

# Synthesis of Multi-period Heat Exchanger Network Considering Characteristics of Sub-periods

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To reduce computational loads for synthesis of multi-period Heat Exchanger Network (HEN), two methods, including a representative period method and a stepwise simplifying method, are proposed by considering the sub-period characteristics. In the representative period method, a representative period is selected, and the optimal HEN structure in this sub-period is obtained by solving a single-period HEN synthesis model. Then, the heat transfer areas and operational parameters are optimized in the fixed optimal structure. On the other hand, the stepwise simplifying method finalizes the multi-period HEN by applying the strategies of single-period HEN optimization, structure merging and multi-stream heat exchanger substitution. Results of the case studies indicate that the representative period method is suitable to solve the problems that feature large number of sub-periods and durations with different length, whereas the stepwise simplifying method can readily reach cost-effective HEN structures that satisfy the multi-period operations without being affected by variations of sub-period durations. Both of these two methods can rapidly facilitate solutions to multi-period HEN synthesis problems at large scale.

## 1. Introduction

In actual production processes, demand and supply shocks, changes in production plans and seasonal shifts may cause cyclic fluctuations of the Heat Exchanger Networks (HENs) (Walmsley et al., 2011), which usually result in problems of multi-period HEN synthesis (Bakar et al., 2013). These problems can be solved by either sequential or simultaneous methods. The former usually calls for decomposition of the design procedure into several steps to reduce the computation effort (Tantimuratha et al., 2001). However, such practice may lead to suboptimal solutions because the energy consumption and capital investment are separately optimized (Jiang and Chang, 2013). In principle, this shortcoming can be overcome by simultaneous methods, and better solutions can be obtained (Chen and Hung, 2004). A review for these methods can be found in Kang and Liu' work (Kang and Liu, 2014a; Kang and Liu, 2014b). However, for the problem of HEN synthesis in multi-period operation, the scale of the problem increase quickly with an increase in the number of process streams and operational periods, which eventually leads to computational difficulties or time-consuming tasks (Escobar et al., 2014).

To reduce the computational loads and time, Escobar et al. (2014) proposed an evolutionary algorithm based on the Lagrangean decomposition. Although the algorithm can effectively solve the multi-period HEN optimization problem, the optimality of the solutions cannot always be guaranteed because the feasible solutions are heuristically constructed in their approach. Jiang and Chang (2013) proposed an approach for multi-period HEN synthesis with timesharing mechanisms (Sadeli and Chang, 2012). In their proposed method, a single-period model on the basis of Yee and Grossmann's work (Yee and Grossmann, 1990) was constructed and solved to produce the optimal design for each period individually. A timesharing strategy was then applied to integrate all single period designs to reduce the overall capital investment, in which the utility consumption in each period was kept at its minimum. Nevertheless, their final HEN structure can only be treated as a preliminary design due to the structure complexity.

In this paper, we proposed two methods for multi-period HEN synthesis by considering characteristics of the sub-period. For multi-period optimization of HEN with large number and significantly different durations of sub-periods, the representative period method is recommended. In this method, the representative

period with the longest duration is selected, and its cost-effective HEN structure is obtained by solving the single-period HEN synthesis model. Then, the heat transfer areas and operational parameters are optimized to meet the requirements of other non-representative sub-periods on the fixed HEN structure achieved by the former step. For the multi-period optimization of HEN with similar or variable durations of sub-periods, the stepwise simplifying method is presented. The method finalizes the multi-period HEN by applying the strategies of single-period HEN optimization, structure merging and multi-stream exchanger substitution. To illustrate the procedure of the proposed methods, we presented a practical example of multi-period HEN synthesis problem. The results obtained in this work were compared with those obtained by the methods in literature, and the effectiveness and advantages of the proposed methods were further verified and demonstrated.

## 2. Two methods for multi-period HEN synthesis considering characteristics of sub-periods

For a multi-period HEN synthesis by considering characteristics of sub-periods, Figure 1 shows the procedure of HEN design by the representative period method and the stepwise simplifying method.

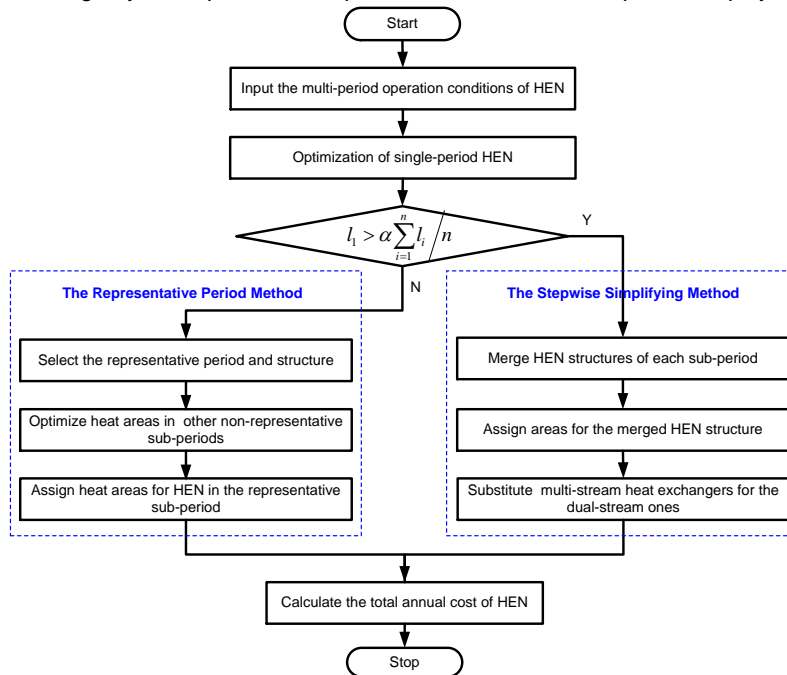


Figure 1: Procedure of the proposed methods for multi-period HEN synthesis

As shown in Figure 1, both of these two methods are based on the single period HEN optimization. One of the two methods for the multi-period HEN synthesis could be employed according to the characteristics of sub-periods after the optimal HEN structure and heat transfer areas in each period which are obtained by solving the single period HEN model, for example Yee and Grossmann's model(Yee and Grossmann, 1990).

In the proposed procedure, for a multi-period HEN with  $n$  sub-periods, assume that the length of each sub-period is different, and the lengths of the sub-periods are ranked in the descending order, i.e.  $l_1, l_2, \dots, l_n$ . If the sub-period with the longest duration  $l_1$  satisfies

$$l_1 > \alpha \sum_{i=1}^n l_i / n, \quad (1)$$

the representative period method is selected to design the multi-period HEN. Otherwise, the stepwise simplifying method is used. In Eq(1)  $\alpha$  is a parameter determined by empirical knowledge.

### 2.1 The Representative Period Method

(1) Select the representative period and structure. The sub-period with the longest duration is selected as the representative period, and its optimal structure is the representative structure.

(2) Optimize the heat transfer areas in the other non-representative sub-periods. The heat transfer areas in the non-representative sub-periods are optimized by solving the single-period HEN synthesis model with fixed representative structure selected in step (1).

(3) Assign the heat transfer areas for the representative structure. The largest heat transfer areas of the same matches in each sub-period are chosen as the final heat-transfer areas after synthesis.

## 2.2 The Stepwise Simplifying Method

(1) Merge the HEN structures of each sub-period. A primary HEN structure is constructed by choosing the heat exchangers with the same matches in different sub-periods, and then the rest of heat exchangers is merged into the primary HEN structure.

(2) Assign the heat transfer areas for the merged HEN structure. The largest heat transfer area of the same matches in each sub-period is selected as the final heat transfer areas after synthesis.

(3) Substitute the multi-stream exchangers for the dual-stream heat exchangers (Xiao et al., 2006). The dual-stream heat exchangers with the same hot or cold stream are combined and replaced by the multi-stream heat exchangers. If the replacement strategies for the same matches are optional, the one with the largest difference on heat transfer areas of multi-streams heat exchangers is chosen.

## 3. Case study

In this section, a multi-period HEN synthesis problem, which is adopted from Jiang and Chang (2013), is used to validate the proposed procedure and the two methods. The results of the proposed methods are compared with the results obtained by the methods in literature. The suitability of the proposed methods is discussed through analysis of the results in two scenarios with different characteristics of sub-periods.

In this example case, the optimal HEN structure and the heat transfer areas in each sub-period are obtained by solving the single period HEN synthesis model proposed by Yee and Grossmann (1990), as listed in Table 1.

Table 1: The optimal structure and the heat transfer areas in three sub-periods

Period 1		Period 2		Period 3	
Match ( $i, j, k$ )	Area/ $m^2$	Match ( $i, j, k$ )	Area/ $m^2$	Match ( $i, j, k$ )	Area/ $m^2$
(2, 1, 2)	200.7	(2, 1, 2)	264.3	(2, 1, 2)	208.2
(1, 1, 1)	66.0	(1, 2, 2)	83.2	(1, 2, 2)	113.3
(1, 2, 2)	60.1	(1, 1, 1)	66.8	(1, 1, 1)	55.3
(2, CU, 3)	36.3	(2, CU, 3)	49.7	(2, CU, 3)	50.8
(HU, 1, 0)	7.3	(HU, 1, 0)	8.1	(HU, 1, 0)	17.7
(2, 2, 2)	/	(2, 2, 2)	14.6	(2, 2, 2)	7.3
(1, CU, 3)	6.9	(1, CU, 3)	/	(1, CU, 3)	/

Table 2: Assignment of heat transfer areas in Scenario 1

Match( $i, j, k$ )	Area / $m^2$			Assigned Area / $m^2$
	Period 1	Period 2	Period 3	
(2, 1, 2)	/	264.3	208.2	264.3
(1, 2, 2)	25.0	83.2	113.3	113.3
(1, 1, 1)	61.9	66.8	55.3	66.8
(2, CU, 3)	36.3	49.7	50.8	50.8
(HU, 1, 0)	11.7	8.1	17.7	17.7
(2, 2, 2)	83.7	14.6	7.3	83.7

### Scenario 1 Durations of sub-periods are significantly different

Assume that the durations of the three sub-periods are 1, 3 and 8 months in a year, respectively. Then, the representative period method is employed to design the multi-period HEN.

According to the method mentioned in section 2.1, the third period is selected as the representative period, and its HEN structure in the period 3 is considered as the representative structure. Then, the heat transfer areas are further optimized to meet the operation conditions in the other non-representative periods on the fixed representative structure. The optimized heat transfer areas in the three periods and the final assignment of heat transfer areas for the representative structure are listed in Table 2.

As shown in Figure 2, there are four process stream heat exchangers, one heater and one cooler in the multi-period HEN structure obtained by the representative period method. The HEN structure is shown in Figure 2. It should be noted that the heat exchanger at match (2, 1, 2) is absent in the first sub-period.

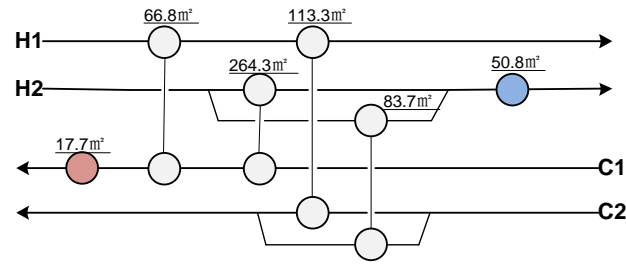


Figure 2: The final structure of multi-period HEN in Scenario 1 obtained by the representative period method

### Scenario 2 Durations of sub-periods are similar or variable

In this scenario, the durations of sub-periods are similar or variable. Table 3 lists the assigned heat transfer areas after merging the structures of the three sub-periods presented in Table 1. The final HEN structure is accordingly shown in Figure 3. The heat transfer units without slashes denote the same matches in each sub-period, whereas the ones with the slashes represent the different matches in the sub-periods. The heat exchanger with the area of  $14.6 \text{ m}^2$  is used in the periods 2 and 3, and the cooler with the area of  $6.9 \text{ m}^2$  is solely used in the first sub-period.

Table 3: Assignment of heat transfer areas in Scenario 2

Match ( $i, j, k$ )	(2, 1, 2)	(1, 1, 1)	(1, 2, 2)	(2, CU, 3)	(HU, 1, 0)	(2, 2, 2)	(1, CU, 3)
Assigned Area/ $\text{m}^2$	264.3	113.3	66.8	50.8	17.7	14.6	6.9

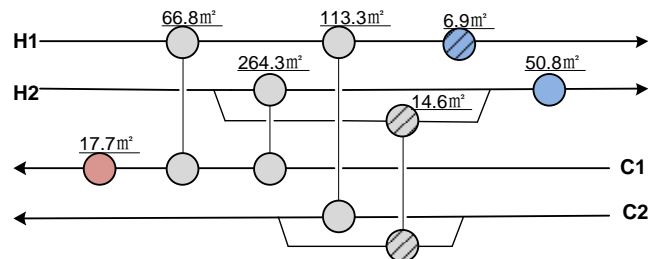


Figure 3: The final structure of multi-period HEN in Scenario 2 obtained by the stepwise simplifying method

Table 4: Comparison of replacement strategies for multi-stream heat exchangers

Replacement strategies	Combined dual-stream exchangers	Area of multi-stream exchangers/ $\text{m}^2$	Capital cost of multi-stream exchangers/ \$	Total capital cost after replacement/ \$
The same hot stream	(1, 1, 1)+(1, 2, 2)	180.1	97,746	224,820
	(2, 1, 2)+(2, 2, 2)	278.9	127,074	
The same cold stream	(1, 1, 1)+(2, 1, 2)	331.1	140,852	220,452
	(1, 2, 2)+(2, 2, 2)	127.9	79,600	

To simplify the HEN structure and reduce the capital cost, the dual-stream heat exchangers with the same hot or cold stream are combined and replaced by the multi-stream heat exchangers. According to the HEN structure in Figure 3, two alternative replacement strategies are available. To select an economically effective strategy, we compared the results obtained by the two strategies, as listed in Table 4.

It can be seen in Table 4 that the total areas of multi-stream heat exchangers in two strategies are the same, and the difference between the two strategies is the heat transfer areas assignment of two multi-stream heat exchangers. The total capital costs for replacement indicate that a larger difference in the

assigned heat transfer areas of the multi-stream exchangers has a greater possibility to reach a lower capital cost for replacement. More details about this assertion can be found in the authors' previous work (Kang and Liu, 2014). For the replacement of multi-stream heat exchangers, the replacement strategy with a larger difference in heat transfer areas of multi-stream heat exchangers should be chosen when the replacement strategies are optional.

Figure 4 shows the final HEN structure after the replacement of multi-stream heat exchangers. The assigned heat transfer areas for each heat exchanger are marked. The two multi-stream heat exchangers are marked with 1 and 2.

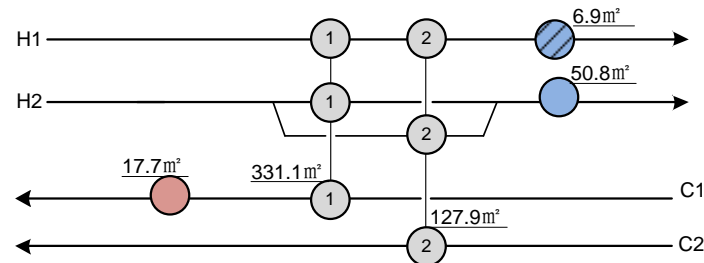


Figure 4: The final optimal HEN structure in Scenario 2 after the replacement of multi-stream exchangers

#### 4. Results and discussion

To study the suitability of the two proposed methods under different characteristics of sub-periods, the stepwise simplifying method in Scenario 1 and the representative period method in Scenario 2 are alternately calculated according to the procedure indicated in Figure 1. All the results obtained by the proposed methods in different scenarios and those in the literature are listed in Table 5.

The methods listed in Table 5 are regarded as the sequential method except for the simultaneous synthesis method that requires solving a complex multi-period HEN model. Thus, the solving difficulties of sequential methods are reduced since the solution of the multi-period HEN model is the most difficult and challenging task as a result of its large dimension.

Table 5: Comparison of the results obtained by the methods in this work and in literature

Methods	Scenarios	No. of heat transfer units	Total areas / m <sup>2</sup>	Capital cost/ \$	Operation cost/ \$.y <sup>-1</sup>	Annual total cost/ \$.y <sup>-1</sup>
Simultaneous	Scenario 1	7	467.5	325,268	188,077	220,604
Synthesis Method	Scenario 2	7	487.4	341,763	171,656	205,833
Structure	Scenario 1	7	534.4	356,458	187,598	223,244
Incorporating Method	Scenario 2	7	534.4	356,458	171,620	207,266
Time-sharing Method	Scenario 1	6	521.1	336,270	187,598	221,225
	Scenario 2	6	521.1	336,270	171,620	205,247
Representative	Scenario 1	6	596.6	382,705	199,781	238,052
Period Method	Scenario 2	6	596.6	382,705	220,355	258,625
Stepwise Simplifying	Scenario 1	5(2MSHE <sup>a</sup> )	534.4	304,298	187,598	218,028
Method	Scenario 2	5(2MSHE <sup>a</sup> )	534.4	304,298	171,620	202,050

<sup>a</sup>MSHE=Multi-Stream Heat Exchanger

It can be seen from Table 5 that the total annual cost of multi-period HEN synthesis obtained by the representative period method is higher than the optimal solution by 9.2 % in Scenario 1 and 28 % in Scenario 2, respectively. Hence, the representative period method could reach a better solution when the durations of the sub-periods are different. In addition, the main advantage of the representative period method is that it can reduce the solving difficulty and computational loads significantly because the representative period method only solves a single-period HEN model, whereas the other methods presented in Table 5 are required to directly solve either a multi-period HEN model or several single-period HEN models. Nevertheless, this method can only obtain a sub-optimal solution. Therefore, if the exact solutions are not required, the representative period method can be applicable to solve the multi-period HEN synthesis problems with large number and different durations.

Among all these methods, the stepwise simplifying method can always obtain the lowest total annual cost of HEN in different scenarios with the different characteristics of sub-periods. In addition, the stepwise simplifying method has all advantages of the time-sharing method, which ensures a cost-effective running in each sub-period without being affected by duration variation of sub-periods. On the other hand, the time-sharing mechanism is bypassed by applying the structure merging and multi-stream exchanger substituting strategies to finalize the HEN structure that satisfies multi-period operations. The multi-period HEN structures obtained by the proposed methods are much simpler and clearer than the method proposed by Jiang and Chang. Subsequently, the stepwise simplifying method proposed in this work can easily produce a cost-effective HEN structure that satisfies the multi-period operations, which is of great significance for quickly and accurately solving the complex multi-period HEN synthesis problem.

## 5. Conclusions

In this work, a representative period method and a stepwise simplifying method are proposed for solving the multi-period HEN synthesis problems. The proposed procedure takes the different characteristics of sub-periods into consideration. A practical example case is used to validate the proposed procedure and to illustrate the effectiveness of the proposed methods by comparing the results obtained by the proposed methods and those in literature. Both of the proposed methods avoid the direct solutions to the multi-period HEN model so as to reduce the computational loads. Moreover, the HEN structures obtained by the proposed procedure are much simpler and clearer.

It is worthy of noting that the solutions to the multi-period HEN synthesis problems obtained by the representative period method are sub-optimal although it takes the least computational efforts. Hence, the representative period method is suitable to solve the multi-period HEN synthesis problem with a characteristic of large number and significantly different durations of sub-periods. In contrast, the stepwise simplifying method can reach better results than those in literature, especially for solving the problem with variable durations of sub-periods. This method can ensure a cost-effective running in each sub-period without being affected by duration variation of sub-periods, which facilitates the solution of multi-period HEN synthesis problems at large scale.

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