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On Coordinating a Loss-Averse OEM Supply Chain with Random Supply and Demand

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This paper establishes a Stackelberg game between a risk-neutral OEM and a loss-averse CM, based on a piecewise-linear utility function and investigates an OEM supply chain consisting of a loss-averse CM and a risk-neutral OEM, with random supply and random demand. The result shows that the loss-averse CM's unique optimal order quantity is always less than the risk-neutral CM's under decentralized decision making. Loss aversion can also affect the order quantity. When an OEM is more loss-averse, the order quantity gets smaller. The OEM supply chain can be coordinated when the order quantity is increased through the CM penalizing the OEM for shortages and sharing a proportion of revenue with the OEM.

1. Introduction

The loss-averse newsvendor model and the impact of loss aversion on ordering decisions have been discussed under various settings including without shortage cost, with shortage cost (Wang and Webster, 2009), with two ordering opportunities (Ma et al., 2012), with asymmetric information (Deng et al., 2013), with option contracts (Chen et al., 2014) and so on. However, only a few of these papers consider supply chain coordination under loss aversion. Wang and Webster (2009) investigate the role of a gain/loss sharing provision in mitigating the loss aversion effect in a risk-neutral manufacturer and a loss-averse retailer system. They found that a gain/loss sharing-and-buyback credit provision can achieve channel coordination. Chen et al. (2014) consider a supply chain consisting of a risk-neutral supplier and a loss-averse retailer. However, it also very rare for papers to consider both random supply and loss aversion. Hence, this study sheds new light on the management of loss-averse OEM supply chains with random supply and stochastic demand.

2. The loss- aversion OEM supply chain model

Consider a supply chain consisting of a loss-averse CM and a risk-neutral OEM. Both manufacturers are independent and make the most favorable decisions according to their own income. Let X be random demand and let f and F denote, respectively, the PDF and CDF of the random demand X. T and X are continuous, differentiable, invertible and independent random variables. Without loss of generality, it is assumed that F(0) = G(0) = 0. c, p and s are exogenous parameters and denote, respectively, the production cost per unit of the OEM, the price of the product per unit and the cost of the salvage value per unit of the CM, it is assumed that p>c>s. For any given realized demand t and supply rate x, the profit function of the CM, denoted by $\pi_b(Q)$, is:

$$\pi_b(Q) = p\cdot\min\{tQ,x\} + s\cdot\left(x-\min\{tQ,x\}\right)^+ - w\cdot tQ = \begin{cases} px - w\cdot tQ & x < tQ, \\ (p-w-s)\cdot tQ + s\cdot x & x \ge tQ. \end{cases} \tag{1}$$

The profit function of the OEM, denoted by $\pi_s(W)$, can be written as:

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$$\pi_s(w) = w \cdot tQ - cQ. \tag{2}$$

Let π_0 denote the CM's reference level at the beginning of the selling season. It is further assumed that the utility function of the risk-neutral OEM is equal to its expected profit function and that the loss-averse CM has the following piecewise-linear utility function:

$$U(\pi) = \begin{cases} \pi - \pi_0, & \pi \geq \pi_0, \\ \lambda(\pi - \pi_0), & \pi < \pi_0, \end{cases} \tag{3}$$

Here, $\lambda \geq 1$ is the CM's loss aversion coefficient, which means that people are more sensitive to losses than to gains of the same size.

If $x<tQ$, let the CM's profit function $\pi_b(Q)=px-w{\cdot}tQ=0$, with the result: $x=wtQ/p$. Since $\pi_b(Q)$ is strictly increasing in x, thus if $x<wtQ/p$, then $\pi_b(Q)<0$ for $x>0$. If $x\geq wtQ/p$, then $\pi_b(Q)\geq 0$ for $x<tQ$. If $x>tQ$, then from the CM's profit function (1), the result is: $\pi_b(Q)=(p-w-s){\cdot}tQ+s{\cdot}x>0$

Lemma 1. If $x<tQ$, let the CM's profit function $\pi_b(Q)=px-w{\cdot}tQ=0$, with the result: $x=wtQ/p$. Since $\pi_b(Q)$ is strictly increasing in x, thus if $x<wtQ/p$, then $\pi_b(Q)<0$ for $x>0$. If $x\geq wtQ/p$, then $\geq\pi_b(Q)0$ for $x<tQ$. If $x\geq tQ$, then from the CM's profit function (1), the result is: $\pi_b(Q)=(p-w-s){\cdot}tQ+s{\cdot}x>0$

Lemma 1 provides the conditions under which the loss-averse CM's profit will be negative or positive, according to the realized supply and demand rate. It shows that if realized demand relative to tQ is relatively low, i.e., $x<wtQ/p$, then the CM will face overage loss. Conversely, if realized demand relative to tQ is relatively high, i.e., $x>wtQ/p$, then the newsvendor will face gains.

According to the above results, by mapping the CM's profit function (1) into its utility function (3), the loss-averse CM's expected utility function, denoted by $E[U(\pi_b(Q))]$, can be written as:

$$\begin{aligned} E\left[U\left(\pi_b(Q)\right)\right] &= \lambda\int_0^1\int_0^{\frac{wtQ}{p}}(px-wtQ)dF(x)dG(t) + \int_0^1\int_{\frac{wtQ}{p}}^{tQ}\left[(p-w-s)tQ+sx\right]dF(x)dG(t) \\ &+\int_0^1\int_{tQ}^{\infty}\left[(p-w-s)tQ+sx\right]dF(x)dG(t) \\ &= E\left[\pi_b(Q)\right]+(\lambda-1)\int_0^1\int_0^{\frac{wtQ}{p}}(px-wtQ)dF(x)dG(t). \end{aligned} \tag{4}$$

The expected utility of a loss-averse CM with random supply and demand is the expected profit plus the expected overage losses, biased by a factor of λ-1. If λ=1, then the CM is risk-neutral and the second term in (4) drops out. The loss-averse CM's decision problem is:

$$\max_{Q>0} E\left[U\left(\pi_b(Q)\right)\right]. \tag{5}$$

While the risk-neutral OEM's decision problem is:

$$\max_{w>0} E\left[\pi_s(w)\right] = \max_{w>0}\left\{w\cdot E(tQ)-cQ\right\}. \tag{6}$$

3. Optimal pricing and ordering under decentralized decision

Theorem 1. $E[U(\pi_b(Q))]$ *is concave in Q. There exists a unique optimal order quantity, denoted by* Q_λ^**, that maximizes the expected utility of the loss-averse CM with random supply and stochastic demand and satisfies the following first-order condition:*

$$(p-s)\int_0^1 t[1-F(tQ_\lambda^*)]dG(t)+(s-w)E(T)-(\lambda-1)w\int_0^1 tF(\frac{wtQ_\lambda^*}{p})dG(t)=0. \tag{7}$$

Proof. The loss-averse CM's expected function $E[\pi_b(Q)]$is first expressed as:

$$E\left[\pi_b(Q)\right]=(p-s)\int_0^1\left[\int_0^{tQ}xdF(x)+\int_{tQ}^{\infty}tQdF(x)\right]dG(t)+(s-w)QE(T) \tag{8}$$

Taking the partial derivative of $E[U(\pi_b(Q))]$with respect to Q, the result is:

$$\frac{\partial E\left[U\left(\pi_b(Q)\right)\right]}{\partial Q}=(p-s)\int_0^1 t[1-F(tQ)]dG(t)+(s-w)E(T)-(\lambda-1)w\int_0^1 tF(\frac{wtQ}{p})dG(t) \tag{9}$$

and:

$$\frac{\partial^2 E\left[U\left(\pi_b(Q)\right)\right]}{\partial Q^2}=-(p-s)\int_0^1 t^2 f(tQ)dG(t)-\frac{(\lambda-1)w^2}{p}\int_0^1 t^2 f(\frac{wtQ}{p})dG(t)<0. \tag{10}$$

That is, $E[U(\pi_b(Q))]$is concave in Q, and a unique production quantity Q_λ^* which maximizes $E[U(\pi_b(Q))]$exists. Let $\frac{\partial E\left[U\left(\pi_b(Q)\right)\right]}{\partial Q}=0$, then it is possible to derive (7).

If the CM is risk-neutral, i.e., λ=1, then the first-order condition (7) reduces to

$$(p-s)\int_0^1 t[1-F(tQ_1^*)]dG(t)+(s-w)E(T)=0. \tag{11}$$

Therefore, the result is $\frac{E\left(T\cdot F(TQ_1^*)\right)}{E(T)}=\frac{p-w}{p-s}$. Okyay et al. (2014) provide similar results for Q_1^*. Hence, the corresponding results in Okyay et al. (2014) are particular cases of our Theorem 1.

To gain more insight, the comparative statics of loss aversion level changes on the loss-averse CM's optimal order quantity and expected utility are investigated.

Theorem 2. *For any $\lambda\geq 1$, the CM's optimal order quantity and expected utility are decreasing in λ.*

Proof. Theorem 2 can be proved by taking the first partial derivatives of (7) with respect to λ and determining its signs.

Let

$$H(\lambda)=(p-s)\int_0^1 t[1-F(tQ_\lambda^*)]dG(t)+(s-w)E(T)-(\lambda-1)w\int_0^1 tF(\frac{wtQ_\lambda^*}{p})dG(t)=0, \tag{12}$$

then it is possible easily to get

$$\frac{\partial H(\lambda)}{\partial\lambda}=-w\int_0^1 tF(\frac{wtQ_\lambda^*}{p})dG(t)<0, \tag{13}$$

By (10), the result is:

$$\frac{\partial H(\lambda)}{\partial Q_\lambda^*}<0. \tag{14}$$

Therefore, by the Implicit Function Theorem, (13) and (14), the result is

$$\frac{\partial Q_\lambda^*}{\partial\lambda}=-\frac{\partial H(\lambda)}{\partial\lambda}\Big/\frac{\partial H(\lambda)}{\partial Q_\lambda^*}<0. \tag{15}$$

That is, the CM's optimal order quantity is decreasing in λ .

When $\lambda=\lambda_1$, the CM's optimal order quantity is $Q_{\lambda_1}^*$. When $\lambda=\lambda_2$, the CM's optimal order quantity is $Q_{\lambda_2}^*$. Without loss of generality, it is assumed that $\lambda_1<\lambda_2$, then by the optimality of $Q_{\lambda_1}^*$, the result is

$$E^{*}_{\lambda_1}\left[U\left(\pi_b(Q^{*}_{\lambda_1})\right)\right]\geq E_{\lambda_1}\left[U\left(\pi_b(Q^{*}_{\lambda_2})\right)\right]. \tag{16}$$

Moreover, since expected utility is decreasing in λ , thus

$$E_{\lambda_1}\left[U\left(\pi_b(Q^{*}_{\lambda_2})\right)\right]\geq E^{*}_{\lambda_2}\left[U\left(\pi_b(Q^{*}_{\lambda_2})\right)\right]. \tag{17}$$

Therefore, by (16) and (17), the result is

$$E^{*}_{\lambda_1}\left[U\left(\pi_b(Q^{*}_{\lambda_1})\right)\right]\geq E^{*}_{\lambda_2}\left[U\left(\pi_b(Q^{*}_{\lambda_2})\right)\right], \tag{18}$$

which implies that the CM's optimal expected utility is decreasing in λ.
Theorem 2 reveals that the loss-averse CM will order less than the risk-neutral one (λ=1). The more loss-averse the CM, the less its order quantity will be. This result also holds in the risk-neutral newsvendor model. Thus, this paper extends the application of the classic newsvendor problem.
The expected profit function of the OEM is:

$$E\left[\pi_s(w)\right]=wQ\int_0^1 tdG(t)-cQ. \tag{19}$$

Then, taking $Q^{*}_{\lambda}(w)$, determined by (7), into the equation (19) and solving the first-order condition with respect to *w*, produces:

$$\frac{\partial E\left[\pi_s(w)\right]}{\partial w}=\left[Q^{*}_{\lambda}(w)+w\frac{\partial Q^{*}_{\lambda}(w)}{\partial w}\right]\int_0^1 tdG(t)-c\frac{\partial Q^{*}_{\lambda}(w)}{\partial w}=0. \tag{20}$$

Let *w** satisfy (20), then *w** is the OEM's optimal outsourcing price and the expected profit of the OEM is currently at its maximum.

4. Supply chain coordination

4.1 Optimal ordering under centralized decision

In reality, every enterprise is one of a number of scattered individuals. Each only focuses on the maximization of its own benefits and does not take global optimization into account in decision-making. Thus, it is difficult to achieve perfect coordination of the supply chain under decentralized decision-making. The balanced order quantity is often not the global optimum under decentralized decision-making, because of the double marginalization effect. As an ideal state, with centralized decisions, the CM and OEM supplier can collaborate closely to coordinate a decision scheme from the overall situation and to maximize the interests of the integrated supply chain.
Under centralized decision-making, the integrated supply chain's profit function, denoted by $\pi_{bs}(Q)$, is:

$$\begin{aligned}\pi_{bs}(Q)&=p\cdot\min\{tQ,x\}+s\cdot\left(x-\min\{tQ,x\}\right)^{+}-cQ\\&=\begin{cases}px-cQ & x<tQ,\\(p-s)\cdot tQ+s\cdot x-cQ & x\geq tQ.\end{cases}\end{aligned} \tag{21}$$

Then, the integrated supply chain's expected profit, denoted by $E[\pi_{bs}(Q)]$, can be written as:

$$E\left[\pi_{bs}(Q)\right]=(p-s)\int_0^1\left[\int_0^{tQ}xdF(x)+\int_{tQ}^{\infty}tQdF(x)\right]dG(t)+sQ\int_0^1 tdG(t)-cQ. \tag{22}$$

Taking the first-order condition of (22) with respect to *Q*, it is possible to get

$$\frac{\partial E\left[\pi_{bs}(Q)\right]}{\partial Q}=s\int_0^1 tdG(t)+(p-s)\int_0^1 t[1-F(tQ)]dG(t)-c=0. \tag{23}$$

By $\partial^2 E[\pi_{bs}(Q)]/\partial Q^2<0$, it is possible to get the supply chain's unique optimal order quantity, denoted by Q^0, which is determined by the first-order condition of (23).

By (23), it is found that the integrated OEM supply chain's profit is only determined by order quantity Q, which has nothing to do with outsourcing price w. The expected profit of the supply chain is at its maximum when the optimal order quantity Q satisfies (23). The following discussion introduces a coordination mechanism. The loss-averse CM's optimal order quantity Q_λ^* under decentralized decision-making is equal to the optimal order quantity Q^0 under centralized decision-making in this mechanism. Thus, it is possible to achieve coordination of the OEM supply chain.

4.2 The contract of coordination

By introducing a contract combining a shortage penalty and revenue sharing, for any given realized demand t and supply rate x, the new profit function of the CM is:

$$\pi_b'(Q)=\phi\Big[p\cdot\min\{tQ,x\}+s\cdot\big(x-\min\{tQ,x\}\big)^+-w\cdot tQ\Big]+m(Q-tQ). \tag{24}$$

Similarly, the OEM's new profit function is

$$\pi_s'(w)=w\cdot tQ-cQ+(1-\phi)\Big[p\cdot\min\{tQ,x\}+s\cdot\big(x-\min\{tQ,x\}\big)^+-w\cdot tQ\Big]-m(Q-tQ). \tag{25}$$

The contract combining a shortage penalty and revenue sharing is just designed to make the transfer payment in the internal supply chain by (24) and (25). The supply chain profit function under centralized decisions is also equal to $\pi_{bs}(Q)$. If the supply chain is coordinated, then the optimal order quantity under decentralized decisions is equal to the optimal order quantity under centralized decisions and the expected profit of the OEM supply is currently at its maximum.

Similar to the proof of Theorem 1, the following is the result for the loss-averse CM.

Theorem 3. *Under the contract combining a shortage penalty and revenue sharing, the expected utility function of the loss-averse CM, denoted by $E[U(\pi_b'(Q))]$, is concave in Q. There also exists a unique optimal order quantity, denoted by $Q_\lambda'^*$, that maximizes the expected utility of the loss-averse CM with random supply and demand and satisfies the following first-order condition:*

$$\phi(p-s)\int_0^1 t[1-F(tQ_\lambda'^*)]dG(t)+(\phi s-\phi w-m)E(T)+m$$
$$-(\lambda-1)\int_0^1(w\phi t-m+mt)F\left(\frac{\phi wt-m-mt}{\phi p}Q_\lambda'^*\right)dG(t)=0. \tag{26}$$

If Q and λ are determined, then (26) is the functions of w, m and ϕ. Let

$$H(w,m,\phi)=\phi(p-s)\int_0^1 t[1-F(tQ_\lambda'^*)]dG(t)+(\phi s-\phi w-m)E(T)+m$$
$$-(\lambda-1)\int_0^1(w\phi t-m+mt)F\left(\frac{\phi wt-m-mt}{\phi p}Q_\lambda'^*\right)dG(t) \tag{27}$$

and

$$G(w,m,\phi)=(1-\phi)(p-s)\int_0^1\Big[\int_0^{tQ}xdF(x)+\int_{tQ}^{\infty}tQdF(x)\Big]dG(t)$$
$$+\big[\phi w+m+(1-\phi)s\big]Q\int_0^1 tdG(t)-(m+c)Q. \tag{28}$$

The contract combining a shortage penalty and revenue sharing is next made to coordinate the OEM supply chain. As the Stackelberg leader, the OEM set the value of the parameter $\{m,\phi\}$ for determining the loss aversion coefficient λ to make the CM in decentralized decision-making choose the supply chain's optimal order quantity Q^0 as its own optimal strategy, i.e. let Q^0 satisfy the first-order condition (26). Then, by selecting an appropriate parameter value *w*, the OEM makes its profit not less than the highest profit in decentralized decision-making. That is, the following equations are established:

$$\begin{cases} H(w,m,\phi)\Big|_{Q_{\lambda}'^{*}=Q^{0}} = 0, \\ G(w,m,\phi)\Big|_{Q=Q^{0}} = \mu, \end{cases} \tag{29}$$

Here, μ is a constant with $\mu \ge E[\pi_S(w^*)]$. The above condition for μ is to ensure that the OEM can actively participate in the contract. Different values of μ represent different profit distributions between the OEM and the CM. Noting that the equations have a degree of freedom, there are hence multiple solutions of the parameters $\{w,m,\phi\}$. By setting different contract parameters, the OEM can adjust the distribution of profits between the CM and the OEM.

5. Conclusions

This paper investigates an OEM supply chain consisting of a loss-averse CM and a risk-neutral OEM, with random supply and random demand. The supply chain model is developed by assuming that the loss-averse CM has a piecewise-linear utility function. The risk-neutral OEM's optimal outsourcing price and the loss-averse CM's optimal order quantity are derived based on a Stackelberg game. The optimal decisions of an integrated supply chain are discussed under centralized decision-making. A contract combining a shortage penalty with revenue sharing is then proposed, in order to coordinate the OEM supply chain. Through model comparison between loss-averse and risk-neutral CMs, it is found that, when supply and demand are random and decisions are decentralized, a loss-averse CM will order less than a risk-neutral CM. The more loss-averse the CM, the lower its order quantity will be. This work reveals that a contract combining a shortage penalty with revenue sharing can effectively coordinate the OEM supply chain. Many coordination programs can be created by setting different contract parameters. Lastly, the impacts of loss aversion and some system parameters on the decisions and performance of the OEM supply chain are analyzed by adopting numerical examples. This study sheds new light on the management of supply chains that operate under random supply and stochastic demand.

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