CHEMICAL ENGINEERING TRANSACTIONS

VOL. 70, 2018

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Timothy G. Walmsley, Petar S. Varbanov, Rongxin Su, Jiří J. Klemeš

**ISBN** 978-88-95608-67-9; **ISSN** 2283-9216

DOI: 10.3303/CET1870012

# Mathematical Programming for Optimal Design of Hybrid Power Systems with Uncertainties

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The hybrid power system (HPS) is an application variant of distributed generation, defined as a system utilising two or more energy sources, such that the power generation is more efficient, reliable and cost-effective, offering a better option than single-source systems. HPSs can be used in urban, rural and remote areas. HPS research has focused on sizing and optimisation, which requires efficient and effective methodologies to ensure reliable power supply and a cost-effective system. This paper presents a mathematical programming technique for the design of off-grid HPSs, taking into account uncertainties in renewable energy resources. The basic model formulation is based on a comprehensive superstructure that includes all possible connections for power allocation. Chance-constrained programming is applied in determining the optimal capacities of power generation and energy storage with a specified system reliability level. A case study is presented to demonstrate the application of the proposed approach.

## 1. Introduction

Renewables have been regarded as one of the key climate change mitigation technologies, expected to contribute 32% of the cumulative emissions reductions in the 2 °C Scenario (2DS) over the period 2013-2050 according to recent IEA analysis (IEA, 2016). Furthermore, renewables will be deployed mainly in the power sector, where wind and solar photovoltaic (PV) have the potential to provide 22% of annual emissions reduction in 2050 under the 2DS (IEA, 2015). Despite the inherent variability and uncertainty of renewables and the resulting fluctuations in their power output, integrating complementary sources such as solar and wind can improve the system efficiency and availability, thus reducing the dependence on backup energy devices (e.g. batteries and diesel generators). This approach has led to the study of different types of hybrid power systems (HPSs), with an emphasis on sizing and optimisation to ensure a cost-effective system.

Process integration (PI) techniques have recently extended to the design of HPSs (Mohammad Rozali et al., 2016). Wan Alwi and co-workers established power pinch analysis (PoPA) for HPS targeting and developed graphical (Wan Alwi et al., 2012) and numerical tools (Mohammad Rozali et al., 2013a). The latter was later extended to consider energy losses from power conversion and storage (Mohammad Rozali et al., 2013b) as well as different types of energy storage systems (Mohammad Rozali et al., 2015). Mathematical programming techniques have also been developed to optimize power allocation in HPSs. Chen et al. (2014a) proposed two transshipment model-based formulations for HPS targeting and design, considering the effect of power losses from transfer and storage; conversion losses are also considered in a later proposed model (Chen et al., 2014b). Lee et al. (2014) presented a superstructure-based optimization model for HPS design with energy loss considerations.

On HPS sizing, Sreeraj et al. (2010) proposed a PI-based methodology to find the minimum battery capacity required for isolated systems, considering also the stochastic nature of the renewable sources and the system reliability requirements using a chance-constrained programming approach. Bandyopadhyay (2011) proposed the use of the grand composite curve representation of stored energy to design isolated PV-battery and wind-battery systems. Mohammad Rozali et al. (2014) applied their previously developed numerical PoPA tool for

Please cite this article as: Lee J.-Y., Lu Y.-C., Aviso K.B., Tan R.R., 2018, Mathematical programming for optimal design of hybrid power systems with uncertainties , Chemical Engineering Transactions, 70, 67-72 DOI:10.3303/CET1870012

sizing an HPS. Norbu and Bandyopadhyay (2017) incorporated chance-constrained programming into the PoPA framework to account for uncertainty.

The development of insight-based PoPA techniques for HPS design and optimisation has gained more attention, and there is less development of complementary mathematical programming techniques. Although useful in setting performance targets and providing high-level insights for the design problem, pinch analysis lacks the capability to effectively address various design constraints and cost trade-offs. Moreover, pinch analysis has less applicability to complex or large-scale problems. In this paper, the mathematical model developed by Lee et al. (2018) is extended for sizing off-grid HPSs considering the variability and uncertainty of renewables. The stochastic nature of renewable power sources is addressed by incorporating chance-constrained programming into the mathematical modelling framework.

## 2. Problem statement

The problem addressed in this paper can be stated as follows.

- An off-grid HPS consists of a set of power sources $i \in \mathrm{I}$ and a set of power demands $j \in \mathrm{J}$. Power sources can be conventional (e.g. diesel) or renewable (e.g. wind and solar) to generate power for demands.
- The availability and power generation of renewable sources are determined by local weather conditions and the equipment installed, assuming conventional sources can be used as backup power supplies. It is also assumed that the load profiles of power demands are available.
- In the light of the variability of renewables and load demands, a set of energy storage systems $s \in \mathrm{S}$ (e.g. batteries) are also given.
- Because power sources and demands as well as energy storage systems can be alternating (AC) or direct current (DC), a power conditioning system would normally be needed. Specifically, the conversion between AC and DC involves a rectifier (AC-DC) and an inverter (DC-AC).
- The objective is to determine the optimal HPS size and configuration that meets the load with a specified level of system reliability.

## 3. Model formulation

Based on the generic model of Lee et al. (2018), the formulation for off-grid HPS design under uncertainties in renewable power sources is presented below, which consists mainly of energy balance equations.

Eq(1) describes the energy balance for power source *i* in time interval *t*. The power generated from source *i* ($p_{it}$) can be sent directly to power demands *j* ($p_{ijt}$) and stored in energy storage systems $s$ ($p_{ist}$) for later use, whilst excess power ($p_{it}^{\mathrm{e}}$) would be dumped.

$$p_{it} = \sum_{j \in \mathbf{J}} p_{ijt} + \sum_{s \in \mathbf{S}} p_{ist} + p_{it}^{\mathrm{e}} \quad \forall i \in \mathbf{I}, t \in \mathbf{T} \tag{1}$$

The power output of a renewable source is given by Eq(2).

$$p_{it} = a_i R_{it} \quad \forall i \in \mathbf{I}, t \in \mathbf{T} \tag{2}$$

where $a_i$ is the total generator area and $R_{it}$ is the power density. For PV arrays, $R_{\mathrm{PV},t} = \eta^{\mathrm{PV}} I_t$, where $\eta^{\mathrm{PV}}$ is the PV system efficiency and $I_t$ is the solar insolation in time interval *t*. For wind turbines, $R_{\mathrm{wind},t} = 1/2\, \rho v_t^3 Cp$, where $\rho$ is the air density, $v_t$ is the wind velocity in time interval *t*, $Cp$ is the power coefficient. Power densities for other renewable sources can also be defined in a similar way.

To account for the variability and uncertainty of renewables, chance-constrained programming is applied. The power available from a renewable source in each time interval is assumed to be a normally distributed random variable with a mean and standard deviation. Thus, the effective power density for a given reliability level $\alpha$ can be calculated using Eq(3).

$$R_{it} = \max\left[0, \left(R_{it}^{\mathrm{mean}} - R_{it}^{\mathrm{std}} Z_\alpha\right)\right] \quad \forall i \in \mathbf{I}, t \in \mathbf{T} \tag{3}$$

where $R_{it}^{\mathrm{mean}}$ is the mean power density of renewable source *i* in time interval *t*, $R_{it}^{\mathrm{std}}$ is the standard deviation of the power density, whilst $Z_\alpha$ is the inverse of the cumulative standard normal probability distribution (with zero mean and unity standard deviation) corresponding to the required confidence level $\alpha$. Note that the power density cannot be negative, and will be taken as zero if $R_{it}^{\mathrm{mean}} - R_{it}^{\mathrm{std}} Z_\alpha < 0$.

Eq(4) describes the energy balance for power demand *j* in time interval *t*. The power required for demand *j* ($P_{jt}$) may come from power sources *i* and energy storage systems $s$ ($p_{sjt}$).

$$P_{jt} = \sum_{i \in \mathbf{I}} p_{ijt} \eta_{ij} + \sum_{s \in \mathbf{S}} p_{sjt} \eta_{sj} \quad \forall j \in \mathbf{J}, t \in \mathbf{T} \tag{4}$$

where $\eta_{ij}$ and $\eta_{sj}$ are efficiency factors accounting for power losses from conversion. These factors are set to the inverter efficiency for DC-AC conversion, to the rectifier efficiency for AC-DC conversion, or to unity (no efficiency losses) if no power conversion is needed for the demand.

Eqs(5) and (6) describe the inlet and outlet energy balances for storage system *s* in time interval *t*, respectively. Energy storage is used to collect surplus power from sources *i* and dispatch it to demands *j* when there is a deficit of power.

$$p_{st}^{\text{in}} = \sum_{i \in \mathbf{I}} p_{ist} \eta_{is} \quad \forall s \in \mathbf{S}, t \in \mathbf{T} \tag{5}$$

$$p_{st}^{\text{out}} \eta_{s}^{\text{out}} = \sum_{j \in \mathbf{J}} p_{sjt} \quad \forall s \in \mathbf{S}, t \in \mathbf{T} \tag{6}$$

where $p_{st}^{\text{in}}$ and $p_{st}^{\text{out}}$ are the total amounts of power charged and discharged before charging/discharging losses. Similar to the efficiency factors in Eq (4), $\eta_{is}$ accounts for power losses from conversion for energy storage, whilst $\eta_s^{\text{out}}$ is the discharging efficiency.

The overall energy balance for storage system *s* is given by Eq(7). It is stated that the amount of energy stored at the end of time interval *t* ($q_{st}$) equals that at the end of the previous time interval ($q_{s,t-1}$) or the initial charge ($q_{s,T}$ for *t* = 1) adjusted by the storage loss (self-discharge) and the amounts charged into ($p_{st}^{\text{in}} \eta_s^{\text{in}}$) and discharged from the system ($p_{st}^{\text{out}}$) during time interval *t*.

$$q_{st} = \left( Y^{\text{op}} q_{s,T}\Big|_{t=1} + q_{s,t-1}\Big|_{t>1} \right) \left( 1 - \sigma_s \Delta_t \right) + \left( p_{st}^{\text{in}} \eta_s^{\text{in}} - p_{st}^{\text{out}} \right) \Delta_t \quad \forall s \in \mathbf{S}, t \in \mathbf{T} \tag{7}$$

where $Y^{\text{op}}$ is a binary parameter indicating the operation mode ($Y^{\text{op}} = 0$ for start-up or the first-day operation; $Y^{\text{op}} = 1$ for normal daily operation), $\sigma_s$ is the hourly self-discharge rate, $\Delta_t$ is the length of time interval *t*, and $\eta_s^{\text{in}}$ is the charging efficiency. In normal operation, the energy stored at the end of the last time interval (*t* = *T*) of a day is taken as the initial charge for the next day. Since the start-up is transient, the design of HPSs will be based on normal operation.

The capacity constraint for energy storage is given in Eq(8). For storage system design, the energy-related capacity ($q_s^{\text{cap}}$) allows for the maximum amount of energy stored during the operation.

$$q_{st} \le q_s^{\text{cap}} \quad \forall s \in \mathbf{S}, t \in \mathbf{T} \tag{8}$$

For designing off-grid HPSs, it is important to determine the minimum generator capacities to meet the load by minimising the power generation cost ($f_{\text{PGC}}$):

$$\min f_{\text{PGC}} = \sum_{i \in \mathbf{I}} AF_i C_i a_i + \sum_{i \in \mathbf{I}} \left( OM_i^{\text{fix}} a_i + OM_i^{\text{var}} H \sum_{t \in \mathbf{T}} p_{it} \Delta_t \right) \tag{9}$$

where $AF_i$ is the annualisation factor of power source *i*, $C_i$ is the capital cost coefficient for power source *i*, $OM_i^{\text{fix}}$ is the fixed operation and maintenance (O&M) cost coefficient for power source *i*, $OM_i^{\text{var}}$ is the variable O&M cost for power source *i*, whilst $H$ is the annual operating time (day). This may be followed by a second step to determine the required storage capacities by minimising the energy storage cost ($f_{\text{ESC}}$):

$$\min f_{\text{ESC}} = \sum_{s \in \mathbf{S}} AF_s C_s \, q_s^{\text{cap}} / DoD_s \tag{10}$$

where $AF_s$ is the annualisation factor of energy storage system *s*, $C_s$ is the capital cost coefficient for energy storage system *s*, and $DoD_s$ is the depth of discharge of energy storage system *s*. In this step, the minimum generator sizes identified earlier ($a_i^*$) are used as upper limits in Eq(11).

$$a_i \le a_i^* \quad \forall i \in \mathbf{I} \tag{11}$$

Alternatively, the objective function may be to minimise the cost of energy produced by the HPS, taking into account the trade-off between power generation and energy storage costs.

Whichever approach is used, sequential or simultaneous, the overall model is a linear programme (LP), which can be readily solved to global optimality without major computational difficulties.

## 4. Illustrative example

A case study is presented in this section to illustrate the proposed approach. The developed model is implemented and solved in GAMS (Rosenthal, 2018) on a Core i7-7500U, 2.70 GHz processor, using CPLEX as the LP solver. All solutions were found with negligible processing time (< 1 CPU s).
This case study, taken from Norbu and Bandyopadhyay (2017), considers a PV-battery system for a remote location in Rinchending, Bhutan. Figure 1 shows the hourly solar insolation data in terms of the mean and standard deviation over an average day of the year. Table 1 presents the key parameters for the PV arrays and battery. The daily load curve of the Rinchending region is shown in Figure 2. Note that the load demand is normalised to 1 kWh according to the pattern of daily electricity use in the Rinchending region, with a peak load of 54.64 W and a minimum load of 27.23 W. Since the case study considers only a single power source and a single energy storage option, the objective functions can be simply to minimise the PV array area and the battery capacity. The minimum PV array area and the corresponding battery capacity for different levels of system reliability ($\alpha$ = 50 - 99 %) are shown in Table 2.

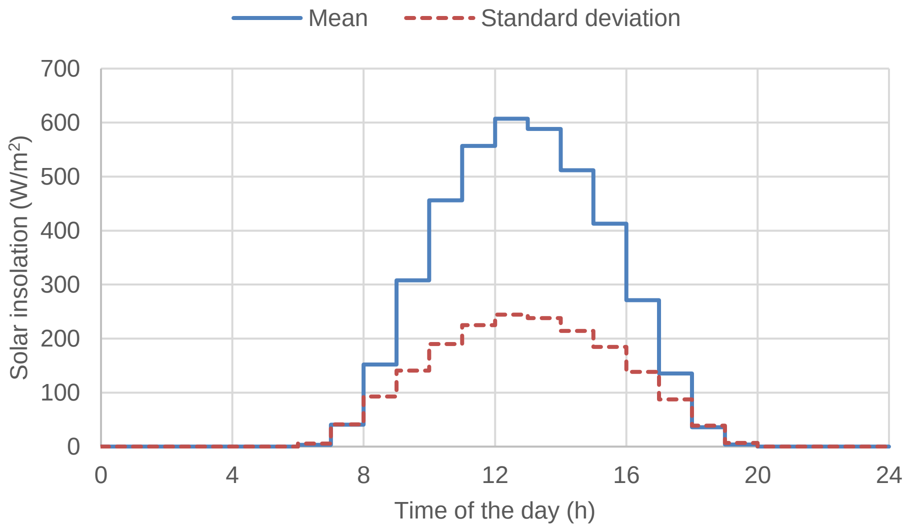


*Figure 1: Hourly solar insolation on the array surface for an average day*

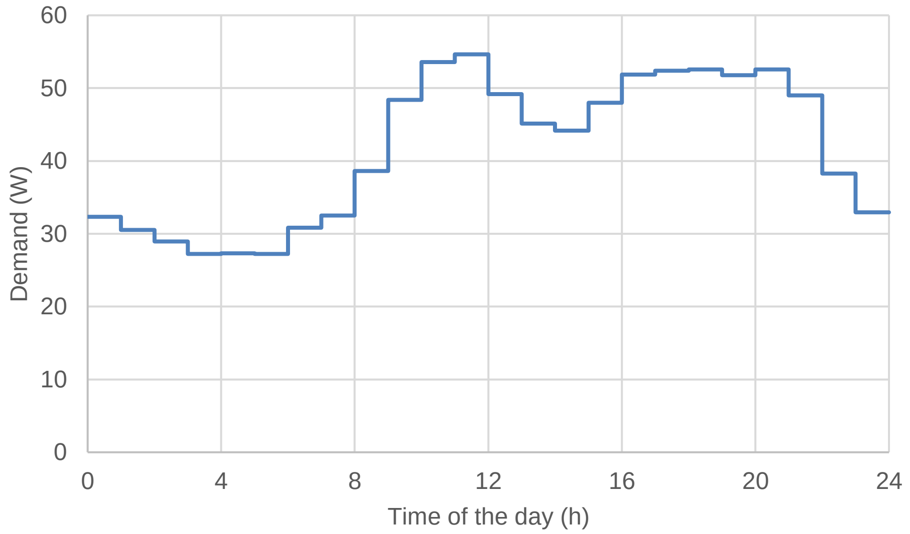


*Figure 2: Demand data for an average day for the location*

*Table 1: Technical data for the case study*

| | |
|---|---|
| PV system efficiency | 15 % |
| Battery charging efficiency | 85 % |
| Battery discharging efficiency | 85 % |
| Battery self-discharge rate | 0 |
| Battery depth of discharge | 70 % |

*Table 2: Results comparison for the case study*

| System reliability ($\alpha$) | $Z_{\alpha}$ | Norbu and Bandyopadhyay (2017) | | This work | |
|---|---|---|---|---|---|
| | | Minimum array area ($m^2$) | Battery size (Wh) | Minimum array area ($m^2$) | Battery size (Wh) |
| 50 % | 0 | 1.95 | 844.4 | 1.95 | 844.4 |
| 60 % | 0.2533 | 2.21 | 855.5 | 2.20 | 855.5 |
| 70 % | 0.5244 | 2.57 | 871.2 | 2.57 | 871.1 |
| 80 % | 0.8416 | 3.18 | 900.5 | 3.17 | 900.4 |
| 90 % | 1.2816 | 4.64 | 950.9 | 4.64 | 950.7 |
| 95 % | 1.6449 | 7.36 | 1,023.2 | 7.33 | 1,022.9 |
| 99 % | 2.3263 | 65.91 | 1,266.1 | 64.17 | 1,266.1 |

*The proposed model involves 169/170 constraints and 195 variables, solved in 0.2 CPU s on average.

It can be seen that both the array area and the required battery size increase with increasing system reliability, especially when the latter increases from 95 to 99 %. This indicates a trade-off between system reliability and economic feasibility. In addition, the results obtained in this work are consistent with those previously reported by Norbu and Bandyopadhyay (2017). However, smaller array areas are found using the proposed model for system reliability levels of 95 and 99 %. The differences could be due to the potential errors in the area determination procedure of Norbu and Bandyopadhyay (2017), in which the minimum PV array area is calculated by linear interpolation. With the larger array areas (7.36 and 65.91 $m^2$), the proposed model shows dumped excess power and a slight reduction in the battery capacity required (for $\alpha$ = 95 %), indicating its capability to identify the minimum generator size more precisely.

## 5. Conclusion

A mathematical model for optimal design and sizing of off-grid HPSs has been developed in this paper. Uncertainties in renewable power sources are considered by incorporating chance-constrained programming into the modelling framework. A case study was solved to demonstrate the application of the proposed model. It is also shown in the results comparison that the proposed model determines the true minimum. Furthermore, compared to PoPA approaches, the modelling framework is flexible in considering cost objectives and more capable of handling complex systems with various types of power sources and demands. Future work involves the use of Monte Carlo simulation to verify the results obtained using the chance-constrained programming-based approach. Parametric uncertainties in load demands as well as technical and economic data will also be addressed by extending the current model.

### Acknowledgments

This research was funded by the Ministry of Science and Technology (MOST) of Taiwan, R.O.C. (Project No. 106-2221-E-027-116). The authors also thank Professor Santanu Bandyopadhyay and Mr. Sonam Norbu for providing the complete data for the case study.

### Nomenclature

Indices and sets:

$i \in \mathbf{I}$ power sources
$j \in \mathbf{J}$ power demands
$s \in \mathbf{S}$ energy storage systems
$t \in \mathbf{T}$ time intervals

Parameters:

$DoD_s$ depth of discharge of energy storage system *s*
$P_{jt}$ power rating of demand *j* in time interval *t*

$R_{it}$ power density of source *i* in time interval *t*
$Y^{\mathrm{op}}$ binary indicating the operation mode (0: start-up; 1: normal)
$\Delta_t$ length of time interval *t*
$\sigma_s$ hourly self-discharge rate of energy storage system *s*
$\eta_s^{\mathrm{in}}$ charging efficiency of energy storage system *s*
$\eta_s^{\mathrm{out}}$ discharging efficiency of energy storage system *s*
$\eta_{ij}$ power conversion efficiency between source *i* and demand *j*
$\eta_{is}$ power conversion efficiency between source *i* and energy storage system *s*
$\eta_{sj}$ power conversion efficiency between energy storage system *s* and demand *j*

Variables:

$a_i$ area/capacity of power source *i*
$p_{it}^{\mathrm{e}}$ excess power from source *i* in time interval *t*
$p_{st}^{\mathrm{in}}$ power charged to energy storage system *s* in time interval *t*
$p_{st}^{\mathrm{out}}$ power discharged from energy storage system *s* in time interval *t*
$p_{it}$ power output of source *i* in time interval *t*
$p_{ijt}$ power from source *i* to demand *j* in time interval *t*
$p_{ist}$ power from source *i* to energy storage system *s* in time interval *t*
$p_{sjt}$ power from energy storage system *s* to demand *j* in time interval *t*
$q_{st}$ energy stored in storage system *s* at the end of time interval *t*
$q_s^{\mathrm{cap}}$ energy-related capacity of energy storage system *s*

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