

Statistical Homogeneity Zoning of a High and Steep Slope Based on Chi-Square Test

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This paper aims to develop an objective and rational method for statistical homogeneity zoning of high and steep slopes. For this purpose, the fracture trace length was introduced to the statistical homogeneity zoning based on the statistical similarity among the samples. Besides, a chi-square contingency table was established for fractured trace length. The established zoning method was applied to a typical high and steep slope near a chemical plant in Jilin Province, China. The results show that the zoning differs greatly among fractures of the same occurrence but different lengths. The research findings lay a solid theoretical basis for the slope stability analysis of chemical plant.

1. Introduction

In the long process of geological formation, it formed a large number of joints and fissures in rock mass system, the joint fissure system can greatly determine the characteristics of fractured rock mass, which as known as discontinuity, inhomogeneity and anisotropy (Wu, 1993). The discontinuity is different between different geological unit, it mainly depends on the change of the fracture parameters. In rock engineering geological problems, the t stitch length, spacing, occurrence, width and the filling material composition of structure flat are uneven and uncertainly distributed in the rock mass (Chen et al., 1996). In statistical sense, the deformation and destruction have significant difference between different occurrence, trace long geological unit. Make the "similar" parameters of rock fracture into unification statistical homogeneity zone, which can greatly reduce work load analysis of rock mass.

So far, the fractured rock mass statistical homogeneity zone research is still few (Lu et al., 2005). First man suggested statistically homogeneous zone calculation is Miller, which make statistical homogeneity zone by number of joints and fissures. Shanley M. Miller took theory of relational tables and schmidt, gave the method of dividing statistical homogeneity zone of rock mass. Mahtab (1984) presented the method of projecting the discontinuity surface on the unit circle to divide the statistical homogeneity zone of rock mass. Chen et al. (1996) improved above methods and use the statistical homogeneous zone division into the practical engineering. Kulatilake Field et al. (1997) used method of fractal to divide the fractured rock mass statistical homogeneity zone. Gao et al. (2010) used rock mass structure theory of Miller, divided the rock mass statistical homogeneity zone of powerhouse based on structure surface occurrence and density. Fan et al. (2003) proposed a method of dividing fractured rock mass statistical homogeneity zone based on structure surface density, which combining with a large hydropower station in southwest China.

The slope stability is critical to the safety of chemical plant. Thus, it is meaningful to perform statistical homogeneity zoning on the slope. The previous research mainly considers the occurrence of fractures, rather than the fracture trace length. To make up for the gap, the fracture trace length was introduced to the statistical homogeneity zoning based on the statistical similarity among the samples. To ensure the objectiveness and rationality of the results, a chi-square contingency table was established for fractured trace length.

2. Methodology

Chi-square test is a kind of method which to compare two or more than two samples' rate (or ratio) of the difference between the significance test. When comparing the difference of samples was caused by the inner factors rather than caused by sampling error, chi-square value is big, meanwhile P is small which is the probability of the difference caused by sampling error sample. Then just say two sample difference is "significant" or "highly significant". Instead, a chi-square value is smaller, the greater the P value, said there was "no significant" difference of two sample.

In practice, a contingency table model can be used to calculate. There are R and C columns model in contingency table, the contingency table is shown as Table 1.

Table 1: Contingency table model

Group Level	1	2	...	C	Total
1	$f_{11}(p_{11})$	$f_{12}(p_{12})$...	$f_{1c}(p_{1c})$	G_1
2	$f_{21}(p_{21})$	$f_{22}(p_{22})$...	$f_{2c}(p_{2c})$	G_2
...
R	$f_{R1}(p_{R1})$	$f_{R2}(p_{R2})$...	$f_{Rc}(p_{Rc})$	G_R
Total	N_1	N_2	...	N_c	N

In the table, f_{Rc} is the R group of the C level of the sample frequency (R=1, 2, ... C=1,2). p_{Rc} is the R group of the C level the percentage (rates) of the sample. C_R is the first sum of frequency of R samples of all levels. N_c is each group first C level is the sum of the sample frequency.

Chi-square value is calculated by the formula below:

$$\chi^2 = N(a_1 + a_2 + \dots + a_c - 1) \tag{1}$$

$$a_c = N_c(f_{1c}p_{1c} + f_{2c}p_{2c} + \dots + f_{Rc}p_{Rc}) \tag{2}$$

When Chi-square value is calculated, according to the degrees of freedom $n = (R - 1) (C - 1)$ to check the card value table, thus discriminant significance of difference between each sample. While $\chi^2 > 20.01$, $P < 0.01$, the samples are highly significant, while $20.01 > \chi^2 > 20.0$, $0.01 < P < 0.05$, the disparity are significant, while $\chi^2 < 20.05$, $P > 0.05$, the disparity are not significant.

The research object is a typical high and steep slope near a chemical plant in Jilin Province, China. The slope was considered as the target of statistical homogeneity zoning. The target slope was divided into 5 sections (Figure 1 ~ 5) and subjected to contingency table chi-square test. During the test, both fracture trace length and occurrence were considered at the same time. The sample areas were tested in pairs to see if there is any significant difference between them. If the chi-square value between two sample areas is lower than the critical chi-square value, there is no significant difference between the two sections. In this case, the two sample areas should be merged as a statistical homogeneous area. Taking slope section I (0 ~ 140 m) as an example, the chi-square test was carried out in the following steps.

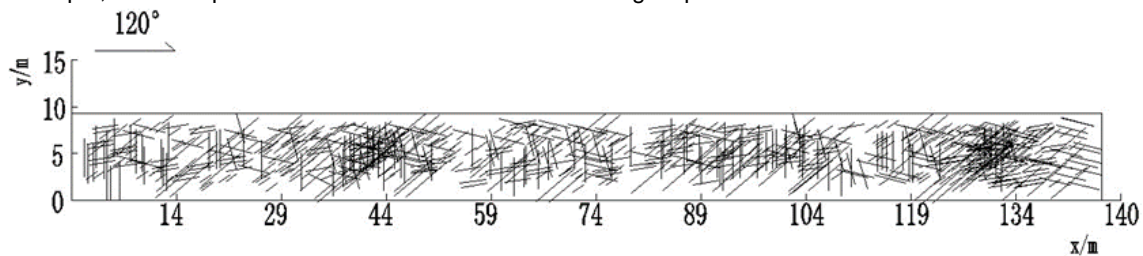


Figure 1: 2-D tracing line graph of section I

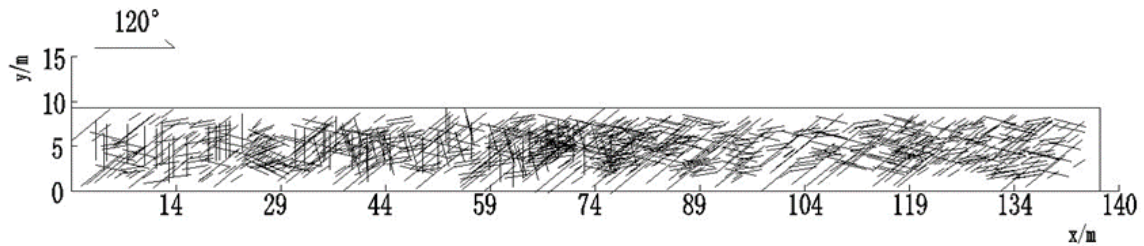


Figure 2: 2-D tracing line graph of section II

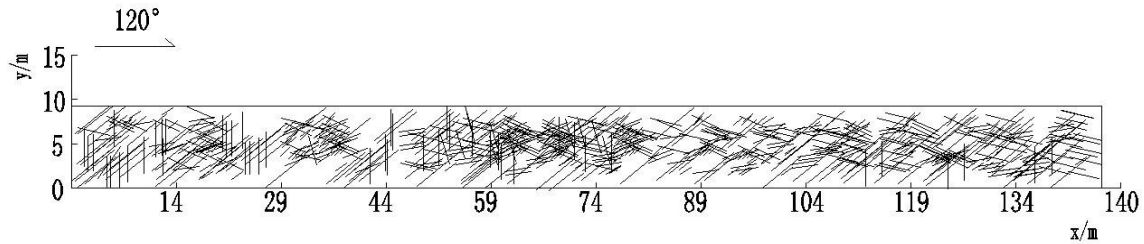


Figure 3: 2-D tracing line graph of section III

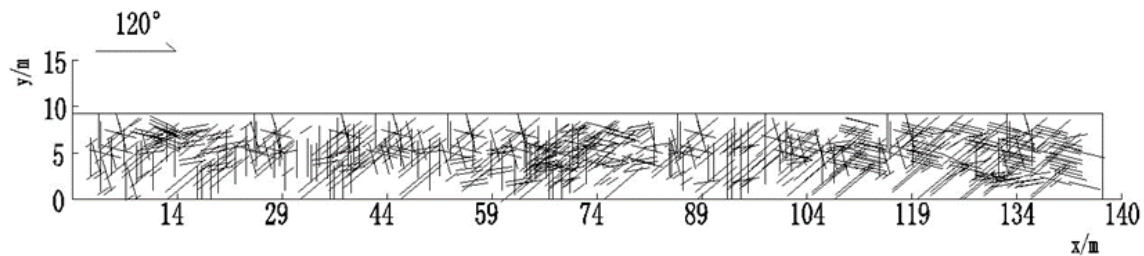


Figure 4: 2-D tracing line graph of section IV

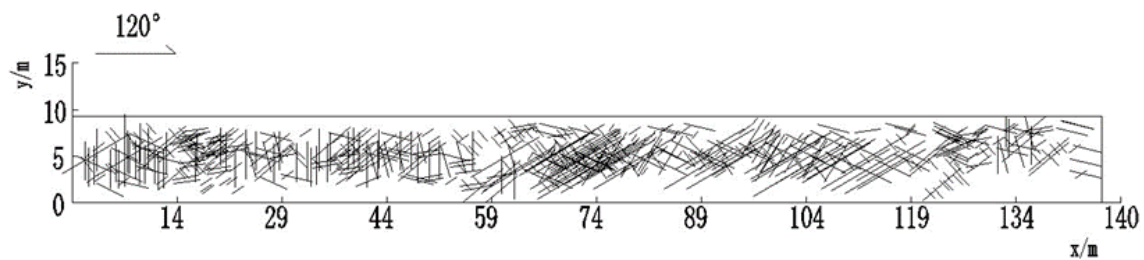


Figure 5: 2-D tracing line graph of section V

(1) Sample selection

To satisfy the sampling frequency $S \geq 50$ of the chi-square test, the samples were collected at an interval of 14 m. As shown in Figure 1, a total of 10 contrast samples were selected, namely (0 m, 14 m), (14 m, 28 m), (126 m, 140 m).

(2) Frequency calculation

The mean length of fracture traces of the contrast samples (0 m, 14 m) and (14 m, 28 m) were calculated separately, and the fracture trace lengths were divided into n different intervals. The number of fractures must exceed 5. Otherwise, the fractures should be added to the next interval. For instance, if the fracture trace length in (0 m, 14 m) falls in the range of (0.5 m, 1.5 m), the fracture length should be divided into two categories: (0.5 m, 1.0 m) and (1.0 m, 1.5 m).

(3) Establishment of contingency table

The fracture samples of (0 m, 14 m) and (14 m, 28 m) are presented in Table 2.

Table 2: Contingency tables chi-square test of 2 samples

Sample	0~0.5	0.5~1	1~1.5	1.5~2	>2
(0m~14m)	5	20	15	10	7
(14m~28m)	3	7	26	7	9

(4) Determination of statistical similarity

The chi-square value and critical chi-square value of the contrast samples were calculated. Based on these values, the statistical similarity was evaluated for the samples.

According to the contingency table (Table 2), the chi-square value of fracture trace length for intervals (0 m~14 m) and (14 m~28 m) was 6.2. In addition, the degree of freedom was 4, the confidence coefficient was 0.05, and the critical chi-square value was determined as 9.49 by looking up the chi-square distribution table.

Through the above analysis, it is clear that the chi-square value was greater than the critical chi-square value for the selected constative samples. Thus, there is statistical similarities between the fractures of (0 m~14 m) and (14 m~28 m). As a result, (0 m~14 m) can be merged with (14 m~28 m), forming a statistical homogenous zone (0 m~28 m). If the selected samples fail to meet the requirements of statistical homogeneity zone, the contrastive samples should be reselected in the step below.

(5) Reselection of contrastive samples

The contrastive samples were reselected, e.g. (14 m, 28 m) and (28 m, 42 m), and subjected to the Steps 2~4 above. The purpose is to determine if the samples have statistical similarity and can be combined into a statistical homogeneous area. All the fracture trace length intervals were compared through the above steps.

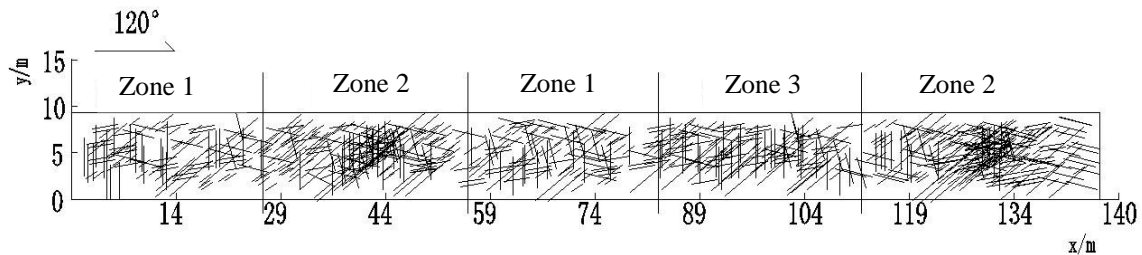


Figure 6: Statistical homogeneous zoning of section I

3. Result Analysis

3.1 Statistical homogeneity zoning of slope section I

Different samples were selected and compared through the above steps. The degree of similarity (i.e. the significance of difference) of these samples are presented in Table 3.

Table 3: Chi-square test results of adjacent samples (Section I)

Contrast sample (m)	Freedom degree	significance level	critical chi-square value	chi-square value	Whether is statistical homogeneous zone
0~14&14~28	4	0.05	9.49	6.2	Y (Zone 1)
14~28&28~42	4	0.05	9.49	11.2	N
28~42&42~56	4	0.05	9.49	5.6	Y (Zone 2)
42~56&56~70	4	0.05	9.49	9.82	N
56~70&70~84	4	0.05	9.49	6.4	Y (Zone 2)
70~84&84~98	4	0.05	9.49	12.3	N
84~98&98~112	4	0.05	9.49	8.5	Y (Zone 3)
98~112&112~126	4	0.05	9.49	11.6	N
112~126&126~140	4	0.05	9.49	5.8	Y (Zone 2)

According to the results in Table 3, the following conclusions can be derived:

(1) For samples (0 m, 14 m) and (14 m, 28 m) and samples (56 m, 70 m) and (70 m and 70 m), the Chi-square value was smaller than the critical chi-square value. The chi-square values of each pair of samples were similar, which satisfies the requirements for sample similarity. In other words, there is no significant

difference between the pairs of samples. Thus, the samples (0 m, 14 m) and (14 m, 28 m), and samples (56 m, 70 m) and (70 m and 70 m) can be allocated to statistical homogeneity zone 1.

(2) For samples (28 m, 42 m) and (42 m, 56 m) and samples (112 m, 112 m) and (126 m and 126 m), the Chi-square value was smaller than the critical chi-square value. The chi-square values of each pair of samples were similar, which satisfies the requirements for sample similarity. In other words, there is no significant difference between the pairs of samples. Thus, the samples (28 m, 42 m) and (42 m, 56 m) and samples (112 m, 112 m) and (126 m and 126 m) can be allocated to statistical homogeneity zone 2.

(3) For samples (84 m, 84 m) and (98 m and 98 m), the Chi-square value was smaller than the critical chi-square value. The chi-square values of the two samples were similar, which satisfies the requirements for sample similarity. In other words, there is no significant difference between the pairs of samples. Thus, the samples (84 m, 84 m) and (98 m and 98 m) can be allocated to statistical homogeneity zone 3.

The above steps were repeated to obtain the results for all samples. The 2D fracture trace lengths and the zoning results of slope section I are illustrated in Figure 1.

3.2 Statistical homogeneity zoning of slope sections II~V

Table 4: Chi-square test results of adjacent samples (Section II~V)

Section	Statistical homogeneous zone 1		Statistical homogeneous zone 2		Statistical homogeneous zone 3	
II	0~18 m	90~110 m	19~55 m	110~140 m	55~90 m	N
III	0~31 m	79~103 m	31~49 m	103~140 m	49~79 m	N
IV	0~28 m	112~140 m	28~56 m	86~112 m	56~86 m	N
V	0~56 m	112~140 m	56~86 m	N	86~112 m	N

Through the above steps of statistical homogeneity zoning, the 2D trace lengths and the zoning results of sections II ~ V are presented in Figures 7~10.

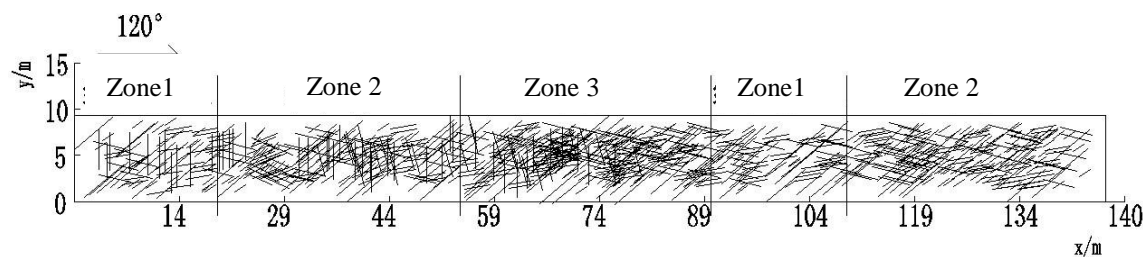


Figure 7: Statistical homogeneous zoning of section II

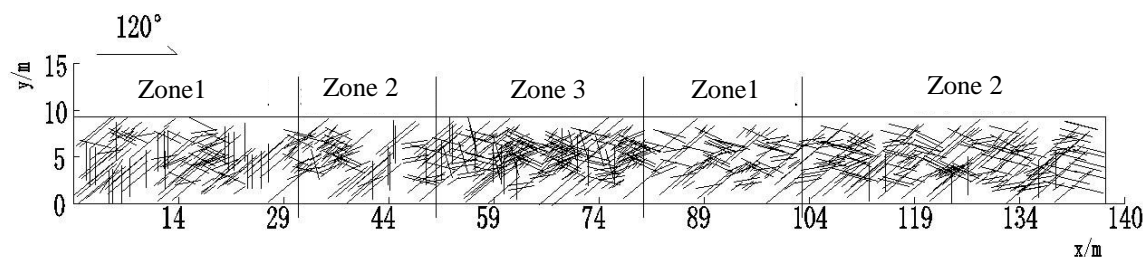


Figure 8: Statistical homogeneous zoning of section III

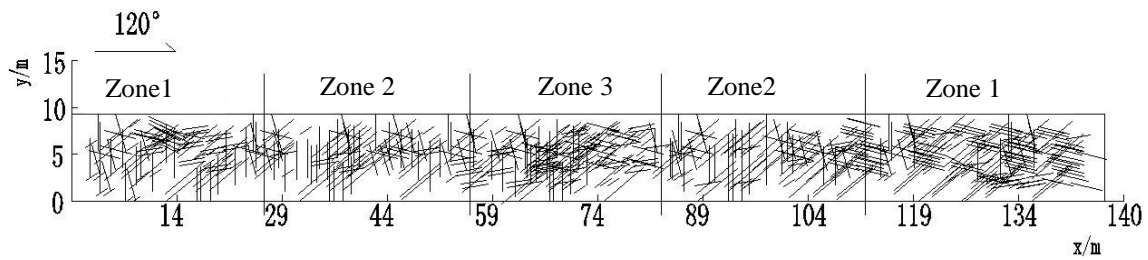


Figure 9: Statistical homogeneous zoning of section IV

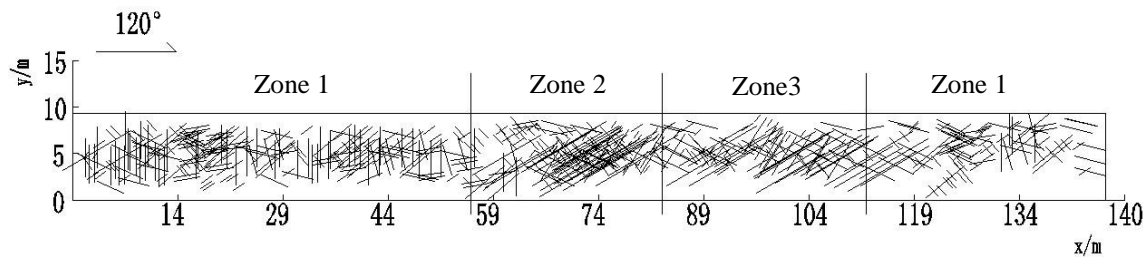


Figure 10: Statistical homogeneous zoning of section V

4. Conclusions

This paper introduces the fracture trace length to the statistical homogeneity zoning of a high and steep slope in a chemical plant. Then, the chi-square test was carried out to complete the statistical homogeneity zoning based on the fracture trace length and occurrence. The results show that the zoning differs greatly among fractures of the same occurrence but different lengths. The research findings lay a solid theoretical basis for the slope stability analysis of chemical plant.

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References

- Wu F.Q., 1993, Statistical Rock Mass Mechanics Theory, University of Geosciences Press, Wuhan, China.
- Chen J.P., Wang Q., Xiao S.F., 1996, An Application of Chi-Square Test Validity of Assumed Distribution in the Engineering Geology, Journal of Jilin University: Earth Science Edition, 26, 332-335, DOI: 10.13278/j.cnki.jjuese.1996.03.016.
- Lu B., Chen J.P., Ge X.R., 2005, Fractal Geometry Study on Structure of Jointed Rock mass, Chinese Journal of Rock Mechanics and Engineering, 24, 461-467, DOI: 10.3321/j.issn:1000-6915.2005.03.016.
- Kulatilake P.H.S.W., Fiedler R., Panda B.B., 1997, Box Fractal Dimension as a Measure of Statistical Homogeneity of Jointed Rock masses, Engineering Geology, 48, 217-229, DOI: 10.1016/S0013-7952(97)00045-8.
- Gao J., Yang C.H., Wang G.B., 2010, Discussion on Zoning Method of Structural Homogeneity of Rock Mass in Beishan of Gansu Province, Rock and oil Mechanics, 31, 588-598, DOI: 10.16285/j.rsm.2010.02.058.
- Fan L.M., Huang R.Q., Ding X.N., 2003, Analysis on Structural Homogeneity of Rock Mass Based on Discontinuity Density, Rock Mechanics and Engineering, 22, 1132-1136, DOI: 10.3321/j.issn:1000-6915.2003.07.016.