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A Method for Group Decision-making with Uncertain Preference Ordinals Based on Probability Matrix

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In this paper, a new method to solve the group decision-making problems is proposed, in which the preference information on alternatives provided by decision makers is in the form of uncertain preference ordinals. This paper firstly gives two new definitions on the probability that the alternative is ranked in each position. Then, in order to process uncertain preference ordinals, two new definitions are used respectively to construct a decision matrix in the form of probabilities. On this basis, a weight probability matrix and a collective probability matrix on alternatives with regard to rank positions are constructed. Finally, an optimization model is built based on the collective probability matrix, and the ranking of alternatives can be obtained by solving the model.

1. Introduction

Multiple criteria decision making (MCDM) is a discipline aimed at supporting decision maker who is faced with numerous and conflicting alternative to make an optimal decision (Pedrycz, 2013; Mardani et al., 2015). While group decision-making (GDM) is decision-making in groups consisting of multiple members. Multiple criteria group decision-making (MCGDM) problem involves a set of feasible alternatives that are evaluated on the basis of multiple, conflicting and non-commensurate criteria by a group of individuals. Meanwhile, it is a problem with extensive theoretical and practical backgrounds in industrial engineering (Soltani et al., 2015; Wijenayake, et al. 2016; Ravindran, 2016).In the real world, due to the complexity and uncertainty of decision-making problems, the limitation of cognition, the estimation inaccuracies and lack of decision makers' knowledge, the preference rankings or ranking ordinals of alternatives provided by decision makers maybe in the form of uncertain preference ordinals. However, few approaches to solve MCGDM problems with preference rankings or uncertain preference ordinals can be found in the existing literature. The existing approaches have made to solve the GDM problems with uncertain preference ordinals on alternatives.

This paper investigates the MCDM problems, where the preference information on alternatives provided by decision makers is in the form of uncertain preference ordinals. In order to solve these problems Fan et al. (2010) gave several definitions on uncertain preference ordinal, constructed a decision matrix in the form of probabilities, and built an optimization model based on the collective probability matrix. Fan's approach can solve the GDM problem effectively. However, it is regarded that each ranking position of an uncertain preference ordinal on alternative has the same probability in Fan's approach but not in line with laws of human cognition. For example, supposing as uncertain preference ordinal of an alternative on interval $[3,7]$, the decision maker often thinks the probability of the alternative ranked in 5th position is higher than in 3th or 7th position (Xu, 2005). In fact, the closer a preference ordinal to the lower bound and upper bound of an uncertain preference ordinal, the smaller the probability that the alternative is ranked in the corresponding position. At the same time, the closer that preference ordinal in the centre of an uncertain preference ordinal, the larger probability that the alternative is ranked in the corresponding position.

Focusing on the above object, this paper will firstly improve Fan's model by giving two new definitions on the probability that the alternative is ranked in each position, which is raised by Wang and Xu (2008). Then, to process uncertain preference ordinals, a matrix in the form of probabilities is constructed. Based on the probability matrix, a weight probability matrix and a collective probability matrix on alternatives with regard to

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rank positions are constructed. Furthermore, an optimization model is built based on the collective probability matrix, and the ranking of alternatives can be obtained by solving the model.

2. Preliminaries

In this section, we will introduce some basic concepts related to multiple criteria group decision making problems, which the preference information on alternatives provided by decision makers is in the form of uncertain preference ordinals.

Definition 1. (Fan et al, 2010) Let Z^+ be the set of positive integer. An uncertain preference ordinal $\tilde{r}$ is expressed in $\tilde{r}=\{r^L, r^L+1, \cdots, r^U\}$, where $r^L, r^L+1, \cdots, r^U \in Z^+$, $r^L \le r^U$, r^L and r^U are the lower bound and upper bound of $\tilde{r}$.For simplicity, we express $\tilde{r}$ as $\tilde{r}=\left[r^L, r^U\right]$.

Remark 1. Let m denotes the number of all alternatives in a GDM analysis as well as the total number of ranking positions. Let $M=\{1,2,\cdots,m\}$ be the set of all the ranking positions, where $1,2,\cdots,m$ denote that the ranking position is the $1st, 2nd, \cdots, mth$, respectively. If k, $k \in M$ represents that the ranking position of an alternative is the kth ,then the smaller k is, the better the corresponding alternative will be. Thus, for uncertain preference ordinal $\tilde{r}=\left[r^L, r^U\right]$, $r^L, r^U \in M$.

Remark 2. Consider an uncertain preference ordinal $\tilde{r}=\left[r^L, r^U\right]$.Let $\tilde{u}^{\tilde{r}}=r^U-r^L+1$, then $\tilde{u}^{\tilde{r}}$ denotes the number of possible ranking positions in $\tilde{r}$ and it is also viewed as the uncertainty degree of $\tilde{r}$. Thus, the greater $\tilde{u}^{\tilde{r}}$ is, the greater the uncertainty degree of $\tilde{r}$ will be. In multiple criteria group decision-making (MCGDM) process, let $S=\{s_1, s_2, \cdots, s_m\}$ $(m \ge 2)$ be a finite set of alternatives $C=\{c_1, c_2, \cdots, c_n\}$ $(n \ge 2)$ be a set of criteria, $\omega=(\omega_1, \omega_2, \cdots, \omega_n)$ is the weight vector of criteria $c_j (j=1,\cdots,n)$,where $\omega_j \ge 0$, $\sum_{j=1}^{n} \omega_j = 1$, Let $E=\{e_1, e_2 \cdots, e_l\}$ $(l \ge 2)$ be the set of decision makers, $\lambda=(\lambda_1, \lambda_2, \cdots, \lambda_l)$ be the weighting vector of decision makers, with $\lambda_t \ge 0$, $\sum_{t=1}^{l} \lambda_t = 1$ .Suppose the decision makers e_t $(t=1,\cdots,l)$ provide their preferences for alternatives in the form of uncertain preference ordinals, then, the following definitions are obtained.

Definition 2. (You et al., 2013) Let Z^+ be the set of positive integer. An uncertain preference ordinal $\tilde{r}_{ij}^{\,t}$ is expressed in $\tilde{r}_{ij}^{\,t}=\{r_{ijt}^L, r_{ijt}^L+1, \cdots, r_{ijt}^U\}$, where $r_{ijt}^L, r_{ijt}^L+1, \cdots, r_{ijt}^U \in Z^+$, $r_{ijt}^L \le r_{ijt}^U$, $\tilde{r}_{ij}^{\,t}$ indicates the preference information that the alternative s_i satisfies the criteria c_j given by the decision maker e_t , r_{ijt}^L and r_{ijt}^U are the lower bound and upper bound of $\tilde{r}_{ij}^{\,t}$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$, $t=1,2,\cdots,l$. Especially if $r_{ijt}^L = r_{ijt}^U$, then $\tilde{r}_{ij}^{\,t}$ reduces to a ranking ordinal. For simplicity, we express $\tilde{r}_{ij}^{\,t}$ as $\tilde{r}_{ij}^{\,t}=\left[r_{ijt}^L, r_{ijt}^U\right]$.

Remark 3. Let m denotes the number of all alternatives in a MCGDM analysis as well as the total number of ranking positions. Let $M=\{1,2,\cdots,m\}$ be the set of all the ranking positions, where $1,2,\cdots,m$ denote that the ranking position is the $1st, 2nd, \cdots, mth$, respectively, if k , $k \in M$ represents that the ranking position of an alternative is the kth ,then the smaller k is, the better the corresponding alternative will be. Thus, for uncertain preference ordinal $\tilde{r}_{ij}^{\,t}=\left[r_{ijt}^L, r_{ijt}^U\right]$, $r_{ijt}^L, r_{ijt}^U \in M$, the smaller r_{ijt}^L or r_{ijt}^U is, the better the ranking position of the alternative will be. If $\tilde{r}=\left[r^L, r^U\right]$, r^L , $r^U \in M$ is an uncertain preference ordinal on an alternative provided by the decision maker. Fan et al (2010) regarded that the alternative could be ranked in position $r^L, r^L+1, \cdots, r^U$ with the same possibility, and gave the definition of the probability vector.

Definition 3. (Fan et al, 2010) Let $\tilde{r}=\left[r^L, r^U\right]$ (r^L , $r^U \in M$) be an uncertain preference ordinal on an alternative provided by the decision maker. Then, the probability vector on $\tilde{r}$ is represented by $\tilde{\dot{p}}_{\tilde{r}}=\left(\dot{p}_{\tilde{r}}^1, \dot{p}_{\tilde{r}}^2, \cdots, \dot{p}_{\tilde{r}}^m\right)$ and the elements of $\tilde{\dot{p}}_{\tilde{r}}$ are given by

$$\dot{p}_{\tilde{r}}^{k}=r^{U}\begin{cases}0, & k=1,2,\cdots,r^{L}-1;\\ \dfrac{1}{\tilde{u}^{\tilde{r}}}, & k=r^{L},r^{L}+1,\cdots,r^{U};\\ 0, & k=r^{U}+1,r^{U}+2,\cdots,m.\end{cases} \tag{1}$$

Where $\dot{p}_{\tilde{r}}^{k}$ ($k=1,2,\cdots,m$) denotes the probability that the alternative is ranked in the kth position, satisfies $\sum_{k=1}^{m}\dot{p}_{\tilde{r}}^{k}=1$ and $0\le\dot{p}_{\tilde{r}}^{k}\le1$， $k=1,2,\cdots,m$.

Definition 4. Let $\tilde{r}=\left[r^{L},r^{U}\right]$ ($r^{L},r^{U}\in M$) be an uncertain preference ordinal on an alternative provided by the decision maker, then, the probability that the alternative is ranked in each position in $\left[r^{L},r^{U}\right]$ is represented by:

$$v_{h}=\frac{C_{n-1}^{h-L}}{2^{n-1}},\ h=L,L+1,\cdots,U,\ n=U-L+1 \tag{2}$$

where $v_{h}\ge0$, $\sum_{h=L}^{U}v_{h}=1$,and $C_{1}^{0}=C_{0}^{0}=1$ .That is, the alternative could be ranked in position $r^{L},r^{L}+1,\cdots,r^{U}$ with possibility $\frac{C_{n-1}^{h-L}}{2^{n-1}}$.

Definition 5. Let $\tilde{r}=\left[r^{L},r^{U}\right]$ ($r^{L},r^{U}\in M$) be an uncertain preference ordinal on an alternative provided by the decision maker, then, the probability that the alternative is ranked in each position in $\left[r^{L},r^{U}\right]$ is represented by:

$$v_{h}'=\frac{e^{-\frac{(h'-u_{n})^{2}}{2\delta_{n}^{2}}}}{\sum_{j=L}^{U}e^{-\frac{(h'-u_{n})^{2}}{2\delta_{n}^{2}}}},h=L,L+1,\cdots,U \tag{3}$$

where $u_{n}=\frac{1}{n}\frac{n(n+1)}{2}=\frac{1+n}{2},\delta=\sqrt{\frac{1}{n}\sum_{h=L}^{U}(h'-u_{n})^{2}},h'=h-L+1$.

Remark 4. It is easy to improve that v_{h} in Definition 4 has the following well-known properties:1) v_{h} is symmetrical, i.e., $v_{h}=v_{n+1-h}(h=1,2,\cdots,n)$. 2) the probability vector is $v=(v_{1},v_{2},\cdots,v_{n})^{T}$,when $n=2k+1$,which satisfy

$$v_{1}\le v_{2}\le\cdots\le v_{\frac{n-1}{2}}\le v_{\frac{n+1}{2}}\ge v_{\frac{n+3}{2}}\ge\cdots\ge v_{n-1}\ge v_{n} \tag{4}$$

when $n=2k$,which satisfy

$$v_{1}\le v_{2}\le\cdots\le v_{\frac{n}{2}-1}\le v_{\frac{n}{2}}=v_{\frac{n}{2}+1}\ge\cdots\ge v_{n-1}\ge v_{n} \tag{5}$$

While v_{h}' in Definition 5 has a same property as v_{h} . Consider an uncertain preference ordinal $\tilde{r}_{ij}^{t}=\left[r_{ijt}^{L},r_{ijt}^{U}\right],r_{ijt}^{L},r_{ijt}^{U}\in M$ and motivated by Definition 4 and Definition 5, we have the following definitions.

Definition 6. Let $\tilde{r}_{ij}^{t}=\left[r_{ijt}^{L},r_{ijt}^{U}\right],r_{ijt}^{L},r_{ijt}^{U}\in M$ .Then, the probability vector on $\tilde{r}_{ij}^{t}$ is represented by $\tilde{p}_{\tilde{r}_{ij}^{t}}=\left(p_{\tilde{r}_{ij}^{t}}^{1},p_{\tilde{r}_{ij}^{t}}^{2},\cdots,p_{\tilde{r}_{ij}^{t}}^{m}\right)$ and the elements of $\tilde{p}_{\tilde{r}_{ij}^{t}}$ are given by

$$\tilde{p}_{\tilde{r}_{ij}^{t}}^{k}=\begin{cases}0,k=1,2,\cdots,r_{ijt}^{L}-1;\\ \dfrac{c_{U-L}^{h-L}}{2^{U-L}},k=r_{ijt}^{L},r_{ijt}^{L}+1,\cdots,r_{ijt}^{U},h=L,\ L+1,\cdots,\ U-L+1;\\ 0,k=r_{ijt}^{U}+1,r_{ijt}^{U}+2,\cdots,m.\end{cases} \tag{6}$$

Where denotes $p^k_{\tilde{r}^t_{ij}}$ denotes the probability that the alternative is ranked in the k th position, such that $\sum_{k=1}^{m} \tilde{p}^k_{\tilde{r}^t_{ij}} = 1$ and $0 \le \tilde{p}^k_{\tilde{r}^t_{ij}} \le 1, k = 1, 2, \cdots, m.$

Definition 7. Let $\tilde{r}^t_{ij} = \left[r^L_{ijt}, r^U_{ijt} \right], r^L_{ijt}, r^U_{ijt} \in M, h = L, L+1, \cdots, U-L+1$. Then, the probability vector on $\tilde{r}^t_{ij}$ is represented by $\tilde{p}_{\tilde{r}^t_{ij}} = \left(p^1_{\tilde{r}^t_{ij}}, p^2_{\tilde{r}^t_{ij}}, \cdots, p^m_{\tilde{r}^t_{ij}} \right)$ and the elements of $\tilde{p}_{\tilde{r}^t_{ij}}$ are given by

$$\tilde{p}^k_{\tilde{r}^t_{ij}} = \begin{cases} 0, & k = 1, 2, \cdots r^L_{ijt} - 1; \\ \dfrac{e^{-\frac{(h'-u_n)^2}{2\delta_n^2}}}{\sum\limits_{j=L}^{U} e^{-\frac{(h'-u_n)^2}{2\delta_n^2}}}, & k = r^L_{ijt}, r^L_{ijt} + 1, \cdots, r^U_{ijt}, h' = h - L + 1; \\ 0\,, & k = r^U_{ijt} + 1, r^U_{ijt} + 2, \cdots, m. \end{cases} \tag{7}$$

where $\tilde{p}^k_{\tilde{r}^t_{ij}}$ denotes the probability that the alternative is ranked in the k th position, such that $\sum_{k=1}^{m} \tilde{p}^k_{\tilde{r}^t_{ij}} = 1$ and $0 \le \tilde{p}^k_{\tilde{r}^t_{ij}} \le 1, k = 1, 2, \cdots, m.$

Remark 5. If $r^L_{ijt} = r^U_{ijt}$ for uncertain preference ordinal $\tilde{r}^t_{ij} = \left[r^L_{ijt}, r^U_{ijt} \right], r^L_{ijt}, r^U_{ijt} \in M$, i.e. , $\tilde{r}^t_{ij}$ reduced to a ranking ordinal, then the elements of probability vector $\tilde{p}_{\tilde{r}^t_{ij}} = \left(p^1_{\tilde{r}^t_{ij}}, p^2_{\tilde{r}^t_{ij}}, \cdots, p^m_{\tilde{r}^t_{ij}} \right)$ on $\tilde{r}^t_{ij}$ are given by

$$\tilde{p}^k_{\tilde{r}^t_{ij}} == \begin{cases} 0, k = 1, 2, \cdots, r^L_{ijt} - 1; \\ 1, k = r^L_{ijt} = r^U_{ijt}; \\ 0, k = r^U_{ijt} + 1, r^U_{ijt} + 2, \cdots, m. \end{cases} \tag{8}$$

Definition 8. Let $\tilde{r}^t_{i1}, \tilde{r}^t_{i2}, \cdots, \tilde{r}^t_{in}$ be n uncertain preference ordinals and $\tilde{p}_{\tilde{r}^t_{i2}} = \left(p^1_{\tilde{r}^t_{i2}}, p^2_{\tilde{r}^t_{i2}}, \cdots, p^m_{\tilde{r}^t_{i2}} \right)$, $\cdots, \tilde{p}_{\tilde{r}^t_{in}} = \left(p^1_{\tilde{r}^t_{in}}, p^2_{\tilde{r}^t_{in}}, \cdots, p^m_{\tilde{r}^t_{in}} \right)$ be the corresponding probability vectors. Let $w = (w_1, w_2, \cdots, w_n)$ be a weight vector, where w_i denotes the weight of $\tilde{r}_i$ such that $\sum_{j=1}^{n} w_j = 1$ and $0 \le w_j \le 1, j = 1, 2, \cdots, n.$ Then, the overall probability vector on $\tilde{p}_{\tilde{r}^t_{i1}}, \tilde{p}_{\tilde{r}^t_{i2}}, \cdots, \tilde{p}_{\tilde{r}^t_{in}}$ is represented by $\tilde{p}_{\tilde{r}^t_{i0}} = \left(p^1_{\tilde{r}^t_{i0}}, p^2_{\tilde{r}^t_{i0}}, \cdots, p^m_{\tilde{r}^t_{i0}} \right)$ and the elements of $\tilde{p}_{\tilde{r}^t_{i0}}$ are given by

$$\tilde{p}^k_{\tilde{r}^t_{i0}} = \sum_{j=1}^{n} w_j p^k_{\tilde{r}^t_{ij}}, k = 1, 2, \cdots, m \tag{9}$$

3. The proposed approach

In this section, this paper will present a handing method for MCGDM problems with uncertain preference ordinals. Firstly, it gives a brief description of the MCGDM problems with uncertain preference ordinals. Then, a probability matrix, the voting information matrix, the collective voting information matrix and an optimization model are constructed. Finally, an algorithm for determining the ranking position of each alternative is given.

Let $S = \{s_1, s_2, \cdots, s_m\}$ $(m \ge 2)$ be a finite set of alternatives, $C = \{c_1, c_2, \cdots, c_n\}$ $(n \ge 2)$ be a set of criteria, whose weight vector is $w = (w_1, w_2, \cdots, w_n)$, where $w_j \ge 0 (j = 1, \cdots, n)$, $\sum_{j=1}^{n} w_j = 1$ and $E = \{e_1, e_2, \cdots, e_l\}$ $(l \ge 2)$ be the set of decision makers whose weight vector is $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_l)$,where $\lambda_t \ge 0, t = 1, \cdots, l$,and $\sum_{t=1}^{l} \lambda_t = 1$.

Suppose the decision makers e_t $(t = 1, \cdots, l)$ provide their preferences for alternatives in the form of uncertain preference ordinals, i.e., $\tilde{r}^t_{ij} = \left[r^L_{ijt}, r^U_{ijt} \right], r^L_{ijt}, r^U_{ijt} \in M$, where $\tilde{r}^t_{ij}$ denotes the preference information that the alternative s_i satisfies the criteria c_j given by the decision maker e_t , $i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, t = 1, 2, \cdots, l$ .The problem concerned in this paper is to rank alternatives or to select the most desirable alternative(s) among a finite set S based on uncertain preference ordinals $\tilde{r}_{ij}^{t}$ .The method is described as follow: Let $\tilde{p}_{ij}^{t}=\left(p_{ij}^{t1},p_{ij}^{t2},\cdots,p_{ij}^{tn}\right)$ be probability vector on uncertain preference ordinals $\tilde{r}_{ij}^{t}$ .It can be determined according to Definition 2.6 or Definition 2.7, where $\tilde{p}_{ij}^{tk}$ denotes the probability that criteria c_j is ranked in the kth position, such that $\sum_{k=1}^{m}p_{ij}^{tk}=1$ and $0\le p_{ij}^{tk}\le 1, i=1,2,\cdots,m;\ j=1,2,\cdots,n.$ For the convenience of analysis, the decision matrix in the form of probabilities based on $\tilde{p}_{ij}^{t}$ is constructed as follows:

$$p^{t}=\left(\tilde{p}_{ij}^{t}\right)_{m\times n}=\begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_m \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} \tilde{p}_{11}^{t} & \tilde{p}_{11}^{t} & \cdots & \tilde{p}_{1n}^{t} \\ \tilde{p}_{21}^{t} & \tilde{p}_{22} & \cdots & p_{2n}^{t} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{p}_{m1}^{t} & \tilde{p}_{m2}^{t} & \cdots & \tilde{p}_{mn}^{t} \end{bmatrix} \end{array} \tag{10}$$

Using Eq.(10), the elements of the ith row of the probability matrix P^{t} and the weight vector w are aggregated to form the weight probability vector on alternative s_i ,which takes the weight of the criteria $c_j\ \left(j=1,2,\cdots,n\right)$ in to consideration, and it is given by

$$q_i^{tk}=\sum_{j=1}^{l}w_j p_{jk}^{tk},\ i,\ k=\ 1,2,\cdots,\ m;\ j=\ 1,2,\cdots,\ n \tag{11}$$

Through Eq.(11), it can be easily seen that $\sum_{k=1}^{\mathrm{m}}q_i^{tk}=1$ .Based on the obtained vectors $q_i^{tk}\ \left(i=1,2,\cdots,m\right)$,the weight probability matrix $Q^{\mathrm{t}}=(q_i^{tk})_{m\times m}$ can be constructed, i.e.,

$$Q^{t}=\left(q_i^{tk}\right)_{m\times m}=\begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_m \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & m \end{array} \\ \begin{bmatrix} q_1^{t1} & q_1^{t2} & \cdots & q_1^{tm} \\ q_2^{t1} & q_2^{t2} & \cdots & q_2^{tm} \\ \vdots & \vdots & \cdots & \vdots \\ q_m^{t1} & q_m^{t2} & \cdots & q_m^{tm} \end{bmatrix} \end{array} \tag{12}$$

The elements of the weight probability matrix Q^{t} and vector λ are aggregated to form the collective probability vector on alternative s_i , i.e., $\tilde{\pi}_i=\left(\pi_i^{1},\pi_i^{2},\cdots,\pi_i^{m}\right)$, where π_i^{k} denotes the collective results by all the decision makers that alternative s_i is ranked in the k th position and it is given by $\pi_i^{k}=\sum_{t=1}^{l}\lambda_t q_i^{tk},\ i,\ k=\ 1,2,\cdots,\ m$.

Based on vectors π_i^{k} ,this paper constructs the following collective probability matrix $\prod$,i.e.,

$$\prod=\begin{array}{c} \\ \mathrm{s}_1 \\ \mathrm{s}_2 \\ \vdots \\ \mathrm{s}_\mathrm{m} \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & \mathrm{m} \end{array} \\ \begin{bmatrix} \pi_1^{1} & \pi_1^{2} & \cdots & \pi_1^{m} \\ \pi_2^{1} & \pi_2^{2} & \cdots & \pi_2^{m} \\ \vdots & \vdots & \cdots & \vdots \\ \pi_m^{1} & \pi_m^{2} & \cdots & \pi_m^{m} \end{bmatrix} \end{array} \tag{13}$$

Based on collective probability matrix $\prod$, this paper attempts to develop a method to determine the ranking position of each alternative that one alternative is only ranked in one ranking position. The description of this method is given below. Let $b_{ik}\ \left(i,k=1,2,\cdots,m\right)$ be 0-1variable, where $b_i^{k}=1$ represents that alternative s_i is ranked in the k th position and $b_i^{k}=0$, otherwise. The total probability that m alternatives are ranked in m positions can be expressed as $\sum_{i=1}^{m}\sum_{k=1}^{m}\pi_i^{k}\,b_i^{k}$, where $\sum_{i=1}^{m}b_i^{k}=1(k=\ 1,2,\cdots,\ m)$ and $\sum_{k=1}^{m}b_i^{k}=1(i=\ 1,2,\cdots,\ m)$ .To rank alternatives or select the best alternative(s), we can construct the following optimization model

$$\max z=\sum_{i=1}^{m}\sum_{k=1}^{m}\pi_i^{k}b_i^{k} \tag{14}$$

Subject to $\sum_{i=1}^{m}b_i^{k}=1,k=1,2,\cdots,m$, $\sum_{i=1}^{m}b_i^{k}=1,i=1,2,\cdots,m$, $b_i^{k}=0\ \ or\ 1,i,k=1,2,\cdots,m$.

Existing mathematical optimization software can be used to solve model (14). In summary, we give an algorithm to determine the ranking position of alternatives and its steps are presented as follows:

step1: Calculate probability vectors $\tilde{p}_{ij}^{t}$ by Eq.(7) or Eq.(8) based on $\tilde{r}_{ij}^{t}, i=1,2,\cdots,n; t=1,2,\cdots,l$.

step2: Construct probability matrix $P^{t}=\left(\tilde{p}_{ij}^{t}\right)_{m\times n}$ based on $\tilde{p}_{ij}^{t}$, $i=1,2,\cdots,m; j=1,2,\cdots,n.$

step3: Construct the weight probability matrix $Q^{t}=\left[q_{i}^{tk}\right]_{m\times m}$ by Eq.(11).

step4: Construct collective probability matrix $\Pi=\left[\pi_{i}^{k}\right]_{m\times m}$ based on $Q^{t}=\left[q_{i}^{tk}\right]_{m\times m}$.

step5: Build the optimization model (14) based on matrix Π and solves it by Hungarian method.

step6: Determine the ranking position of each alternative based on the obtained optimal solution(s) of model (14) and record the probability of ranking position of alternative based on matrix Π .

4. Conclusion

In multiple criteria group decision-making situations that the decisions makers can not give the exact value, the decision makers may be suitably expressed with preference ordinals. Focusing on this problem, this paper proposes a new method to solve the MCDM problems that the preference information is in the form of uncertain preference ordinal. It improves the method proposed by Fan, and is more in line with the law of general human cognition. First, it develops two normal distribution-based methods to determine the probability that the alternative is ranked in each position. Then, in order to process uncertain preference ordinals, a matrix in the form of probabilities is constructed. Furthermore, a weight probability matrix and a collective probability matrix on alternatives with regard to ranking positions are constructed. Finally, an optimization model is built based on the collective probability matrix, and the ranking of alternatives can be obtained by solving the model. The methods proposed in this paper may also be used in MCDM problems with other preferences information formats, e.g., interval and set-values, uncertain linguistic variables, etc.

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