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Transient Free Convection MHD Flow Past an Accelerated Vertical Plate with Periodic Temperature

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A theoretical model is performed to study the effect of magnetic field on transient free convection flow of an electrically conducting fluid past an accelerated vertical plate with periodic temperature. The governing momentum and energy equations are solved numerically using a full implicit finite difference method. An analytical solution using eigenfunction expansion method is carried out for temperature profile in case of constant plate temperature. The results are compared with the numerical ones and a good agreement is achieved.

The effect of different physical parameters on the transient velocity and temperature, such as Grashof's number, magnetic parameter, Prandtl number and temperature frequency is studied. It is found that the velocity increases with increase in Gr and t, while it decreases with increase in Pr and M. It is found also that the periodicity of the plate temperature does not affect the velocity profile.

1. Introduction

Magnetohydrodynamic (MHD) flow of a viscous incompressible electrically conducting fluid past moving vertical plates has many industrial and engineering applications. Recently many problems have been analyzed to study the effect of magnetic field on the natural convection flow past a vertical plate with various conditions.

Some of studies are mentioned here. Chaudhary et al. (2006) analyzed the free convection flow of a viscous incompressible fluid past a vertical accelerated plate with constant heat flux in the presence of transverse magnetic field. Deka and Neog (2009) studied the transient natural convection fow past a vertical accelerated plate, immersed in a viscous thermally stratified fluid. Analytical solution of unsteady flow past an accelerated vertical plate with constant temperature has been carried out by Uwanta and Yale (2014). Mahender and Rao (2015) examined the influence of unsteady MHD mass transfer flow past a vertical porous fixed plate in presence of viscous dissipation and heat source.

It is noticed that the surface temperature is varied in many fluid flow problems. Hence, many studies were presented to discuss the MHD flow with variable temperature. Sinha et al. (2017). presented an exact solution to the problem of the natural convective flow of an optically thin viscous incompressible electrically conducting fluid past a vertical plate with constant velocity with ramped wall temperature. The unsteady MHD free convection flow of an incompressible viscous fluid over a vertical plate with ramped temperature was studied by Shah et al. (2018). Rajput and Kumar (2011) studied an MHD flow past an impulsively started vertical plate with constant velocity and variable temperature. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. Neog and Das (2012) have studied the effect of magnetic field on transient free convection flow of an electrically conducting fluid over an impulsively started isothermal vertical plate with variable temperature and with chemical reaction. Muthucumaraswamy et al. (2013) have studied MHD flow past an incompressible fluid past a uniformly accelerated infinite vertical plate with variable heat and mass transfer, in the presence of rotation. The plate temperature was raised linearly with time. Hemamalini and Kumar (2015) studied the effect of the transient flow of an incompressible viscous fluid past a uniformly accelerated infinite vertical porous plate in the presence of the variable temperature. MHD flow past an impulsively started oscillating vertical plate with variable temperature and constant mass diffusion in the presence of Hall current is studied was studied by Rajput and Kanaujia (2016). Paul (2017) presented an analytical solution of one-dimensional unsteady laminar boundary layer MHD flow of a viscous incompressible fluid past an exponentially accelerated infinite vertical plate in presence of transverse magnetic field. The motion of the plate was a rectilinear translation with an arbitrary time-dependent velocity.

In the previous studies the temperature is considered to be varied linearly with time. Other studies have been carried out considering different surface temperature profiles. Javaherdeh et al. (2015) presented a numerical solution of two-dimensional steady laminar free convection flow with heat and mass transfer past a moving vertical plate in a porous medium subjected to a transverse magnetic field. The temperature at the plate surface were assumed to follow a power law type of distribution. An analytical study of the transient hydromagnetic and thermal behaviour of free convection flow past a fixed vertical plate embedded in a porous medium with oscillating temperature has been presented by Chaudhary and Jain (2008).

The purpose of this paper is to study the transient free convection magnetohydrodynamic flow past an accelerated vertical plate with periodic plate temperature. A numerical analysis of the dimensionless equaions is performed and verified by an analytical solution for constant plate temperature.

2. Mathematical formulation

Consider a two-dimensional MHD flow of an incompressible electrically conducting viscous fluid past an accelerated infinite vertical plate. A conductive liquid with a density ρ, a dynamic viscosity μ, and an electrical conductivity σ fills the region around the plate. The coordinates x, y are aligned, respectively, along the plate axis, and width. The plate is placed in a uniform transverse magnetic field of flux density B_o in the y direction. When applying the magnetic field with a presence of periodic plate temperature, the transient governing equations of the MHD flow that present the fluid motion and temperature are:

$$\frac{\partial u}{\partial t} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{1}$$

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} \tag{2}$$

With initial and boundary conditions:

$t \le 0$: $u = 0$ $\quad T = T_\infty$ for all y

$t > 0$: $\quad u = \left(\frac{u_0^3}{\nu}\right)t \quad T = T_w + \epsilon(T_w - T_\infty)cos\varpi t \quad at\ y = 0$

$$u \to 0 \quad T \to T_\infty \quad as\ y \to \infty \tag{3}$$

Where T, T_w and T_∞ denote the temperature, wall temperature and initial fluid temperature respectively, u is the velocity component in x direction, K is the thermal conductivity, C_p is the specific heat, ϖ is the frequency of oscillation and ε is a small reference parameter.

The governing equations can be written in dimensionless form using the following non-dimensional quantities:

$$Y = \frac{yu_0}{\nu},\ \tau = \frac{tu_0^2}{\nu},\ \omega = \frac{\nu\varpi}{u_0^2},\quad U = \frac{u}{u_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$M = \frac{\sigma\nu B_0^2}{\rho u_0^2},\ Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\quad Pr = \frac{\mu c_p}{k} \tag{4}$$

Where τ, ω, U, θ, M, Gr and Pr are dimensionless time, dimensionless frequency, dimensionless velocity, dimensionless temperature, magnetic parameter, Grashof number and Prandtl number, respectively.

The dimensionless equations becomes:

$$\frac{\partial U}{\partial \tau} = Gr\theta + \frac{\partial^2 U}{\partial Y^2} - MU \tag{5}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{6}$$

The corresponding boundary conditions can be specified as follows:

Initially, $\tau \le 0$: $U = 0 \quad \theta = 0$ for all Y

$$\tau > 0:\ U = \tau \quad \theta = 1 + \epsilon cos\omega t \quad at\ Y = 0 \tag{7}$$

$$U \to 0 \quad \theta \to 0 \quad as\ Y \to \infty$$

3. Numerical analysis

The finite difference technique is applied to solve the dimensionless momentum and energy equations. A uniform grid consists of 1000 nodes in the y direction is used.
The method with a large amount of time steps is used. At each time the implicit solutions using Crank-Niclson technique are directly obtained from the use of Thomas algorithm.

4. Code validation

The numerical solution can be verified by comparing it with analytical solution in case of constant wall temoerature (ε=0). The analytical eigenfunction expansion method is used to solve the energy equation.
The dimensionless time dependent energy equation can be written as:

$$\frac{\partial\theta}{\partial\tau} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial Y^2} \tag{8}$$

The boundary conditions for constant wall temperature can be written as follows:

Initially, $\tau \leq 0$: $\theta = 0$ for all Y

$\tau > 0$: $\theta = 1 \quad at\ Y = 0$ (9)

$\theta \to 0 \quad as\ Y \to \infty$

Since the boundary conditions are non homogeneous, we can convert them to homogeneous by introducing:

$$F(Y,\tau) = \theta(Y,\tau) - \left(1 - \frac{Y}{\infty}\right) \tag{10}$$

Then the energy equation becomes:

$$\frac{\partial F}{\partial\tau} = \frac{1}{Pr}\frac{\partial^2 F}{\partial Y^2} \tag{11}$$

The boundary conditions can be specified as follows.

Initially, $\tau \leq 0$: $F = \left(\frac{Y}{\infty} - 1\right)$ for all Y

$\tau > 0$: $F = 0 \quad at\ Y = 0$ (12)

$F \to 0 \quad as\ Y \to \infty$

Let $F(Y,\tau) = \emptyset(Y)\delta(\tau)$

Taking the derivatives and substituting into Eq(11) yields the eigenvalue problem as:

$$\frac{d^2\emptyset}{dY^2} + \lambda\emptyset = 0 \qquad \emptyset(0) = \emptyset(\infty) = 0 \tag{13}$$

Where λ is the separation constant. The solution of the above equation will be

$$\emptyset_n(Y) = sin\frac{n\pi}{\infty}Y \tag{14}$$

With eigenvalues

$$\lambda_n = \left(\frac{n\pi}{\infty}\right)^2 \tag{15}$$

Then the general solution is:

$$F(Y,\tau) = \sum_{n=1}^{\infty} B_n\, sin\left(\frac{n\pi Y}{\infty}\right) e^{\frac{-\left(\frac{n\pi}{\infty}\right)^2}{pr}\tau} \tag{16}$$

Which will satisfy the non-homogeneous initial condition $F(Y,0) = \left(\frac{Y}{\infty} - 1\right)$.

Hence

$$B_n = \frac{-2}{n\pi}$$

Where $n = 1,2, \dots\dots, \infty$

Then the final form of solution is

$$\theta(Y,\tau) = \left(\sum_{n=1}^{\infty} \frac{-2}{n\pi} sin\left(\frac{n\pi Y}{\infty}\right) e^{\frac{-\left(\frac{n\pi}{\infty}\right)^2}{pr}\tau} \right) + \left(1 - \frac{Y}{\infty}\right) \tag{17}$$

5. Results and discussion

In this section, the effect of different parameters on the transient velocity and temperature profiles is discussed. The numerical solution using crank-Niclson technique for the velocity and temperature profiles is computed for different values of magnetic parameter, Prandtl number, Grashof number, and temperature frequency. The following parameter values are used to get the results: Gr = 5, Pr = 7, M = 1, ω = 0.2, ε = 0.001, τ = 0.6.

Figure 1a illustrate the effect of Grashof number on the dimensionless velocity profile. It is seen that the effect of increasing Gr is to increase the velocity component U when all other parameters are held constant. Also it is seen that as we move away from the plate the effect of *Gr* is not that significant.

The effect of Magnetic field on the velocity profile has been studied in Figure 1b. It is seen that the increase in the applied magnetic intensity contributes to the decrease in the velocity. Further, it is seen that the magnetic effect does not contribute significantly as we move away from the plate.

Figures 2a and 2b show the effect of Prandtl number on both the velocity and temperature profiles. It is noticed that increasing Pr will decrease the velocity and temperature.

The effect of Prandtl number on the velocity profiles has been illustrated in Figure 2a. The dispersion in the velocity profile is found to be more significant for smaller values of Pr and not that significant at higher values of Prandtl number.

The effect of Prandtl number on the temperature field has been illustrated in Figure 2b. It is observed that as the Prandtl number increases, the temperature in the fluid decreases. Also, as we move away from the plate, the Prandtl number has a significant influence on the temperature for smaller values of Prandtl number.

The transient velocity and temperature profiles have been showed in Figures 3a and 3b for different locations on the fluid. It is noticed that the effect of increasing time is to increase the velocity and temperature. It is noticed also that only the temperature has the oscillatory behavior nearby the plate and it decays as moving away from the plate. It is seen also that the periodicity of the plate temperature doesnot affect the velocity profile.

To study the effect of plate temperature frequency on the transient temperature profile, the pulsed volume must be cleared and so ε is taken to be 0.2. Figure 4a explains that at high frequency, the temperature seems to be continuous instead of pulsating behavior.

A comparison of the transient temperature results that comes from Crank-Niclson solution (Fully implicit method) with analytical solution in the case of ε=0 for temperature profile is shown in Figure 4b. It is seen that the results are in very good agreement with each other.

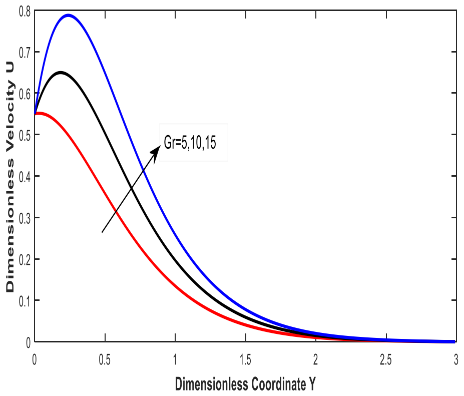

Figure 1a: Velocity profile for different Grashof number

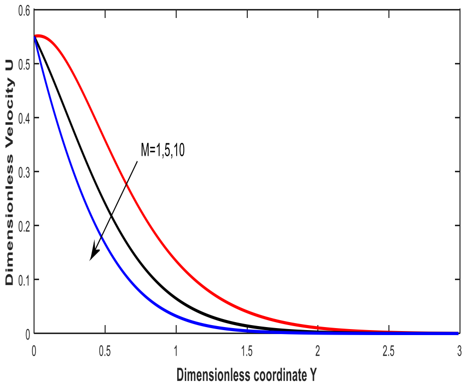

Figure 1b: Velocity profile for different magnetic parameter

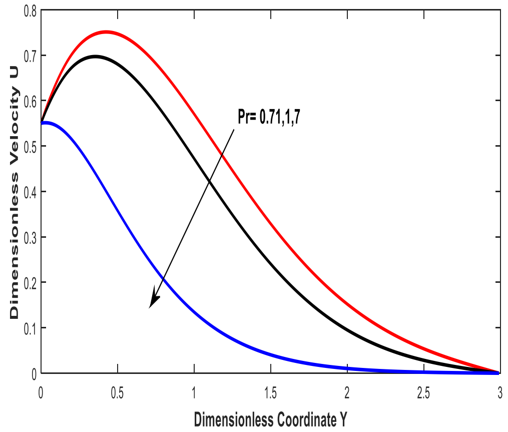

Figure 2a: Velocity profile for different Prandtl number

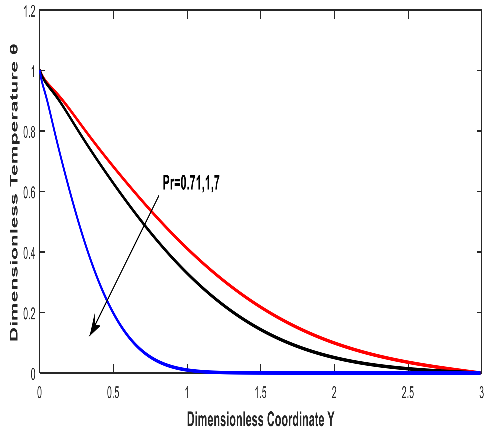

Figure 2b: Temperature profile for different Prandtl number

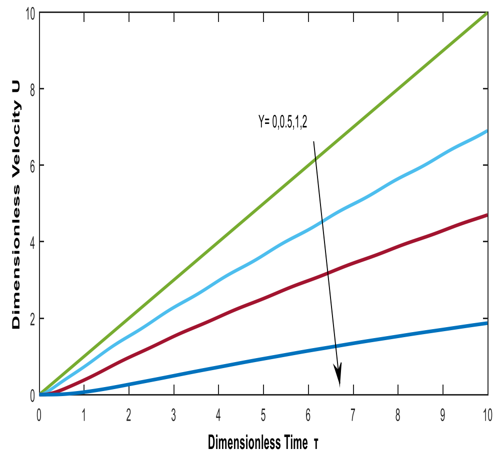

Figure 3a: Transient velocity at different points on the Y coordinate

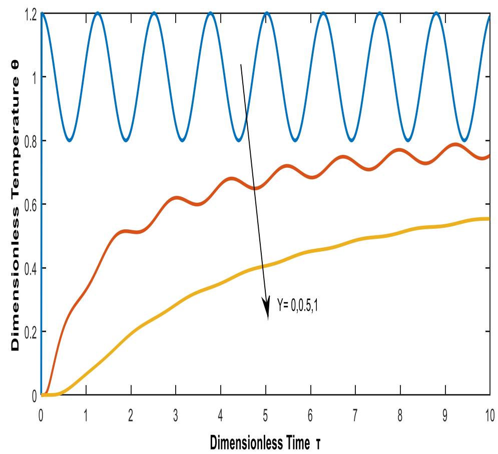

Figure 3b: Transient temperature at different points on the Y coordinate

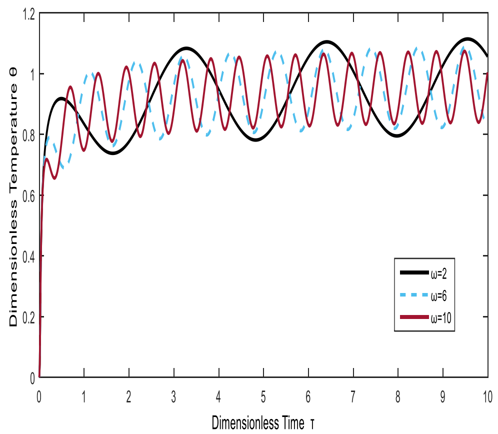

Figure 4a: Transient temperature for different values of temparature frequency

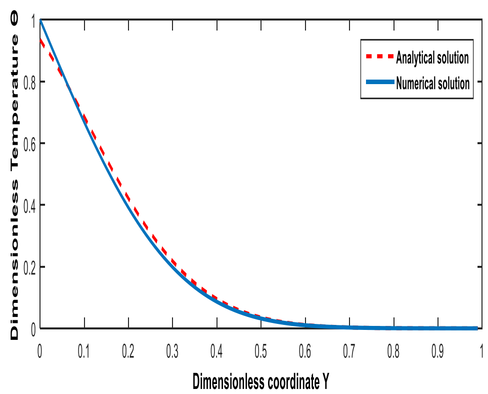

Figure 4b: Comparison of temperature numerical and analytical results

6. Conclusions

A numerical method is performed to predict the velocity and temperature distribution in a free convection flow past an accelerated vertical plate with periodic temperature.

The effect of Grashof number, magnetic parameter, Prandtl number, and temperature frequency on the transient velocity and temperature profiles is studied. The conclusions can be summarized in the following points:

1. It is seen that the velocity decreases as the magnetic parameter increases, which means that a slightly conductive fluid is not affected by the presence of magnetic field.
2. It is seen that increasing the Grashof number will cause an increase of the velocity around the plate.
3. It is seen that increasing the Prandtl number will decrease the velocity and temperature.
4. The periodic behavior of the plate temperature is reflected on transient temperature profile, while this behavior is not observable in velocity profile.

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