

Deflection Analysis of RC Beams Strengthened by External Pre-stressed Steels

Jun Li

School of Civil Engineering and Architecture, Changsha University of Science and Technology, Changsha, China.
 ljsbm523@126.com

Strengthening method of using external pre-stressed steels has many advantages. It gets the recognition of researchers and is widely used in the strengthening and rebuilding projects. However, calculation theories about the method are still lagging behind the practice. It is difficult to put the theories about unbounded pre-stressed steels into construction of external pre-stressed projects because of cross-section deformation incoordination and quadratic effect. This paper takes the design theory and method of external pre-stressed beam in normal use stage as the main line, uses the iterative method by the relationship of equivalent load-deformation-stress increment, numerical simulation the deflection and reverse arch value of the beam. On the basis of the influence of quadratic effect, the equivalent reduction factor of the external pre-stressed steels and whether steels being unloaded or not, a simplified calculation formula for the beam rigidity at using stage is proposed. By compared with the experimental dates, this method is verified feasible in practical application.

1. Introduction

The wide application of external pre-stressed steels makes the deflection analysis of bending members very important under normal conditions. Because the pre-stressed steels and concrete are non-bonded, the pre-stressing effect on the beam is only through the steering block and the anchoring point. So there is no longer a simple deformation coordination relationship between external pre-stressed steels and concrete. For one section between two anchorage points, the strain of external pre-stressed steels and the concrete is no longer equal. So the related method of bonded pre-stressed concrete cannot be used to calculate (Xiong, 2005; Lv and Meng, 1998). In addition to the cross sections of anchor points and the steering block, the vertical displacements of pre-stressed steels and concrete are not equal, so the calculation formula of non-bonded pre-stressed concrete members cannot be completely applied (Deng, 1998; Li, 2004).

At present, a large number of experiments and theoretical studies have been carried out on the calculation of beam rigidity home and abroad (Alkhairi, 1991),

Tan, Kiang-Hwee and Ng Chee-Khoon(1997)). But there are few researches on the external pre-stressed concrete beam rigidity calculation. For the deflection calculation of the external pre-stressed beams, the deflection formula of the non-bonded pre-stressed concrete beams is mainly used (Xiong et. al, 2004). But we know the deflection calculation of external pre-stressed structure is closely related to the stress increment of pre-stressed steels. The bending deformation can reduce the effective eccentricity of the steels in the beam. This can cause the quadratic effect which will influence the accurate calculation of the deflection. The influence is even greater when the span depth ratio is high.

2. Calculation of short-term deflection of external of external pre-stressed concrete beam

In the process of pre-stressing, we can use the equivalent load to analyze the strengthening effect of external pre-stressed concrete members. Therefore, the external pre-stressed reinforced concrete beam under the action of external load can be regarded as the concrete components under the common action of external load and pre-stressed load, as shown in Figure 1.

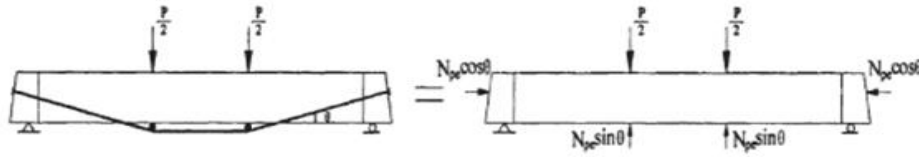


Figure 1: Equivalent

2.1 Calculation of Short-term Rigidity B_s of External Pre-stressed Concrete Beam

In the case of no cracks, the performance of the external pre-stressed concrete beam is approximately elastic. But in the calculation of rigidity, the non-elastic deformation of concrete should be taken into account, so take $B_s = 0.85E_c I_0$. We can use the method of short-term rigidity of bias component to calculate the short-term rigidity of external pre-stressed concrete beam. And it can be calculated by the following formulas:

1) Members without cracks:

$$B_s = 0.85E_c I_0 \tag{1}$$

2) Members with cracks:

$$B_s = \frac{E_s A_s h_0^2}{\psi \left(1.15 - 0.4 \frac{h_0}{e} \right) + 0.2 + \frac{\sigma_E \rho}{1 + 3.5 \gamma_f}} \tag{2}$$

As the strengthened beam with external pre-stressed steels, the calculation of N_{p0} is the same as the post-tensioning method beam:

$$N_{p0} = (\sigma_{con} - \sigma_1) A_p + \alpha_E \sigma_{pc} A_p - \sigma_{15} A_p \tag{3}$$

2.2 Component Deformation and External Pre-stressed Steels Stress

Based on the experiment analysis, Du Jinsheng of Tsinghua University proposed the stress increment analysis method based on structural deformation (Du and Liu, 2003). The method can be used to calculate the stress increment of non-bonded pre-stressed steels under normal and limit states. This method established the relationship of stress increment and the deflection of the beam, which can reflect the mechanism of the structure.

Taking point A at the fixed end of the beam as the origin, using the beam deformations at the anchorage and turning position, the new coordinate position of the pre-stressed steels can be obtained. Then the deformation and stress of the pre-stressed steels can be obtained.

The rotations of point A and B of the concrete beam are θ_A and θ_B , vertical deformation of point C and D is ΔC and ΔD , θ is the angle between the line of external pre-stressed steels and the beam axis.

The strain increment of external pre-stressed steels $\Delta \epsilon_p$ is:

$$\Delta \epsilon_p = \Delta l_p / l_p \tag{4}$$

By $\Delta \epsilon_p$ and the obtained stress-strain relationship, the stress of external pre-stressed steels can be determined.

$$\sigma_p = E_p \left(\frac{\sigma_{pe}}{E_p} + \Delta \epsilon_p \right) \tag{5}$$

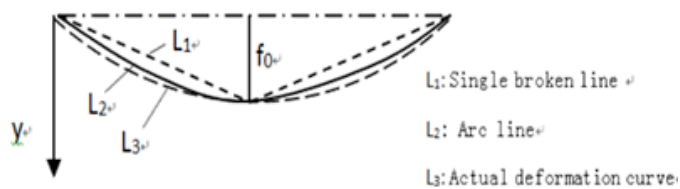


Figure 2: Deformation Curve

Wang Yunfei from Southeast University uses the broken line and arc line to calculate the stress increment respectively, as shown in figure 2. He reaches the conclusion by comparison that the deviation is less than 3%, and the actual deformation curve of the beam is between single line and arc line (He ZQ., 2007). According to the single line model, θ_A and θ_B approximately is: $\theta_A = \theta_B = 2f_0/L$.

2.3 Deflection Calculation of External Pre-stressed Concrete Beam

Under the action of external load, the deflection of external pre-stressed concrete beam can be calculated by using the method of elastic homogeneous materials beam. There into the maximum deformation of the midspan f can be calculated as follows:

$$f = SML^2/B_s \quad (6)$$

On the basis of determining the rigidity of the concrete beam, we can calculate the deformation and then get the stress increment of external pre-stressed steels. The calculation steps are as follows:

1. Gradingly apply external loads, and determine the moment increment of the concrete beam in grade i , $\Delta M_{ki} = M_i - M_{i-1}$;
2. According to the M_i under load i , obtain the bending rigidity of concrete beam by formula (1) and (2).
3. Using the method of structural mechanics, calculate the increment of deflection and angle by ΔM_k . According to the deformation, using formula (4) and (5), get the steel stress increment $\Delta\sigma_{p1}$.
4. Calculate the equivalent load increment by $\Delta\sigma_{p1}$. Then get the inverse moment increment caused by equivalent load ΔM_{pe} . Making $\Delta M_{ki} = \Delta M_{ki} - M_{pe}$, calculate the deformation and the steel stress increment $\Delta\sigma_{p2}$ again.
5. If $\Delta\sigma_{p2}$ is closes to $\Delta\sigma_{p1}$ enough, end the computation; Otherwise repeat step 3 and 4 until two external steel stress increments are close to each other enough.

3. Iterative Method for Simulation of Reinforcement Process

3.1 Program Flow Chart

According to the process, the whole strengthening process of the external pre-stressed concrete beam is programmed. The calculation process is shown in figure (3).

3.2 Test Analysis

Test results of 4 external pre-stressed concrete beams are used to verify the deformations which are gained by the iterative method using the load-deformation-stress increment relationship. The test dates are obtained from the experiment of Jia Huilin of Southeast University (Jia, 2007). The basic information of selected components is shown in Table 1. The four beams section size is 3600×200×400(mm), normal steels are 2 ϕ 16, pre-stressed steels are 2×7-15.2, loading mode is two points loading. Loading-deflection curves are shown in figure (4).

From the results of the above analysis, the calculated values are close to the experimental values. Based on the eccentric compression rigidity formula and the relationship of loading-deflection-stress increment, the iterative method for calculating the deflection of the beam is feasible, and the results can meet the accuracy requirement after iteration 3 times.

But the values got by this method vary greatly with experimental ones after the steels yielding. The bibliography points out that formula (2) need to be modified when compressed steels reach the yielding strength or 2‰ of the corresponding calculation deformation. At the same time, there is a large mutation at the critical point of the magnitude and small eccentric compression. In addition, the deviation of the calculated value and the test value is also large when e_0/h_0 is small. It may be due to the deviation of the geometric center of gravity and physical center of gravity in small eccentric compression.

Table 1: Experimental Information of Selected Components

No.	Steels arrangement	Reinforcement procedure
L1	cross	Reinforce when crack width is 0.1mm
L2	Double turning line	Reinforce when crack width is 0.1mm
L3	cross	Reinforce when crack width is 0.2mm
L4	cross	First tension then loading

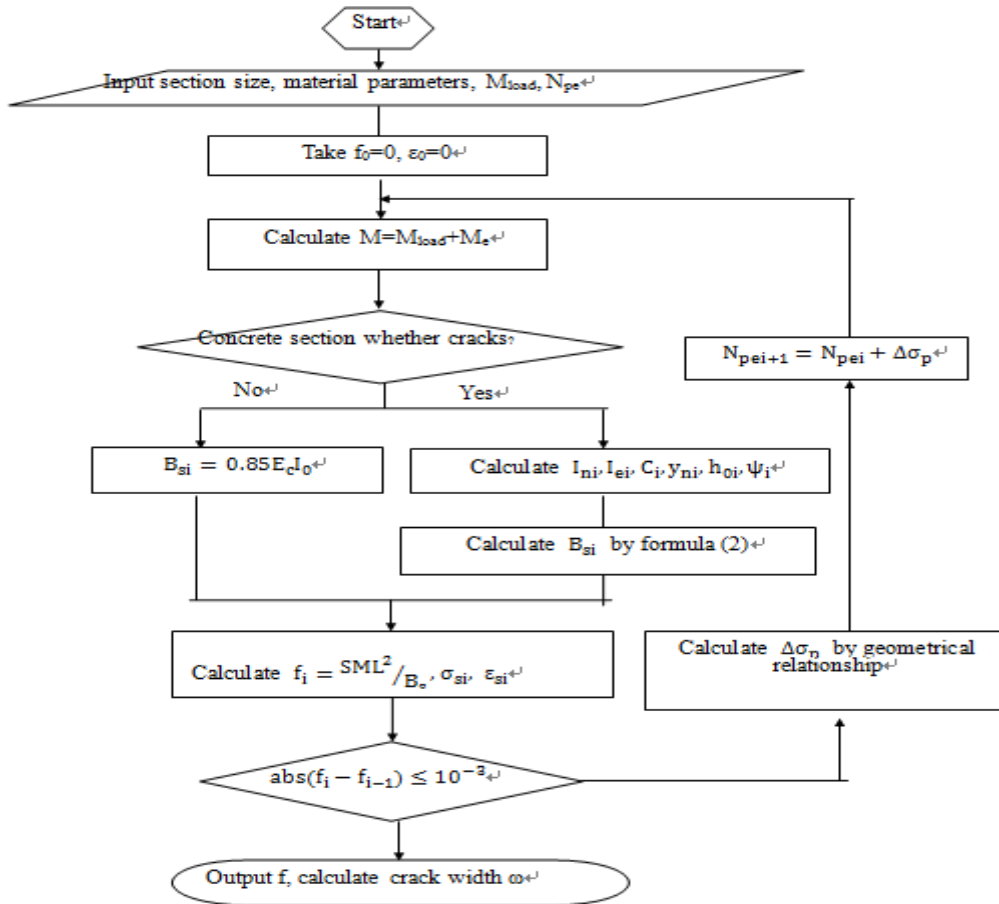


Figure 3: Program Flow Chart

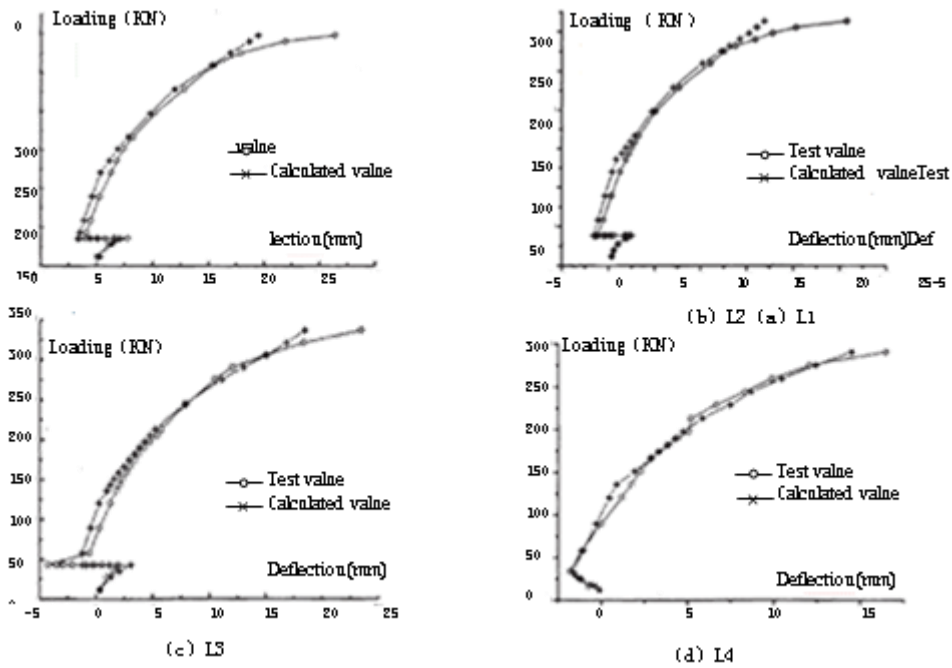


Figure 4: Loading-deflection Curve

4. Improvement about the deflection calculation formula of the beam strengthened by external pre-stressed steels

The method above can be used with high accuracy to simulate the performance of the normal using stage of the beam with external pre-stressed steels. But if it is used in engineering, the calculation process is complicated, so it needs to be improved to facilitate the engineering application.

In order to simplify the calculation of the deflection of the strengthened beams after the cross section cracking, basing on the rigidity formula of eccentric compression, considering the influence of the quadratic effect, deformation uncoordinated and whether unloading to the beam rigidity, we carry on the correction to formula (2).

4.1 Consider the Influence of Quadratic Effect on External Pre-stressed Concrete Beam Rigidity

When considering the quadratic effect, the deflection cannot be calculated by the formula of external pre-stressed concrete members (Zhang F., et. al, 2006). Eccentric distance reduction of external pre-stressed steels equal to the relative upward displacement of pre-stressed steels of the corresponding section Δ , so changes in the loading stage of external pre-stressed steels eccentric distance and highly effective can be expressed as:

$$e_{ps} = e_0 \cdot \Delta \quad (7)$$

$$d_{ps} = d_{ps0} - \Delta \quad (8)$$

Due to the configuration of non-pre-stressed steel beam, load- deflection curve showed a linear relationship before the steel yielding. So Δ can be calculated by span moments based on the elasticity theory. According to the stress of external pre-stressed concrete beams, Δ can be calculated by the following formula:

$$\Delta = k_1 \frac{ML^2}{E_c I_e} - k_2 \frac{\sigma_p A_p e_p L^2}{E_c I_e} \quad (0 \leq M \leq M_{cr}) \quad (9)$$

If consider the beam is simply supported, three-point loading is commonly used; k_1, k_2 can be calculated with the method of structural mechanics. The calculation of k_2 also needs to consider the influence of the layout of external pre-stressed steels and the number of the steering blocks. It can be determined by the values in Table 2.

Table 2: Coefficient K_1, K_2

Steering block		Coefficient	
Number	Position	k_1	k_2
0	-	23/216	1/8
1	Middle	0	0
2	Three-point	1/72	$\cos\alpha/72 + L\sin\alpha/216e_p$

Considering the effect of quadratic effect of external pre-stressing, short-term rigidity B_s of external pre-stressed concrete beam can be calculated approximately by formula (10):

$$B_s = \frac{(E_s A_s + E_p A_p) h_0^2}{\psi \left(1.15 - 0.4 \frac{h_0}{e + \Delta} \right) + 0.2 + \frac{6\alpha E_p \rho}{1 + 3.5\gamma_f}} \quad (10)$$

4.2 Consider the Impact of External Pre-stressed Steels on Beam Rigidity Reduction

Considering the contribution of applying pre-stress to rigidity by $E_p A_p$, and the contribution of longitudinal tensile steels to rigidity by general steels ratio ρ , the formula (20) can be further revised. The equivalent longitudinal tension steels ratio can be calculated by formula (11).

$$\rho_e = \frac{A_s + \alpha A_p}{bh_0} \quad (11)$$

Therefore, formula (10) is modified as follows:

$$B_s = \frac{(E_s A_s + \alpha E_p A_p) h_0^2}{\psi \left(1.15 - 0.4 \frac{h_0}{e + \Delta} \right) + 0.2 + \frac{6\alpha E_p \rho_e}{1 + 3.5\gamma_f}} \quad (12)$$

It should be emphasized that the value α in formula (12) is obtained by the test results of the non- bonded partially pre-stressed concrete beam at using stage. So the formula does not apply to the calculation of the bearing capacity limit (Niu, 2000).

4.3 Consider the Effect on the Rigidity of External Pre-stressed Concrete Beam when Unloaded or not

In order to facilitate the engineering application, for the strengthened beams without unloaded steels imagine an unloaded steels state, at this time the effective pre-stress is N_{pe0} . Under the same load, pre-stress increment is $\Delta\sigma_p$. That is, the effective pre-stress is $N_{pe0} = N_{pe} - A_p\Delta\sigma_p$ at an imaginary unloaded and strengthened state. From the previous discussion, the stress increment $\Delta\sigma_p$ is a function related to deflection. Therefore, when calculating the deflection of the non-unloaded of concrete beam in external pre-stressed steels, convert the effective pre-stress into one under the unloaded condition, and substitute it into formula (12) to calculate.

5. Conclusion

From the analysis of the stress state of the external pre-stressed beam, it can be seen that the bending state plays a decisive role in the short term rigidity of the beam, while the bending state can be regarded as the eccentric compression state as well. Therefore, the short-term rigidity of external pre-stressed concrete can be calculated by using the rigidity formula of the concrete eccentric compression member.

Based on the rigidity formula of ordinary concrete beam with eccentric compression, we worked out the corresponding program by using the equivalent load-stress increment-rigidity relationship, and use this method to calculate the inverse arch value in the stretching process of external pre-stressed steels.

On the basis of the above analysis, in consideration of the quadratic effect, the uncoordinated deformation of external pre-stressed steels and concrete cross section, and whether the steels being unloaded or not. This paper modifies the calculation formula of deflection during the subsequent loading process. By calculation comparison, the rigidity correction formula presented in this paper is in good agreement with the experimental values, which can be used to calculate the deflection under the normal using state.

Acknowledgment

The project was supported by the Open Fund of Hunan Province University Key Laboratory of Bridge Engineering (Changsha University of Science & Technology, China, No.14KD07), the Key Discipline Project of Civil Engineering of Changsha University of Science & Technology, China (No.15ZDXK17), the General Project of Hunan Provincial Department of Education, China (No.15C0050).

References

- Alkhairi F.M., 1991, On the Flexural Behaviour of Concrete Beams Pre-tressing with Unbonded Internal and External Tendons, The University of Michigan, USA.
- Deng X.H., 1998, Mechanical Properties Study of External Pre-stressed Beams, Tongji University.
- Du J.S., Liu X.L., 2003, Research on the Stress Change of Non-bonded Pre-stressed Tendons Based on Structural Deformation, Chinese Journal of Civil Engineering, 36, (8), 12-19.
- He Z.Q., 2007, Stress Increment Unified Algorithm of RC Beams by External Pre-stressed Tendons Based on Deflection, China Civil Engineering Journal.
- Jia H.L., 2007, Cross Type in Vitro Pre-stressed Reinforcement Beam Experimental Study, Southeast University.
- Li G.P., 2004, Bridge Pre-stressed Concrete Technology and Design principle, People's Communications Press.
- Lv Z.T., Meng SP., 1998, Modern Pre-stressed Design, China Building Industry Press, 23-26.
- Niu B., 2000, Limit State Analysis of Pre-stressed Concrete Beam in Vitro, Chinese Journal of Civil Engineering, 33 (3): 15-7.
- Tan K.H., Ng C.K., 1997, Effect of Deviator and Tendon Configuration on Behaviour of External Pre-stressed Beams, ACI Structural Journal, Jan-Feb, 13-22.
- Xiong X.Y., 2005, External Pre-stressed Structure Design, China Building Industry Press, 1-5.
- Xiong X.Y., Lei L.Y., 2004, Design of External Pre-stressing Structure, Industrial Architecture, 34 (7).
- Zhang F., Xu D., Xi G.H., 2006, Study on the Quadratic Effect of External Pre-stressed Steel Bundle in Elastic Stage, Structural Engineer, 22 (6).