

# Mathematical Modelling and Optimization for Integrated Production and Energy System in an Iron and Steel Plant

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In this paper, we address a simultaneous optimization problem of integrated production and energy system in an iron and steel plant. We develop a novel multi-period mixed integer linear optimization model incorporating many realistic operational features such as industrial boilers, steam turbines, combined heat and power generation units and waste heat recovery and power generation units. A case study of a real iron and steel plant demonstrates that compared to the realistic operational strategy, the total operational cost is reduced by 6.25 % using the proposed model.

## 1. Introduction

The iron and steel industry is one of the energy-intensive industries with high CO<sub>2</sub> emissions. As reported (Zhang, 2008), the energy cost in the iron and steel industry accounts for about 20% of the total operating cost. In recent years, iron and steel industries are facing great pressure to reduce their operating cost, improve energy efficiency, reduce CO<sub>2</sub> emissions, and become more competitive in the global market, which drives them to seek advanced techniques to improve their planning operations.

In an iron and steel plant, production units such as coke ovens, sintering furnace, blast furnaces, hot stoves, basic oxygen furnaces are usually integrated with its energy system. Optimal planning of such integrated system has several advantages such as increasing profit margin, decreasing operating cost, improving energy efficiency, and reducing CO<sub>2</sub> emissions. However, optimal planning of such integrated system is not trivial since it involves many production units, fuel boilers, steam turbines, combined heat and power (CHP) units and waste heat and energy recovery and generation (WHERG) units, resulting in many operations including steelmaking, rolling, steam and power generation, byproduct gas storage and distribution, and burner switching operations. More important, generation rates of byproduct gases and demands of byproduct gases, steam and electricity from production units vary from time to time, increasing the complexity of such integrated system. The burner switching operations involve the monitoring of number of burners whose status changes at each time, leading to a combinatorial problem.

Many research efforts have been made on planning and scheduling of production processes such as steelmaking and continuous casting processes (Li et al., 2012) and cold rolling process (Tang et al., 2016) without considering the energy system. Since an iron and steel plant is energy intensive, several researchers have proposed some methods for the calculation of energy consumption (Ansari and Seifi, 2012) and energy efficiency (Wei et al., 2007). Several energy saving measures for CO<sub>2</sub> emissions mitigation were investigated and their impact on productive efficiency were analyzed (Zhang and Wang, 2008). These energy saving measures were evaluated from a system perspective in relation to each other without optimization (Johansson and Söderström, 2011). Although some researchers have focused on the optimization of byproduct gas system considering effect of penalty factors (Zhao et al., 2015), the turn-down ratio of boiler burners (de Oliveira Junior et al., 2016), suitable capacity for buffer users (Yang et al., 2017), and steam and power generation system (Zeng and Sun, 2015) in an iron and steel plant, several realistic features are missing such as CHP and WHERG units for steam and electricity generation, minimum heating value requirements, ramp rates variations, piecewise constant demand profiles of byproduct gases, steam and electricity. In this paper,

we develop a multi-period mixed integer linear programming (MILP) model for simultaneous optimization of integrated production and energy system in an iron and steel plant. Many realistic features such as byproduct gas distribution and storage, steam and power generation system, CHP and WHERG units, dedicated byproduct gasholders, boiler burner switching operations, minimum heating values, ramp rate variations, and piecewise constant demand profiles of byproduct gases, steam and electricity are incorporated into the model. The computational results demonstrate that the proposed model solves an industrial example to optimality within 2 CPU second. The total operational cost is reduced by 6.25% using the proposed model compared to that from actual operation.

## 2. Problem definition

Figure 1 illustrates a schematic diagram of integrated production and energy system in a typical iron and steel plant. There are total  $U$  ( $u = 1, 2, \dots, U$ ) production units,  $I$  ( $i = 1, 2, 3, \dots, I$ ) fuel boilers,  $J$  ( $j = 1, 2, \dots, J$ ) steam turbines,  $K$  ( $k = 1, 2, \dots, K$ ) CHP units, and  $M$  ( $m = 1, 2, \dots, M$ ) WHERG units. The fuel boilers in a CHP unit are denoted as  $I_k$ .  $Q$  ( $q = 1, 2, \dots, Q$ ) types of energy sources such as by-product gas, coal, natural gas, and fuel oil can be used in fuel boilers and some production units as fuels. By-product gases generated from coke ovens, blast furnace, and basic oxygen furnaces are called COG, BFG, and LDG, respectively, which are included into a set  $\mathbf{Q}^g$ . By-product gases are either provided for production units, boilers, and CHP units as fuels, or stored in their dedicated gasholders. Total  $R$  ( $r = 1, 2, \dots, R$ ) levels of steam are generated from fresh water in boilers, CHP and WHERG units, and consumed in production units and turbines. The electricity generated from turbines, CHP and WHERG units is supplied for production units. While excess electricity is sold to the grid, insufficient electricity is purchased from the grid. The entire problem is defined as follows,

**Given:** (1) Byproduct gases data including their types, heating values, and generation rate and demand profiles; (2) Data on dedicated gasholders including their capacities, normal inventory levels, threshold inventory levels for low and high operational regions; (3) Data on fuel boilers including their inlet flow rate and steam generation rate limits, thermal efficiency, and minimum heating values; (4) Data on burners including suitable byproduct gases, feed rates, and initial status; (5) Data on steam turbines including suitable steams, thermal efficiency, limits on feed rates, steam and power generation rates; (6) Data on CHP units including thermal efficiency, limits on feed rates, steam and power generation rates, minimum heating values, (7) Data on WHERG units including amount of heat recovered, thermal efficiency, steam and power generation rate limits, (8) Data on steam and electricity demand profiles, steam enthalpy, and electricity energy content; (9) Economic data including coal, natural gas and electricity purchase cost, electricity sale price, penalty coefficients for byproduct gas emissions and burner switching operations, penalty coefficients for deviations of normal inventory levels and violations of threshold inventory levels of low and high operational regions in gasholders, maintenance cost for steam and power generation units and the planning horizon.

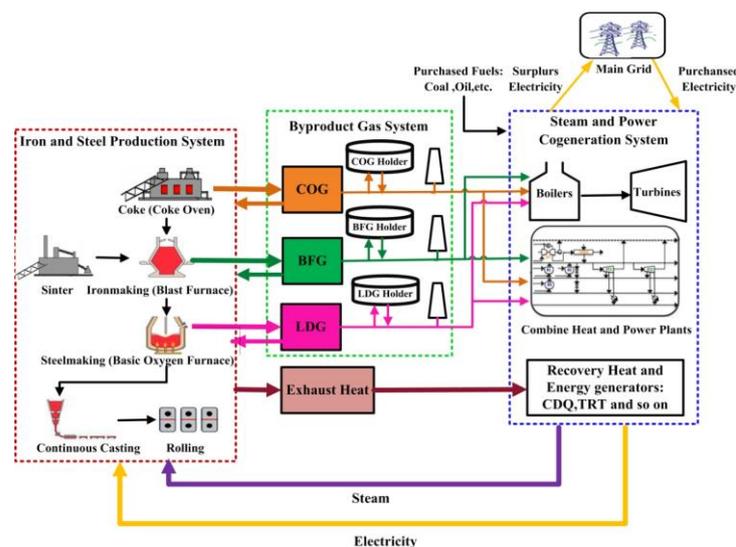


Figure 1: A typical integrated production and energy system in an iron and steel plant

**Determine:** (1) Selection of byproduct gases, coal, and natural gas in boilers; (2) Optimal distribution of byproduct gases among production units, boilers, and CHP units; (3) Inventory profiles of gasholders; (4)

Power generation plan; (5) Detailed operational plan for burners in boilers

**Assumptions:** (1) All parameters are deterministic; (2) Byproduct gas generation rates are piecewise constant; (3) Demands of byproduct gases, steam, and electricity in production units are piecewise constant. Our objective is to minimize total operating cost including purchase cost of coal, natural gas, and cooling water, electricity cost, equipment maintenance cost, and some penalties such as penalty for byproduct gas emissions and penalty for burner switches.

### 3. Mathematical formulation

The entire planning horizon is divided into  $T$  periods ( $t = 1, 2, \dots, T$ ) based on the piecewise constant generation rate and demand profiles of byproduct gases, steam and electricity. The length of each period  $t$  is denoted as  $\tau_t$ .

#### 3.1 Utility generation model

(1) Fuel boiler operational model

$$\sum_{r=1}^R (F_{irt} \cdot \tau_t \cdot H_r^{stm} - F_{irt} \cdot H^w \cdot \tau_t) = \eta_i \cdot \left[ \sum_{q=1}^Q (F_{iqt} \cdot \tau_t \cdot HV_q) \right] \quad \forall i \notin \mathbf{l}_k, t \quad (1)$$

$$F_{ir}^{min} \leq F_{irt} \leq F_{ir}^{max} \quad \forall i \notin \mathbf{l}_k, r, t \quad (2)$$

$$F_{iq}^{min} \leq F_{iqt} \leq F_{iq}^{max} \quad \forall i \notin \mathbf{l}_k, t, q \in \mathbf{Q}^g \quad (3)$$

$$\sum_{q \in \mathbf{Q}^g} (F_{iqt} \cdot \tau_t \cdot HV_q) \geq \left[ \sum_{q \in \mathbf{Q}^g} (F_{iqt} \cdot \tau_t) \right] \cdot HV_i^{min} \quad \forall i \notin \mathbf{l}_k, t \quad (4)$$

(2) Steam turbine operational model

$$F_{jt}^{in} \cdot \tau_t = \sum_r (F_{jrt}^{in} \cdot \tau_t) = \sum_r (F_{jrt}^{out} \cdot \tau_t) + F_{jt}^{exh} \cdot \tau_t \quad \forall j, t \quad (5)$$

$$P_{jt} \cdot \tau_t \cdot HC^P = \eta_j \cdot \left[ \sum_{r=1}^R (F_{jrt}^{in} \cdot \tau_t \cdot H_r^{stm}) - \sum_{r=1}^R (F_{jrt}^{out} \cdot \tau_t \cdot H_r^{stm}) - F_{jt}^{exh} \cdot \tau_t \cdot H_j^{exh} \right] \quad \forall j, t \quad (6)$$

$$F_{jr}^{in,min} \leq F_{jrt}^{in} \leq F_{jr}^{in,max} \quad \forall j, r, t \quad (7)$$

$$F_{jr}^{out,min} \leq F_{jrt}^{out} \leq F_{jr}^{out,max} \quad \forall j, r, t \quad (8)$$

$$P_j^{min} \leq P_{jt} \leq P_j^{max} \quad \forall j, t \quad (9)$$

(3) CHP operational model

$$P_{kt} \cdot \tau_t \cdot HC^P + \sum_{r=1}^R (F_{krt} \cdot \tau_t \cdot H_r^{stm} - F_{krt} \cdot \tau_t \cdot H^w) = \eta_k \cdot \left[ \sum_{i \in \mathbf{l}_k} \sum_{q=1}^Q (F_{iqt} \cdot \tau_t \cdot HV_q) \right] \quad \forall k, t \quad (10)$$

$$F_{kq}^{min} \cdot \tau_t \leq \left( \sum_{i \in \mathbf{l}_k} F_{iqt} \right) \cdot \tau_t \leq F_{kq}^{max} \cdot \tau_t \quad \forall k, t, q \in \mathbf{Q}^g \quad (11)$$

$$F_{kr}^{min} \leq F_{krt} \leq F_{kr}^{max} \quad \forall k, r, t \quad (12)$$

$$P_k^{min} \leq P_{kt} \leq P_k^{max} \quad \forall k, t \quad (13)$$

$$\sum_{i \in \mathbf{l}_k} \sum_{q \in \mathbf{Q}^g} (F_{iqt} \cdot \tau_t \cdot HV_q) \geq HV_k^{min} \cdot \left[ \sum_{i \in \mathbf{l}_k} \sum_{q \in \mathbf{Q}^g} (F_{iqt} \cdot \tau_t) \right] \quad \forall k, t \quad (14)$$

(4) WHERG operational model

$$P_{mt} \cdot \tau_t \cdot HC^P + \sum_{r=1}^R (F_{mrt} \cdot \tau_t \cdot H_r^{stm} - F_{mrt} \cdot \tau_t \cdot H^w) = \eta_m \cdot E_{mt}^{in} \quad \forall m, t \quad (15)$$

$$P_{mt}^{\min} \leq P_{mt} \leq P_{mt}^{\max} \quad \forall m, t \quad (16)$$

$$F_{mrt}^{\min} \leq F_{mrt} \leq F_{mrt}^{\max} \quad \forall m, r, t \quad (17)$$

### 3.2 Power, steam and by-product gas balances

$$\sum_{j=1}^J (P_{jt} \cdot \tau_t) + \sum_{k=1}^K (P_{kt} \cdot \tau_t) + \sum_{m=1}^M (P_{mt} \cdot \tau_t) + P_t^{\text{imp}} = P_t^{\text{dem}} + P_t^{\text{exp}} \quad \forall t \quad (18)$$

$$\sum_{i \in I_k} (F_{it} \cdot \tau_t) + \sum_{k=1}^K (F_{krt} \cdot \tau_t) + \sum_{m=1}^M (F_{mrt} \cdot \tau_t) + \left[ \sum_{j=1}^J (F_{jrt}^{\text{in}} \cdot \tau_t) - \sum_{j=1}^J (F_{jrt}^{\text{out}} \cdot \tau_t) \right] = D_{rt}^{\text{dem}} \quad \forall r, t \quad (19)$$

$$Inv_{qt} = Inv_{q(t-1)} + F_{qt}^{\text{Gen}} \cdot \tau_t - \sum_{u=1}^U (F_{uqt} \cdot \tau_t) - \sum_{i=1}^I (F_{iqt} \cdot \tau_t) - Q_{qt}^{\text{emission}} \quad \forall q \in \mathbf{Q}^g, t \quad (20)$$

### 3.3 Gasholder operational model

$$Inv_q^{\min} \leq Inv_{q,t} \leq Inv_q^{\max} \quad \forall q \in \mathbf{Q}^g, t \quad (21)$$

$$Inv_{qt} - Inv_q^N = SInv_{qt}^d - SInv_{qt}^r \quad \forall q \in \mathbf{Q}^g, t \quad (22)$$

$$Inv_q^L - SInv_{qt}^L \leq Inv_{qt} \leq Inv_q^H + SInv_{qt}^H \quad \forall q \in \mathbf{Q}^g, t \quad (23)$$

### 3.4 Burner operational constraints

$$F_{iqt} \cdot \tau_t = F_{iq} \cdot N_{iqt} \cdot \tau_t \quad \forall i, q \in \mathbf{Q}^g, t \quad (24)$$

$$\Delta N_{iqt} \geq N_{iqt} - N_{iqt(t-1)} \quad \forall i, q \in \mathbf{Q}^g, t \quad (25a)$$

$$\Delta N_{iqt} \geq N_{iqt(t-1)} - N_{iqt} \quad \forall i, q \in \mathbf{Q}^g, t \quad (25b)$$

$$\Delta N_{iqt} = ibn1_{iqt}^+ + ibn2_{iqt}^+ + ibn3_{iqt}^+ \quad \forall i, q \in \mathbf{Q}^g, t \quad (26)$$

$$ibn1_{iqt}^+ \geq ibn2_{iqt}^+ \geq ibn3_{iqt}^+ \quad \forall i, q \in \mathbf{Q}^g, t \quad (27)$$

$$ibn0_{iqt} + ibn1_{iqt}^+ = 1 \quad \forall i, q \in \mathbf{Q}^g, t \quad (28)$$

$$ibn0_{iqt} + ibn2_{iqt}^+ \leq 1 \quad \forall i, q \in \mathbf{Q}^g, t \quad (29)$$

$$ibn0_{iqt} + ibn3_{iqt}^+ \leq 1 \quad \forall i, q \in \mathbf{Q}^g, t \quad (30)$$

where,  $ibn0_{iqt}^+$  is 0-1 continuous variables denoting if no burner changes its status in  $t$ ;  $ibn1_{iqt}^+$ ,  $ibn2_{iqt}^+$ ,  $ibn3_{iqt}^+$  are binary variables denotes if one burner, two burners and three burners change status in  $t$ .

### 3.5 Byproduct gas constraints for production units

$$F_{uq}^{\min} \leq F_{uqt} \leq F_{uq}^{\max} \quad \forall u, q \in \mathbf{Q}^g, t \quad (31)$$

$$F_{uqt} \cdot \tau_t \geq D_{uq}^{\text{dem}} \quad \forall u, q \in \mathbf{Q}^g, t \quad (32)$$

$$\sum_{q \in \mathbf{Q}^g} (F_{uqt} \cdot \tau_t \cdot HV_q) \geq HV_u^{\min} \cdot \sum_{q \in \mathbf{Q}^g} (F_{uqt} \cdot \tau_t) \quad \forall u, t, HV_u^{\min} > 0 \quad (33)$$

$$\sum_{q \in \mathbf{Q}^g} (F_{uqt} \cdot \tau_t \cdot HV_q) \geq E_{ut}^{\text{Dem}} \quad \forall u, t, E_{ut}^{\text{Dem}} > 0 \quad (34)$$

### 3.6 Objective function

$$\begin{aligned}
 TC = & \sum_{t=1}^T \sum_{i=1}^{I+K} \sum_{q \in Q^g} (C_q \cdot F_{iqt} \cdot \tau_t) + \sum_{t=1}^T \sum_{i=1}^{I+K} (C^w \cdot F_{i,t}^w \cdot \tau) + \sum_{t=1}^T (C_t^{exp} \times P_t^{exp} \times \tau_t) - \sum_{t=1}^T (C_t^{imp} \times P_t^{imp} \times \tau_t) \\
 & + \sum_{t=1}^T \sum_{q \in Q^g} [(W_q^{emission} \cdot Q_{qt}^{emission})] + \sum_{t=1}^T \sum_{q \in Q^g} [(W_q^L \cdot SInv_{qt}^L + W_q^H \cdot SInv_{qt}^H + W_q^{d'} \cdot SInv_{qt}^{d'} + W_q^{d''} \cdot SInv_{qt}^{d''})] \\
 & + \sum_{t=1}^T \sum_{i=1}^I \sum_{q \in Q^g} [W_q^{SW} (ibn1_{iqt}^+ + ibn2_{iqt}^+ + ibn3_{iqt}^+) + W_q^{2S} \cdot ibn2_{iqt}^+ + W_q^{3S} \cdot ibn3_{iqt}^+] \\
 & + \sum_{t=1}^T \sum_{i=1}^I (C_i^M \cdot F_{it} \cdot \tau_t) + \sum_{t=1}^T \sum_{j=1}^{J+K+M} (C_j^{PM} \cdot P_{jt} \cdot \tau_t)
 \end{aligned} \tag{35}$$

We complete our mixed-integer linear optimization model (MILP) denoted as **M**, which comprises the objective function TC and constraints Eq(1) – Eq(34). The binary variables are used to model switching operations of burners in fuel boilers, and decisions on electricity sale and purchase.

### 4. Computational results

We use the proposed model **M** to solve an industrial example from an iron and steel plant in China. This industrial plant consists of 4 coke ovens, 2 blast furnaces, and 5 basic oxygen furnaces. The energy system is made up of 4 boilers (B1 - B4), 2 steam turbines (TB1 - TB2), 2 CHP units (CHP1 - CHP2), and 2 CDQ units (CDQ1 - CDQ2). Three levels of steams are generated from the boilers, which are high-pressure steam S1 (3.5 MPa), medium-pressure steam S2 (1.0 MPa) and low-pressure steam S3 (0.4 MPa). Three types of byproduct gases are generated, which are COG, BFG, and LDG. Prices of natural, coal, fresh water are 3.5 ¥/m<sup>3</sup>, 500 ¥/t and 10 ¥/t. The horizon is about 6 h and divided into 6 identical periods based on energy demand profiles. This example is solved using CPLEX 12.6.1.0/GAMS 24.4.2 on a Dell Inspiron15 5000 of Intel Core i7-4510U CPU 2.0 GHz with 8GB RAM memory running Windows 7. The optimization model contains 438 binary variables, 918 continuous variables, and 2,293 constraints. The optimal solution of ¥1,285,132 is obtained within 2 CPU s. The optimal distribution of by-product gases in production units and gasholders are illustrated in Figure 2. It is noted that the inventory level of each dedicated gasholder at any time is maintained around its normal operation level. The comparative results from the proposed approach and the actual operation are given in Table 1. The total cost of ¥1,370,252 from the actual operation is reduced by 6.25 % compared to that of ¥1,285,132 using the proposed model. The purchase cost of electricity and coal from the market, and the penalty cost for gasholder deviation from the normal level and burner switches are significantly reduced using the proposed model, although the equipment maintenance cost is slightly increased.

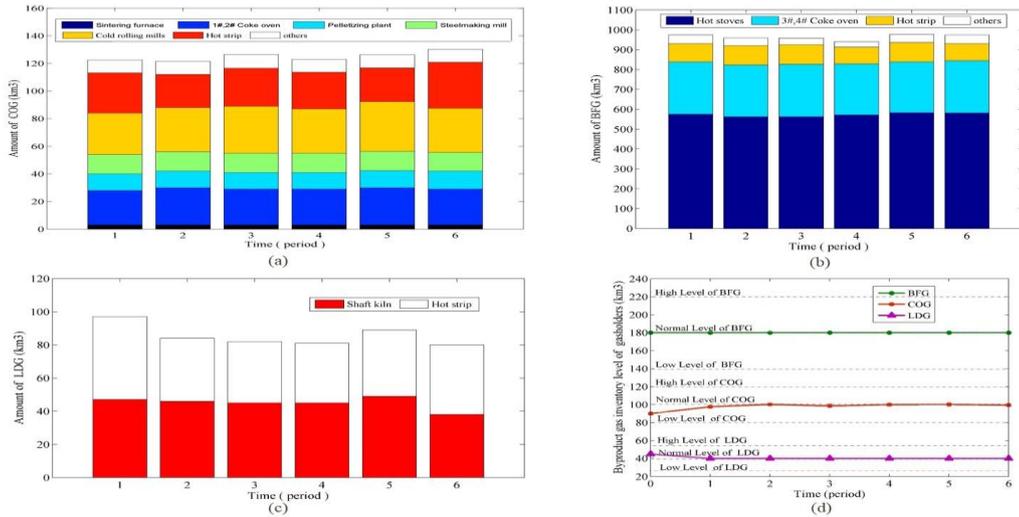


Figure 2: The optimal distribution of byproduct gases in production units and gasholders: (a) COG, (b) BFG, (c) LDG, (d) Inventory profiles of gasholders

Table 1: Comparative results from the proposed model and actual operation

Item	Actual Operation	Proposed Model
Coal cost (¥)	858,560	804,776
Natural gas cost (¥)	0	0
Water cost (¥)	80,246	72,585
Penalty for byproduct gas emissions (¥)	0	0
Penalty for violation of high levels in gasholders (¥)	0	0
Penalty for violation of low levels in gasholders (¥)	0	0
Penalty for deviation of normal levels in gasholders (¥)	15,045	5,089
Penalty for burner switches (¥)	36,800	24,400
Equipment maintenance cost (¥)	316,197	327,588
Electricity purchase cost (¥)	63,404	50,694
Electricity sale revenue (¥)	0	0
Total cost (¥)	1,370,252	1,285,132

## 5. Conclusions

In this paper, a novel multi-period MILP planning model was developed for simultaneous optimization of integrated production and energy system in an iron and steel plant. The proposed model incorporated many realistic operational features such as byproduct gas distribution, boilers and turbines, CHP and WHERG units, dedicated byproduct gasholders, burner switching operations, minimum heating values, ramp rate variations, and piecewise constant demand profiles of byproduct gases, steam and electricity. The results indicate that a reduction of 6.25 % in total operational cost was successfully achieved compared to that from actual operation.

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