works, the proposed PSO method shows a superior performance in overcoming local optima and improving efficiency.

2. HENS problem formulation

The general simultaneous MINLP formulation first presented by Yee and Grossmann (1990) can be written as shown in Eq(1), based on the stage-wise superstructure representation. The stage-wise superstructure consists

of several stages. At each stage, hot and cold streams are split to allow the potential existence of a heat exchanger to match any hot-cold pair of streams, and at the end of the stage, a mixer combines all split streams back into on single stream for the next stage without the assumption of isothermal mixing. For simplicity in the presentation, the utilities are placed on the extreme ends of the superstructure and no stream bypass exists. The number of stages can be selected at random or set by the designer.

$$\begin{aligned} \text{Minimize } & \left\{ \sum_{i=1}^{HN} \sum_{j=1}^{CN} \sum_{k=1}^{KN} \left[CF_{ij} z_{ijk} + C \left(Ae_{i,j,k} \right)^{B_{ij}} \right] + \sum_{i=1}^{HN} \left[CF_{i,CU} z_{cu_i} + C_{i,CU} \left(Acu_i \right)^{B_{i,CU}} \right] \right. \\ & \left. + \sum_{j=1}^{CN} \left[CF_{HU,j} z_{hu_j} + C_{HU,j} \left(Ahu_j \right)^{B_{HU,j}} \right] + \sum_{i=1}^{HN} CCUq_{cu_i} + \sum_{j=1}^{CN} CHUq_{hu_j} \right\} \\ \text{Subject to } & \left\{ \begin{aligned} & \left(tph_{i,j,k,in} - tph_{i,j,k,out} \right) Fph_{i,j,k} = q_{i,j,k} \\ & \left(tpc_{j,i,k,out} - tpc_{j,i,k,in} \right) Fpc_{j,i,k} = q_{j,i,k} \\ & Fh_i \cdot t_{i,k} = \sum_{j=1}^{CN} Fph_{i,j,k} \cdot tph_{i,j,k,out} \\ & Fc_j \cdot t_{j,k} = \sum_{i=1}^{HN} Fpc_{j,i,k} \cdot tpc_{j,i,k,out} \\ & i = 1, 2, \dots, HN; j = 1, 2, \dots, CN; k = 1, 2, \dots, KN \end{aligned} \right. \end{aligned} \quad (1)$$

where q , q_{hu} and q_{cu} are the heat load of the stream matches, hot utilities and cold utilities respectively; tph and tpc are branch temperatures of hot and cold streams relatively; Fph , Fpc are branch heat capacities of hot and cold streams relatively; Fh , Fc are total heat capacities of hot and cold streams relatively; The subscript in and out represent inlet and outlet of heat exchangers relatively; CF is fixed charge for exchangers; C is area cost coefficient; B is exponent for area cost; Ae , Acu , Ahu are area of heat exchangers, heaters and coolers respectively; z , z_{cu} , z_{hu} are binary variables representing the existence of heat exchangers, heaters and coolers respectively; The subscript HU and CU identify hot utility and cold utility; i represents index for hot process stream, j represents index for cold streams, k represents index for stages in the superstructure; HN is the number of hot process streams, CN is the number of cold process streams, KN is the number of stages.

3. Particle swarm optimization

3.1 The particle swarm optimization algorithm

In the standard PSO, each individual is called a particle and a group of particles forms a swarm. Assume that a swarm consists of m particles, each of which is a D -dimensional vector denoted by $X_{id} = (x_{i1}, x_{i2}, \dots, x_{id}) (i = 1, 2, \dots, m)$. The particles' fitness could be calculated through position vectors using objective function. $Pbest_{id} = (pbest_{i1}, pbest_{i2}, \dots, pbest_{id})$ represents the best position of the i^{th} particle and $Gbest_i = (gbest_1, gbest_2, \dots, gbest_d)$ indicates the best position in the swarm. Let v_{id} be the velocity vector for the particle $i (i=1, 2, \dots, m)$, then the update of each particle and its velocity can be carried out by the following equations:

$$v_{id}(t+1) = w(t+1)v_{id}(t) + c_1 r_1 (Pbest_{id}(t) - x_{id}(t)) + c_2 r_2 (Gbest_i - x_{id}(t)) \quad (2)$$

$$x_{id}(t+1) = x_{id}(t) + v_{id}(t+1) \quad (3)$$

where c_1 and c_2 are two positive constants, called the cognitive and social acceleration coefficients, respectively; r_1 and r_2 represent uniform random numbers between 0 and 1; w is an inertia weight controlling the influence of previous velocity on the new velocity.

3.2 Equality constrain handing process

The usual way of dealing with constraints is by penalizing infeasible solutions. However, the performance of usual handling methods is not satisfactory for the HENS problems with heavy equality constraints. The point of this section is thus to propose an adequate method of handing equality constrains to improve the optimization efficiency. The first important work is to identify the independent variables, and then the reduced variables should be solved directly through the equality constraints. For HENS problems, identifying the reduced variables is a very complex work. In our work, through analyzing equality constraints, the split heat capacity flows and temperatures are assigned to independent variables. Therefore, these variables can be generated randomly and optimized through algorithm directly. The dependent variables should be expressed and calculated as follows:

(1) Heat load for each exchanger

Based on split stream heat flows and outlet temperatures of the cold stream, the heat loads between the hot stream i and the cold stream j in stage k is determined by Eq(4), which is the potential heat exchanging load when exchanger minimum temperature approach (EMAT) is set for each exchanger.

$$\begin{aligned}
 & \text{if } Fph_{i,j,k} \geq Fpc_{i,j,k} \\
 & q_{i,j,k} = \min \left(\begin{array}{l} Fph_{i,j,k} (tph_{i,j,k,in} - Th_{i,out}), \\ Fpc_{i,j,k} (tpc_{i,j,k,out} - Tc_{j,in}) \end{array} \right) \\
 & \text{else} \\
 & q_{i,j,k} = \min \left(\begin{array}{l} Fph_{i,j,k} (tph_{i,j,k,in} - Th_{i,out}), \\ Fpc_{i,j,k} (tpc_{i,j,k,out} - Tc_{j,in}), \\ \frac{tph_{i,j,k,in} - EMAT - tpc_{i,j,k,out}}{\frac{1}{Fph_{i,j,k}} - \frac{1}{Fpc_{i,j,k}}} \end{array} \right) \\
 & \text{end}
 \end{aligned} \tag{4}$$

(2) The stream temperatures at each stage

The outlet temperatures of stages can be calculated through the equality constraints of energy balances for each mixer by Eq(5)-(6).

$$th_{i,k,out} = \frac{\sum_{j=1}^{CN} Fph_{i,j,k} \cdot tph_{i,j,k,out}}{Fh_i} \tag{5}$$

$$tc_{j,k,out} = \frac{\sum_{i=1}^{HN} Fpc_{i,j,k} \cdot tpc_{i,j,k,out}}{Fc_j} \tag{6}$$

In most cases, the calculated temperature difference exists between adjacent stages due to independent optimization for split heat capacity flows and temperatures, which can not satisfy the equality constraints. To get feasible solutions, the split temperatures need to be adjusted reasonably according to constraints. In this work, the minimum temperature between two adjacent stages is chosen as the final value. Then the corresponding split temperatures should be reduced by the calculated temperature difference while other parameters of stream matches can be guaranteed.

4. Case studies

The proposed particle swarm algorithm coupled with a novel adaptive constraint-handling technique in this work has been applied to two medium-scale HENS problems reported in the published literature. The algorithm was implemented in a Matlab program running under Windows 7 system on Intel core i5 2.4GHz having 4 GB RAM.

4.1 Case 1

This case originally proposed by Linnhoff and Flower (1978) consists of five hot and five cold streams. It is also a popular problem studied by many researchers. The input data is summarized in Table 1. To get a better cost-optimal solution, there are many critical factors that need to be investigated and be set reasonably. In this case, the number of stages is set to 3 and the potential number of branches in each stage is set to 2. EMAT is originally set to 5 K and can be arranged to suit constraints in the searching procedure.

Through numerous experiments, the optimal solution can be found reliably when the population size of PSO is set to 100 and the maximum iteration times is set to 5,000. The final structure was found after average computation time of 1,550 s, shown in Figure 1. It can be observed that the actual EMAT is 17.25 K, which is greater obviously than the initial value. In the optimal structure, the splitting matches H3-C3 and H3-C5 only exists in stage 1 with an isothermal mixing, although the non-isothermal mixing is considered in this work. The total annual cost for the obtained network is \$43,422, which is only slightly larger than that obtained by Lin and Miller. The comparison between the results obtained from this paper and those obtained by other researchers is presented in table 2.

Table 1: Problem data for case 1

Stream	$T_{in}(K)$	$T_{out}(K)$	$F(kW \cdot K^{-1})$
H1	433	366	8.79
H2	522	411	10.55
H3	544	422	12.56
H4	500	339	14.77
H5	472	339	17.73
C1	355	450	17.28
C2	366	478	13.90
C3	311	494	8.44
C4	333	433	7.62
C5	389	495	6.08
Steam	509	509	-
Cold water	311	355	-

Cost data

Exchanger annual cost: $145.63A^{0.6}$

Cost of hot utility: $37.64 \$ \cdot kW^{-1} \cdot y^{-1}$

Cost of cold utility: $18.12 \$ \cdot kW^{-1} \cdot y^{-1}$

Heat transfer coefficient for all matches involving utilities : $1.136 kW \cdot m^{-2} \cdot K^{-1}$

Heat transfer coefficient for all matches involving steam : $0.852 kW \cdot m^{-2} \cdot K^{-1}$

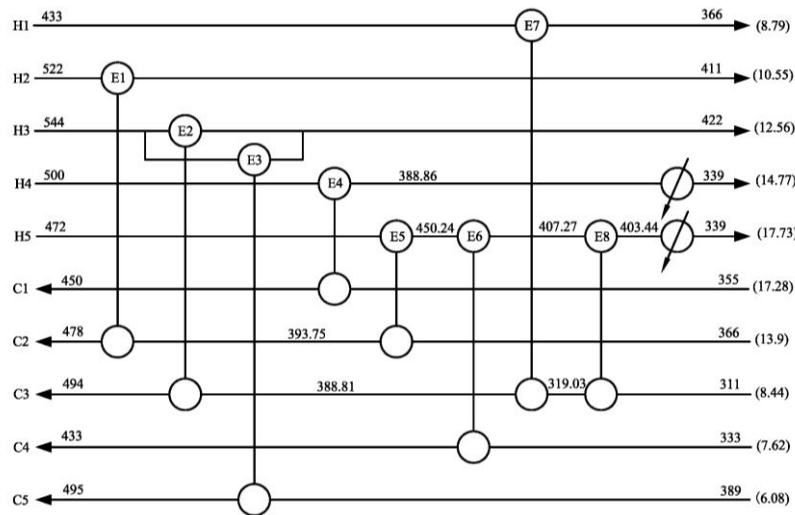


Figure 1: Optimal network for case 1

Table 2: Optimal results for case 1

Method	No. of units	qcu(kW)	Total annual cost(\$)
Linnhoff and Flower (1978)	10	1,975	43,934
Lewin et al. (1998)	10	1,879	43,452
Lin and Miller (2004)	10	1,879	43,329
Pariyani et al. (2006)	10	1,879	43,439
Yerramsetty and Murty (2008)	10	1,879	43,538
This work	10	1,879	43,422

4.2 Case 2

This case is the popular Aromatics plant problem, studied by many researchers, which involves four hot and five cold streams with hot oil and cold water as utilities. The supplying data for the problem is presented in Table 3. The maximum number of stages is initially set to 4. The maximum number of split-stream for each stream is assigned to 2. EMAT is fixed temporarily at 5 K for this problem. The parameters of the PSO are the same as those of case 1.

Table 3: Problem data for case 2

Stream	$T_{in}(K)$	$T_{out}(K)$	$F(kW \cdot K^{-1})$	$h(kW \cdot m^{-2} \cdot K^{-1})$
H1	600	313	100	0.50
H2	493	433	160	0.40
H3	493	333	60	0.14
H4	433	318	400	0.30
C1	373	573	100	0.35
C2	308	437	70	0.70
C3	358	411	350	0.50
C4	333	443	60	0.14
C5	413	573	200	0.60
Hot oil	603	523	-	0.50
Water	288	303	-	0.50

Cost data

Exchanger annual cost: $2,000+70A$

Cost of hot utility: $60 \$ \cdot kW^{-1} \cdot y^{-1}$

Cost of cold utility: $6 \$ \cdot kW^{-1} \cdot y^{-1}$

Table 4: Optimal results for case 2

Methods	No. of units	Total heat exchanger area (m^2)	q_{hu} (MW)	q_{cu} (MW)	Total annual cost (M\$)
Linhoff and Ahmad (1990)	13	17,400	25.31	33.03	2.960
Zhu et al. (1995)	14	16,630	26.22	33.94	2.970
Lewin (1998)	12	17,050	25.09	32.81	2.936
Yerramsetty and Murty (2008)	15	16,536	25.88	33.60	2.942
Luo et al. (2009)	14	--	--	--	2.9223
This work	14	18,159	23.96	31.68	2.9271

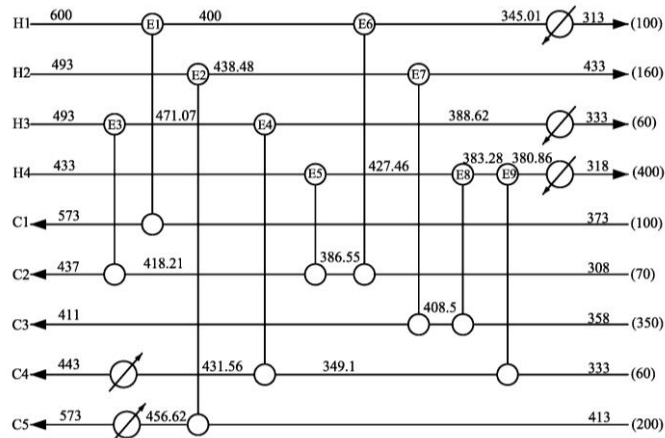


Figure 2. Optimal network for case 2

The output data show that after 1,200 generations the solution has already converged to the minimum, and the average computation time is 850 s. The obtained structure is shown in Figure 2, which has 14 heat exchangers, two heaters and three coolers, and the total annual cost is \$2,927,064. It can be observed that no splitting match exists in this configuration. The actual EMAT is 14.79 K. The comparison between the costs of the networks is presented in Table 4.

5. Conclusions

A robust particle swarm algorithm coupled with a novel adaptive constraint-handling technique was proposed for solving non-convex HENS problems with equality constraints. The approach is easy to implement and its computational cost is considerably reduced through effective constraints handling strategy. The results

encourage further research of the application of PSO or improved PSO to some large HENS problems since the method performs better than many other algorithms during two cases studies. As shown in this study, the proposed method is a viable alternative for conventional HEN synthesis methods and has a good potentiality for solving real-world, industrial-sized HEN synthesis problems. However, It is worth noting that the performance of algorithm and the efficiency of the constraint handling strategy are key factors and need to be further investigated to improve this method.

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