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Research on the Application of Polynomial Direct Fitting to Platinum High-precision Temperature Measurement

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Although many methods can be used to conduct platinum resistor non-linear rectification, they cannot be put into wide use because of shortcomings such as narrow temperature range and complicated calculation of rectification. This article conducts fitting algorism based on three temperature ranges of platinum resistor within temperature range and obtains cubic and quartic polynomial coefficients via the method of adjusting weight function using least square method after a brief summary of the regular method and problems existing in platinum resistor non-linear function inversion calculation. Meanwhile, fitting algorism error distribution graph is also demonstrated. The maximum error is only ±0. 003℃ when applying quartic multinomial fitting algorism within the range of 0-650℃. Finally, the advantages of multinomial direct fitting is further testified an analysis of C language calculation precision and data instances.

1. Introduction

Temperature is a physical quantity that represents objects' degree of hotness or coldness. It serves as an important parameter in industrial production and scientific experiments. Characterized by features such as stable performance, wide range of temperature measurement, easy calibration and good interchangeability, platinum resistor has been widely used in temperature measurement. Stipulated in International Temperature Scale ITS-90, specially structured platinum resistor is used as standard thermometer between -259℃ and 961. 78℃.

Because non-linear relationship exists between platinum resistor's resistance value and temperature, non-linear operation (non-linear rectification) is required when temperature is reappeared. Based on scaling function and scaling table, non-linear operation is an indispensable part of high-accuracy temperature measurement with various methods. Although Newton Iteration Method can be used to solve platinum resistor scaling function and satisfactory results can be obtained after second iteration, iteration takes too long and internal memory is occupied too much. When Neural Network Approach is applied to conduct polynomial fitting for three times between 0℃ and 600℃ divided into six parts, the error is less than 0. 02℃. The temperature is calculated on the basis of the resistor value via radical sign operation's analytic expression [Ping Yang]. The error ≤0. 02℃ is obtained within the range of 0℃-150℃ via symmetric function non-linear method and the error ≤0. 05℃is achieved within the range of 48℃-50℃ via least square method.

Intelligent digital instrument whose core unit is an 8 bit single-byte-character single chip microcomputer is generally used to conduct high-accuracy temperature measurement. Therefore, the simple and direct algorithm used in studying the relationship between platinum resistor and temperature occupies little space and is valuable.

2. Non-linearity of Platinum Resistor (PT100)

Pt100 is the most frequently used temperature repetition element among Pt 10, Pt 100 and Pt 1000.
The relationship between platinum resistor and temperature within the range of 0℃-850℃ is:

$$R_t = R_0(1 + at + bt^2) \tag{1}$$

In the equation
a=3. 90 802E-3/℃

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b=-5. 80195E-7/℃
R (100℃) / R (0℃) =1. 385 00
Factor (1) indicates the relationship between the temperature and the resistor. Therefore, the temperature calculated via resistor value is an inversion process.
If resistor value R_F is set corresponding to upper limit of temperature range t_F and the temperature and resistor value within the measurement range is linear, the non-linear error is:

$$e_R = R_0(1+at+bt^2) - \left[\frac{R_F - R_0}{t_F - t_0} \bullet t + R_0\right] \quad (2)$$

Derivatives of formula (2) are calculated and ordered to be 0:

$$\frac{de_R}{dt} = aR_0 + 2bR_0 t - \left[\frac{R_F - R_0}{t_F - t_0}\right] = 0$$

then:

$$t = [(R_F - R_0)/(t_F - t_0) - aR_0]/2bR_0$$

The highest temperatures and resistor values of non-linear errors are 325℃/6. 127Ω and 425℃/ 10.478Ω within the temperature ranges of 0-650℃ and 0-850℃, respectively. As is shown in graph 1, the reduced non-linear is about 27℃.

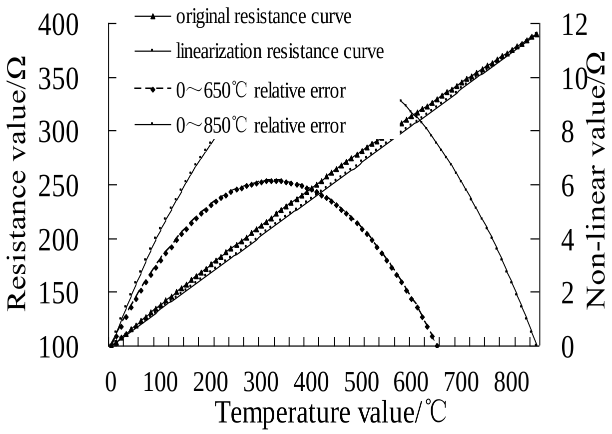

Figure 1: Non-linear errors distribution diagram within the range of 0-650℃ and 0-850℃

3. Least Square Method and Weight Function

Discrete points (x_i, y_i) (i=0, 1, 2, ……, m) .
Approximate function curve S (x) is constructed and the inherent law of the data is described, that is, selecting appropriate function class (collection)

$$\Phi = span\{\phi_0, \phi_1, \phi_2, \cdots, \phi_n\}$$

$$= \{a_0\phi_0 + a_1\phi_1 + a_2\phi_2, \cdots + a_n\phi_n \,|\, a_i \in R\}$$

φ_0, φ_1, φ_2, ……, φ_n is n+1 linearly independent continuous function on [a, b]. Find a function

$$S*(x) = \sum_{i=0}^{n} a_i^* \phi_i(x) \quad \text{(n<m)}$$

making the deviation (residual error) of S^* (x) and y=f (x) on the above-mentioned m+1 points satisfy

$$\delta_i = \delta^*(x_i) - y_i \quad \text{(i=0, 1, 2, ……, m)}$$

$$\|\delta\|_2^2=\sum_{i=0}^{m}\delta_i^2=\sum_{i=0}^{m}\omega(x_i)[S^*(x_i)-y_i]^2$$

$$=\min_{s\in\Phi}\sum_{i=0}^{m}\omega(x_i)[S^*(x_i)-y_i]^2 \quad (3)$$

among which

$$\delta=(\delta_1,\delta_2,\cdots\cdots\delta_m)^T$$

$$S(x)=\sum_{i=0}^{n}a_i\phi_j(x)$$

ω (x) ≥0 is the weight function between [a, b]. The function S^* (x) of satisfiable formula (3) is least square solution.

Least square method aims at least square error (residual sum of squares). Linear models chosen according to shapes such as composite function, growth function, logarithmic function exponential function or logic function can be used when fitting curves. However, methods such as Taylor's series are still used for polynomial in program compilation, therefore, only polynomial is most suitable for sequential algorithm.

Although increasing the orders of polynomial can improve fitting effect in polynomial fitting, it may lead to ill-conditioned equations. If segment fitting is adopted, moving least square can be used to improve fitting precision.

Draw a t-R curve at certain temperature intervals for formula (1), and then conduct r-T fitting. Fitting based on temperature range is done in order to facilitate sequential algorithm. Considering approaching original function to the maximum or error distribution accompanies original function and showing symmetry, order constant whose weight function ω (x) is "1" to conduct fitting. Observe error distribution, adjust corresponding nodes' weight of weight function, and then fit again. Such process is repeated until error distribution approaches symmetry.

4. Platinum Resistor Polynomial Fitting

Polynomial can be loop calculation based on product-addition, which occupies small internal memory and has rapid calculation. See formula (4)

$$t_{(r)}=a_0+(a_1+(a_2+(a_3+a_4\bullet r)\bullet r)\bullet r)\bullet r \quad (4)$$

$t_{(r)}$ is temperature value, r is actually measured platinum resistor value. Once platinum resistor value is measured, temperature value can be calculated via formula (3).

When using platinum resistor to measure temperature, 0-650℃ is considered as standard temperature range, 650℃-850℃ is extension range and 0-850℃ is global range. Therefore, fitting calculations are conducted based on three ranges.

According to above principles, coefficients of polynomial (4) can be obtained via Matlab, see graph 1.

Concerning global range, maximum absolute value error of cubic polynomial is 0. 1567℃, which satisfies on-site temperature control. Maximum absolute value error of quartic polynomial after fitting is 0. 0249℃, which meets the precision requirement of 0. 1℃.

When it comes to standard temperature range, maximum absolute value errors of cubic polynomial and quartic polynomial are 0. 0320℃ and 0. 0024℃ respectively. Quartic polynomial satisfies the precision requirement of 0. 005℃.

Regarding extension range, maximum absolute value errors quadratic polynomial and cubic polynomial are 0. 0194℃ and 0. 0053℃ respectively.

As is shown in graph 1, cubic polynomials in these ranges can meet precision requirements of most engineering temperature measurements. Concerning measurement requirements, quartic polynomial can be applied. Extension range in graph 1 also shows that with regard to the same order, the smaller the range is, the higher the precision is when applying least square fit.

Table 1: Cubic polynomial and quartic polynomial graph within three temperature ranges

range /℃	order	coefficients of each polynomial						
		a_0	a_1	a_2	a_3	a_4	$+e_{max}$	$-e_{max}$
0-850	3	-2. 4942627E+02	2. 4371743E+00	4. 3341924E-04	1. 3815819E-06		0. 1567	-0. 1452
	4	-2. 4668008E+02	2. 3765713E+00	8. 8576133E-04		1. 4772388E-09	0. 0249	-0. 0242
0-650	3	-2. 4864735E+02	2. 4216850E+00	5. 2163889E-04	1. 2304235E-06		0. 0320	-0. 0312
	4	-2. 46389305E+02	2. 37230798E+00	9. 01867745E-04		1. 42268923E-09	0. 0024	-0. 0024
650-850	2	-1. 6886298E+02	1. 8044518E+00	2. 0658125E-03			0. 0179	-0. 0194
	3	-2. 5720370E+02	2. 5450480E+00		1. 9173044E-06		0. 0048	-0. 0053

In order to better show fitting effect, graph 2 and 3 indicate error distributions in global and standard ranges.

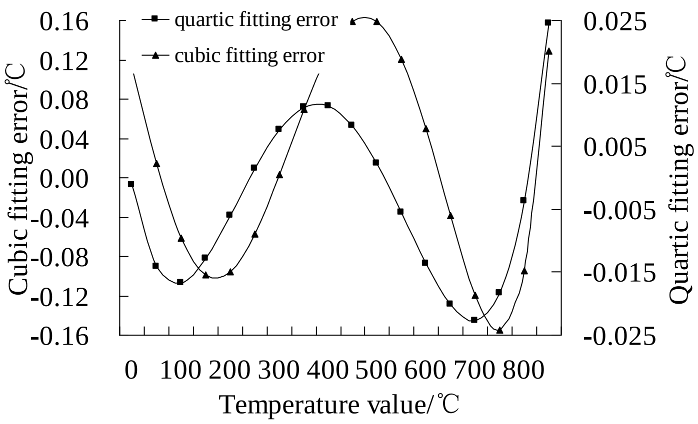

Figure 2: Error distribution of cubic and quartic fitting within the range of 0-850℃

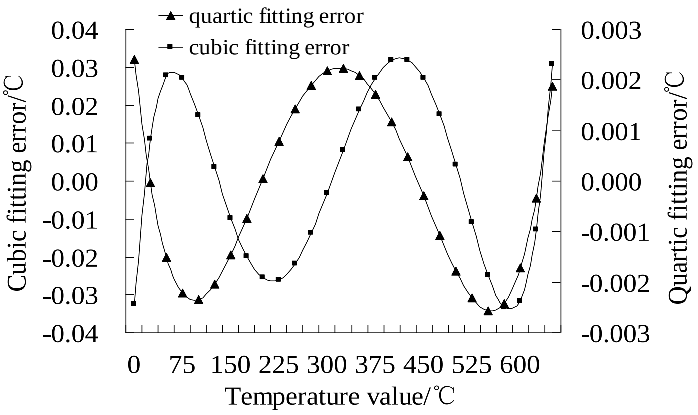

Figure 3: Error distribution of cubic and quartic fitting within the range of 0-650℃

5. C Language and Calculation Speed

C language is the widely used program in intelligent instrument using Single Chip Microcomputer and System On Chip. C language itself has no precision standard. Real variable is classified into three groups, namely, float, double, and long double, of which float takes up only 4 bits and has the highest calculation speed. In System On Chip, float is universally used in calculation considering its small internal memory.
Set standard 8051 Single Chip Microcomputer and 12MHz oscillation as an example, check with C language at Keil platform and conduct quartic polynomial fitting at 0-650℃. Formula (5) is obtained via calculating coefficients in graph 1 in formula (4). The calculation results and EXCEL calculation results are shown in graph 2.

$$t_{(r)} = -246.38931 \quad (5)$$
$$+(2.3723080$$
$$+(9.0186775\text{E-}04$$
$$+(0+1.4226892\text{E-}09 \bullet r) \bullet r) \bullet r) \bullet r$$

Table 2: C language and EXCEL calculation comparison

Platinum resistance/Ω	nominal temperature /℃	EXCEL calculation/℃	C language calculation/℃	
		temperature	Temperature	error
100. 000	0. 000	0. 002439	0. 002457	-0. 0025
119. 395	50. 000	49. 99779	49. 99779	0. 0022
138. 500	100. 000	99. 99869	99. 99870	0. 0013
157. 315	150. 000	150. 0011	150. 0011	-0. 0011
175. 840	200. 000	200. 0029	200. 0030	-0. 0030
194. 074	250. 000	250. 0009	250. 0009	-0. 0009
212. 019	300. 000	300. 0007	300. 0007	-0. 0007
229. 673	350. 000	349. 9977	349. 9977	0. 0023
247. 038	400. 000	399. 9985	399. 9985	0. 0015
264. 112	450. 000	449. 9981	449. 9980	0. 0020
280. 896	500. 000	499. 9993	499. 9993	0. 0007
297. 390	550. 000	550. 0012	550. 0013	-0. 0013
313. 594	600. 000	600. 0018	600. 0018	-0. 0018
329. 508	650. 000	649. 9975	649. 9976	0. 0024

Program fitting calculation time is 1470MT (Machine Period), 1487MT and 1488MT with the resistor value being 100. 000Ω, 212. 019Ω and 329. 508Ω respectively, thus proving the high calculation speed of formula (4).
Table 2 also demonstrates the calculation correctness of Keli C.

6. Conclusions

Fitting algorism based on three ranges within temperature range of platinum resistor is conducted and cubic and quartic polynomial coefficients are obtained via the method of adjusting weight function using least square method after a brief summary of the regular method and problems existing in platinum resistor non-linear function inversion calculation. Meanwhile, fitting algorism error distribution graph is demonstrated. The maximum error is only ±0. 003℃ when applying quartic multinomial fitting algorism within the range of 0-650℃. With the advantages of simple algorithm, small internal memory, and rapid arithmetic speed, multinomial fitting algorism is especially suitable for single chip microcomputer or SOC using C language programming. The precision and speed of C language using single precision real type variable operations further testifies the advantages of multinomial fitting. The multinomial coefficients provided in this article can be directly applied to platinum resistor temperature measurement. This method can also be applied to thermocouple temperature measurement.

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