



Controlling Chaos in Permanent Magnet Synchronous Motor

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This paper mainly analyses chaotic phenomenon of PMSM when PMSM in turn off and put forward an effective way to control chaos of PMSM by feedback in practice. Testing different parameters of motor have what effect on chaos controlling of PMSM. Finally, verifying the validity of way.

1. Introduction

Since permanent magnet synchronous motor, PMSM have many advantages than traditional motor in that it high efficiency, high power density and high torque current and so on. At the same time, PMSM expose a defect that PMSM represent complex chaotic phenomenon when PMSM in turn on or turn off. Once motor in chaos, motor torque changes randomly and the motor speed oscillate within a range (Carroll and Pecora (1991), Chen G, Dong X (1998), A.S. Elwakil (2002)). Permanent magnet synchronous motor (PMSM) is of great interest, particularly for industrial applications in the low-medium power range, since it has superior features such as compact size, high torque/weight ratio, high torque/inertia ratio and absence of rotor losses. Over the past few years, the secure and stable operation of PMSM, which is essential requirement of industrial automation manufacturing, has been received considerable attention. (Dedieu et al. (1993), Abolhassan Razminia et al. (2011), Duan LiXia et al. (2009), Halle K S et al (1993)). It is found that with certain system parameter values, the PMSM is experiencing chaotic behaviour. (Hindmarsh (2005) and Zheng (2010)). Hindmarsh has controlled chaos in PMSM by backstepping technique (Hindmarsh (2005)). However, there is a drawback in conventional backstepping method called the problem of explosion of terms' caused by the repeated differentiations of virtual input. The main aim of this work is to find a proper and applicable method for controlling chaos in PMSM. On the other hand, recently, most works on chaos control and synchronization have noticed the important role of the stability time; e.g., Li et al applied a terminal sliding mode controller to synchronize a class of chaotic systems with mismatched parametric uncertainties Li et al. (2005), Mohammad Ataei et al (2006) reported and Yu and Peng (2005). Mohamed Zribi et al (2008) finite-time chaos control and synchronization for unified chaotic systems with uncertain parameters.

Due to PMSM is multivariable, nonlinear and strongly coupled, controlling chaos in motor is very difficult. With development of chaos, there are many means of controlling and analysis chaos. The OGY method is a basic methodology for controlling chaos but finding a reasonable parameter is not often simple. A neuro-fuzzy controller (NFC) is suitable for control of systems with uncertainties and nonlinearities.

Some ways also are used in chaos controlling of PMSM including impulsive parametric perturbations, adaptive backstepping, linear and nonlinear controller (Peng et al. (2008), Mohammad Ataei et al (2010)).

Over the past few years, the secure and stable operation of PMSM, which is essential requirement of industrial automation manufacturing, has been received considerable attention. It is found that with certain system parameter values, the PMSM is experiencing chaotic behaviour, see, for instance, the works by Marat Rafikov and José Manoel Balthazar (2004). Chaotic behaviour in PMSM, which appears mainly intermit-tent ripples of torque, low-frequency oscillations of ro-tational speed of motor, can extremely destroy the sta-bility of the motor even induce the collapse of driven system. The existence of chaotic behaviour in PMSM is highly undesirable for its performance. Thus, controlling chaos in PMSM is a significant job. However, to the best of our knowledge, analytical investigation into controlling chaos in PMSM is few and the existing controlled methods are defective and unacceptable to practical use. For example, in our previous work, we suggested that the chaos in PMSM can be controlled by the entrainment and migration control strategy (Hindmarsh (2005)). But, this control strategy does not permit the control objective to be any part of trajectory of the controlled system and the control law cannot put into effect until the states of the system enter into the domain

of attractions, which may not be consistent with the requirement of the application. So, this method is not desirable to practical use; Hindmarsh has controlled chaos in PMSM by backstepping technique (Chen (1998)). However, there is a drawback in conventional back-stepping method called the problem of 'explosion of terms' caused by the repeated differentiations of virtual input.

The main aim of this work is to find a proper and applicable method for controlling chaos in PMSM. On the other hand, recently, most works on chaos control and synchronization have noticed the important role of the stability time; e.g., Mohammad et al applied a terminal sliding mode controller to synchronize a class of chaotic systems with mismatched parametric uncertainties (Mohammad et al. (2010)), Yu and Peng (2005) proposed finite-time chaos control and synchronization for unified chaotic systems with uncertain parameters. To control the undesirable chaos in PMSM, we present a finite-time chaos control based on the finite-time stability theory. Simulation results indicate that the proposed control law is very effective.

This paper devise a project to how to achieve aim in theory and test different parameter of PMSM have what effect on chaos controlling of PMSM.

2. Model of PMSM

PMSM base on the d-q axis can be described as follows [15-21].

$$\begin{cases} \frac{di_d}{dt} = (u_d - R_1 i_d + \omega L_q i_q) / L_d \\ \frac{di_q}{dt} = (u_q - R_1 i_q - \omega L_d i_d - \omega \psi_f) / L_q \\ \frac{d\omega}{dt} = [n_p \psi_f i_q + n_p (L_d - L_q) i_d i_q - T_L - \beta \omega] / J \end{cases} \quad (1)$$

where i_d , i_q and ω are state variables and motor angular frequency respectively; u_d and u_q are direct-axis and quadrature-axis stator voltage components, respectively; J is the polar moment of inertia; T_L is external load torque; β is the viscous damping coefficient; R_1 is stator winding resistance; L_d and L_q are the direct-axis and quadrature-axis stator inductors, respectively; ψ_f is permanent-magnet flux; n_p is the number of pole-pairs.

Applying transformation form [], $x = \lambda \tilde{x}$, and $t = \tau \tilde{t}$, where $x = [i_d \quad i_q \quad \omega]^T$, $\tilde{x} = [\tilde{i}_d \quad \tilde{i}_q \quad \tilde{\omega}]^T$,

$$\lambda = \begin{bmatrix} \lambda_d & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \lambda_q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \lambda_\omega \end{bmatrix}^T = \begin{bmatrix} b k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 1/\tau \end{bmatrix}, \quad b = \frac{L_q}{L_d}, \quad k = \frac{\beta}{n_p \tau \psi_f}, \quad \tau = \frac{L_q}{R_1}.$$

The transformed model of PMSM can be described as a set of equations in the following nondimensionalized form.

$$\begin{cases} \frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega} \tilde{i}_q + \tilde{u}_d \\ \frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q - \tilde{\omega} \tilde{i}_d + \gamma \tilde{\omega} + \tilde{u}_q \\ \frac{d\tilde{\omega}}{d\tilde{t}} = \sigma (\tilde{i}_q - \tilde{\omega}) + \xi \tilde{i}_d \tilde{i}_q - \tilde{T}_L \end{cases} \quad (2)$$

where $\gamma = \frac{n_p \psi_f^2}{R_1 \beta}$, $\sigma = \frac{L_q \beta}{R_1 J}$, $\tilde{u}_d = \frac{n_p L_q \psi_f u_d}{R_1^2 \beta}$, $\xi = \frac{L_q \beta^2 (L_d - L_q)}{L_d J n_p \psi_f^2}$, $\tilde{T}_L = \frac{L_q^2 T_L}{R_1^2 J}$, $n_p = \mathbf{1}$.

When $L_d = L_q = L$, the transformed model of PMSM is expressed as follows.

$$\begin{cases} \frac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega} \tilde{i}_q + \tilde{u}_d \\ \frac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q - \tilde{\omega} \tilde{i}_d + \gamma \tilde{\omega} + \tilde{u}_q \\ \frac{d\tilde{\omega}}{d\tilde{t}} = \sigma (\tilde{i}_q - \tilde{\omega}) + \tilde{T}_L \end{cases} \quad (3)$$

where \tilde{i}_d and \tilde{i}_q are the transformed direct-axis stator and quadrature-axis stator currents respectively; $\tilde{\omega}$ is transformed angular speed of the motor; \tilde{u}_d and \tilde{u}_q are transformed direct-axis and quadrature-axis stator voltage components, respectively; \tilde{T}_L is the transformed external load torque; σ and γ are system parameters. If $\tilde{u}_d = \tilde{u}_q = \tilde{T}_L = \mathbf{0}$, system (3) can be thought as that the external inputs of system (3) are

removed after a period of the operating of the system. Supposing $x_1 = \tilde{i}_d$, $x_2 = \tilde{i}_q$ and $x_3 = \tilde{\omega}$, system (3) is rewritten as

$$\begin{cases} \dot{x}_1 = -x_1 + x_3 x_2 \\ \dot{x}_2 = -x_2 - x_3 x_1 + \gamma x_3 \\ \dot{x}_3 = \sigma(x_2 - x_3) \end{cases} \quad (4)$$

3. Analyse of chaos of system (4)

For system (4), there are two parameters, γ and σ , except x_1 , x_2 and x_3 . The values of σ and γ have effect on chaos of PMSM. For system (4), due to $\sigma > 0$, $\Delta V = \frac{dx_1}{dx_1} + \frac{dx_2}{dx_2} + \frac{dx_3}{dx_3} = -(\sigma + 2) < 0$.

Basing on dissipation of system, we can draw a conclusion that system (4) is chaotic system. Fig. 1 show the evolution of the largest Lyapunov exponent base γ under the condition that $\sigma > 0$. From Fig. 1, we can see that the largest Lyapunov exponent change from negative to positive with the increasing of parameter γ .

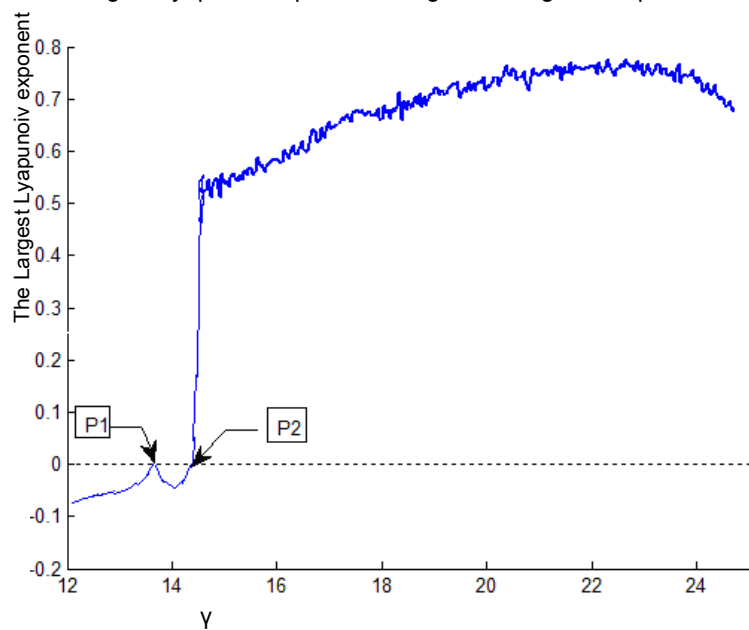


Figure 1: The evolution of the largest Lyapunov exponent

The largest Lyapunov exponent is an important index for estimating chaos of system, every dynamic system has a spectrum of Lyapunov exponents which determines how length, area and volumes change in phase space.

In sum, any bounded motion in a system contains at least one positive Lyapunov exponent is defined as chaotic, nonpositive Lyapunov exponents indicate periodic motion.

From Fig. 1, we can see that the system (4) state change from non-chaos to chaos when $\gamma = 14.3$. Fig. 2 are phase portrait and time response when $\gamma = 12$ for system (4). Fig. 3 are phase portrait, Poincare map and frequency spectrum when $\gamma = 14.3$. Fig.4 are phase portrait, Poincare map and frequency spectrum when $\gamma = 17$.

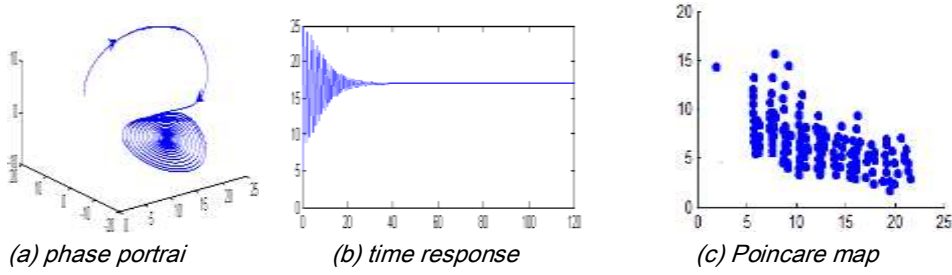


Figure 2: The state of system (4)

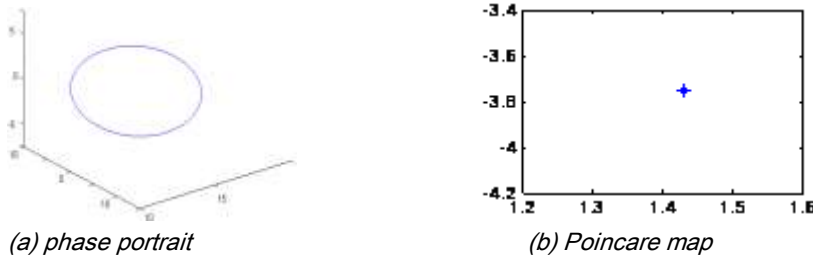


Figure 3: The phase portrait of system (4)

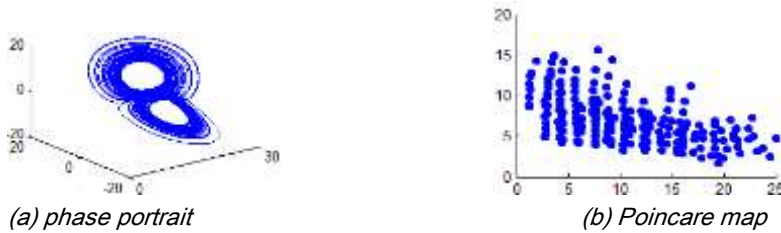


Figure 4: The phase portrait and Poincare map of system (4)

From Fig. 2, Fig. 3, and Fig. 4, we can draw a conclusion that with the increasing of γ , the chaos of system (4) change from non-chaos to chaos.

4. Chaos controlling

Aim of chaos controlling is that chaotic motion have to be changed into periodic motion for improving performance of PMSM, this section propose a way base on feedback mechanism to control chaos of PMSM.

System (4) is selected as the drive system when $\gamma = \gamma_1$ and system (4) is selected as the response system when $\gamma = \gamma_2$.

$$\begin{cases} \dot{x}_1 = -x_1 + x_3 x_2 \\ \dot{x}_2 = -x_2 - x_3 x_1 + \gamma_1 x_3 \\ \dot{x}_3 = \sigma(x_2 - x_3) \end{cases} \tag{5}$$

$$\begin{cases} \dot{y}_1 = -y_1 + y_3 y_2 \\ \dot{y}_2 = -y_2 - y_3 y_1 + \gamma_2 y_3 \\ \dot{y}_3 = \sigma(y_2 - y_3) \end{cases} \tag{6}$$

A control signal is introduced into system (6) as a feedback controller to synchronize system (5) and system (6). The control signal can be devise a linear, the control signal was incorporated into system (6) to yield coupled system (7),

$$\begin{cases} \dot{y}_1 = -y_1 + y_3 y_2 + a(x_1 - y_1) \\ \dot{y}_2 = -y_2 - y_3 y_1 + \gamma_2 y_3 + b(x_2 - y_2) \\ \dot{y}_3 = \sigma(y_2 - y_3) + c(x_3 - y_3) \end{cases} \quad (7)$$

where a , b and c are feedback gain, basing on theory of dissipation of system,

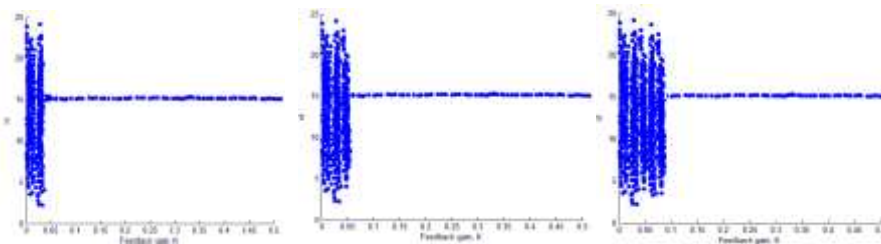
$$\Delta V = \frac{dy_1}{dt} + \frac{dy_2}{dt} + \frac{dy_3}{dt} = -2 - \sigma - a - b - c,$$

due to system (7) is chaotic system, $\Delta V < 0$, $\sigma > -2 - a - b - c > 0$, $a + b + c < -2$.

From system (3) and system (4), we learn x_3 denote $\tilde{\omega}$, x_1 denote \tilde{i}_d , x_2 denote \tilde{i}_q . We can know from system (7) that $a + b + c < -2$ by calculate theoretic. If we check carefully system (7), we can find that $a \neq 0$, $b \neq 0$, $c \neq 0$, the system (6) can reach control aim effectual. When $a \neq 0$, $b \neq 0$, $c \neq 0$, and $\gamma = 18$, $a + b + c < -2$, the stable equilibrium point motion occurred, setting $k = a = b = c$, when $k > \max(\lambda_1, \lambda_2, \lambda_3)$, the system can be controlled to stable state the quickest than other value of k . The system (7) can be showed as follows,

$$\begin{cases} \dot{y}_1 = -y_1 + y_3 y_2 + k(x_1 - y_1) \\ \dot{y}_2 = -y_2 - y_3 y_1 + \gamma_2 y_3 + k(x_2 - y_2) \\ \dot{y}_3 = \sigma(y_2 - y_3) + k(x_3 - y_3) \end{cases} \quad (8)$$

system (8) exhibits chaotic motion, Fig. 4, when $k = 0$ and $\gamma = 17$. The feedback gains k are varied from 0.0 to 0.5 for observing variety of k have what effect on chaotic control of system (8). Fig. 4 shows bifurcation diagram of system (8) base on i_d and k , at the same time, $\gamma = 15$, $\gamma = 17$ and $\gamma = 19$, respectively.



(a) $\gamma = 15$ chaos motion (b) $\gamma = 17$ chaos motion (c) $\gamma = 19$ chaos motion

Figure 4: Chaos motion of system (8)

Fig. 4 shows that chaos motion of system (8) appear at about $k < 0.051$, chaos critical value is 0.051, the stable equilibrium point motion occurs when $k \geq 0.051$. In sum, we can draw a conclusion from Fig.4 that chaos critical value become more and more big, from about 0.045, 0.051 to 0.057, with the increasing of γ under the condition that transformation of k change from 0 to 0.5 and $\gamma \geq 14.3$.

5. Conclusions

This paper investigates mainly the complex nonlinear behaviors when PMSM in turn off, the parameter of PMSM is a critical factor to decide PMSM is whether chaos or not. Feedback technique is an effective way to control chaos. According to analysis above, choosing feedback gain k base on γ can control chaos of PMSM effectively.

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