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A Linear Mathematical Model to Determine the Minimum Utility Targets for a Batch Process

Nitin Dutt Chaturvedi[a], Zainuddin Abdul Manan*[a], Sharifah Rafidah Wan Alwi[a], Santanu Bandyopadhyay[b]

[a]Process Systems Engineering Centre (PROSPECT), Faculty of Chemical Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Malaysia.
[b]Department of Energy Science and Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India
zain@cheme.utm.my

This paper presents a mathematical model to determine the minimum utility targets for a batch process. The model was developed based on the source-demand classification of process streams where each stream was simultaneously treated as a source at its shifted supply temperature and as a demand at its shifted target temperature. The proposed model was formulated as a linear programming model (LP) and hence, guarantees the global optimal solution. The mathematical model can be used to calculate the utility targets for any fixed-schedule batch process. As the formulation is linear solutions, it can be guaranteed to be globally optimal. Applications of the proposed mathematical formulation on two illustrative examples demonstrate significant energy saving potential. Potential reductions of 52 % in hot utility and 48 % in cold utility have been estimated for the first example. Similarly, reductions of 71 % in hot utility and 65 % in cold utility can be potentially achieved for the second example.

1. Introduction

Heat integration has become an essential part of industrial process design due to rising energy demands and growing concern on environmental sustainability. A large percentage of industrial processes are operated in the batch mode. Calculation of utility targets for a batch process is more complex than for a continuous process due to the need to consider time as an additional variable in a batch process. Insight-based methodologies (Wang et al., 2014) and mathematical optimization approaches (Stamp and Majozi, 2011) have been proposed for energy conservation in batch process. Du et al. (2013) proposed an insight based method for heat exchanger network synthesis for batch process. Rossi et al. (2014) proposed an mathematical optimisation based algorithm to simultaneously optimise the batch operation time as well as utlity cost requiremments. Mathematical optimizations approaches typically result in nonlinear formulations with integer variables. Fernández et al. (2012) presented a review of various methodologies for energy conservation in batch processes. In this paper, a linear programming model has been proposed to calculate the minimum utility targets for a fixed scheduled batch process. The model was developed based on the source-demand stream classification. Instead of using the conventional classification of hot and cold streams, the model simultaneously treats each stream as a source at its shifted supply temperature and as a demand at its shifted target temperature. Such classification reduces the complexity of model.

2. Problem statement

The problem addressed in this section can be stated as follows:
Given the:

(i) Starting and ending times for each stream
(ii) heating and cooling duties for the relevant tasks
(iii) operating temperatures of heat sources and heat sinks
(iv) minimum allowable temperature difference

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The objective is to determine minimum hot and cold utility requirements for a batch process

3. Mathematical model

The mathematical model to calculate the minimum utility requirement for a batch process comprises the following sets, variables, parameters and constraints. The constraints consist of equations related to availabilities, heat balances and capacity rate balances.

Sets

$ST = \{st | st = hot or cold stream\}$

Parameters

$T_s(st)$	Shifted outlet temperature of stream '*st*'
$T_d(st)$	Shifted inlet temperature of stream '*st*'
$t_{sts}(st)$	Time at which stream '*st*' starts
$t_{ste}(st)$	Time at which stream '*st*' ends

Continuous variables

$hu(st)$	Hot utility requirement for stream (st)
$cu(st)$	Cold utility requirement for stream (st)
$h(st,st')$	Heat capacity for heat supply from stream (st) to the stream (st')
$z_u(st,st')$	Fraction of heat capacity available for heat supply from stream (st) to the stream (st')
$z_y(st,st')$	Limiting value of fraction of heat capacity available for heat supply from stream (st) to the stream (st')

Constraints

This formulation uses source-demand breakdown of streams whereby each stream is treated simultaneously as a source at its supply temperature and as a demand at its target temperature. Bandyopadhyay and Sahu (2010) proposed a modified problem table algorithm (MPTA) for utility targeting for a continuous process using such stream breakdown. The methodology was proven to yield results equivalent to those generated using the problem table algorithm of Linnhoff and Flower (1978).

Availability Constraints:

A source stream may be fully available, partially available or not available for a demand. This depends on the time of operation of the sources and demands. For partially available streams the fraction of heat capacity available for heat supply from stream (st) to the stream (st') is actually the fraction of time when the source is available to supply heat to the demand, to the total duration of source. Eq(1) and Eq(2) expresses the availability constraints. Note that, the variable $z_y(st,st')$ is introduced to make sure $z_u(st,st')$ is zero when a source stream is completely unavailable to supply a demand stream.

$$z_u(st,st') \le \frac{t_{se}(st) - t_{ss}(st')}{t_{se}(st) - t_{ss}(st)} z_y(st,st') \qquad \forall\, st, st' \in ST \qquad (1)$$

$$z_y(st,st') \times (t_{se}(st) - t_{ss}(st')) \ge 0 \qquad \forall st, st' \in ST \qquad (2)$$

The fraction should be between 0 and 1. This constraint is imposed using Eq(3) and Eq(4).

$$z_u(st,st') \le 1 \qquad \forall st, st' \in ST \qquad (3)$$

$$z_y(st,st') \le z_u(st,st') \qquad \forall st, st' \in ST \qquad (4)$$

Heat balance constraints: Eq(5) expresses the heat balance for demand and Eq(6) limits source availability to a demand.

$$MCp(st')T_d(st',) = hu(st') - cu(st') - \sum_{st \in ST} h(st,st')T_s(st) \qquad \forall st, st' \in ST \qquad (5)$$

$$h(st,st') \le MCp(st) z_u(st,st') \qquad \forall st, st' \in ST \qquad (6)$$

Capacity rate constraints: Eq(7) satisfies the capacity rate balance for a demand and Eq(8) sets the limits on the maximum availability of the capacity rate of a source, to supply to various demands.

$$MCp(st') = \sum_{st \in ST} h(st,st') \qquad \forall st, st' \in ST \qquad (7)$$

$$MCp(st) \geq \sum_{st' \in ST} h(st, st') \qquad \forall st, st' \in ST \qquad (8)$$

Objective:
The objective is to minimize the total external utility requirement.

$$Objective = \sum_{st \in ST} hu(st) + \sum_{st \in ST} cu(st) \qquad (9)$$

4. Illustrative examples

The proposed mathematical model can be applied to calculate the minimum utility requirements in different batch heat exchanger networks (HENs). The applicability of the proposed mathematical model is demonstrated using two examples. The models were solved by the GAMS/ XPRESS solver on the computer (Intel(R) Core(TM) i5 (3 GHz) and 2 GB RAM).

4.1 Example 1: Two-product batch plant

The applicability of proposed mathematical model is illustrated by the two product batch plants (Kondili et al., 1993) represented by the flow sheets in Figure 1. Product 1 and Product 2 are to be produced from raw materials Feed A, Feed B and Feed C. Three reactions occur in two reactors (RR1 and RR2). Figure 2 shows one of the possible schedules for an 8 h time horizon to maximize production (Majozi, 2010). The total production (Product 1 and Product 2) is 151.3 units comprising of 70.9 units of product 1 and 80.4 units of product 2. The heating and cooling requirements are listed in Table 1. The hot utility and cold utility requirements without heat integration are 27.5 kWh and 29.9 kWh.

Table 1: Data pertaining to energy requirements

Task (*i*)	T_{in} (°C)	T_{out}(°C)	Cp (kJ/kg°C)
Heating (H)	50	70	2.5
Reaction 1 (R1)	100	70	3.5
Reaction 2 (R2)	70	100	3.2
Reaction 3 (R3)	100	130	2.6
Separation (S)	130	55	2.8

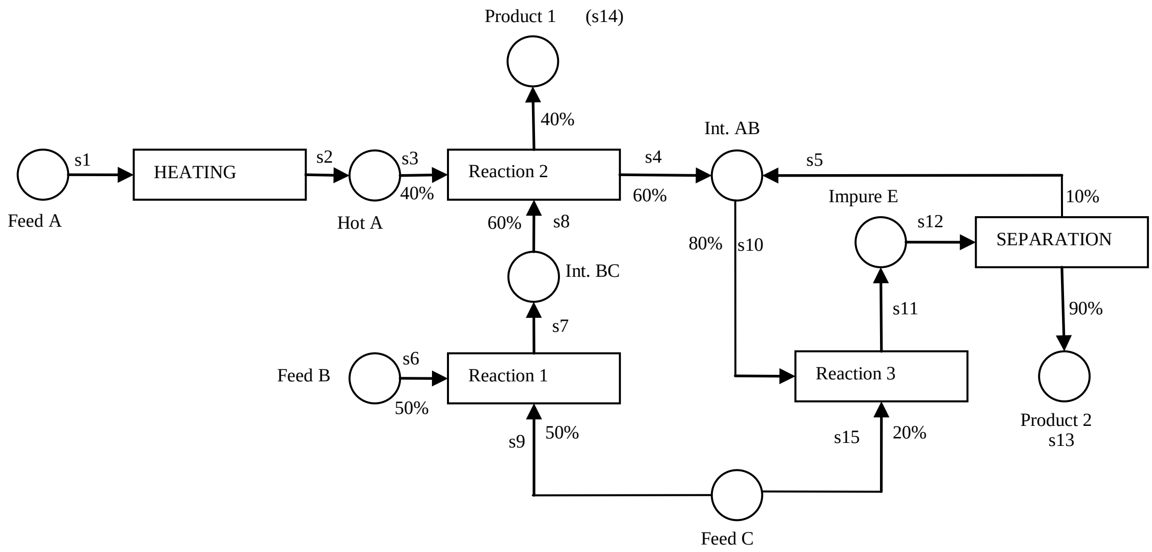

Figure 1: Flow sheet corresponding to the two-product batch plant.

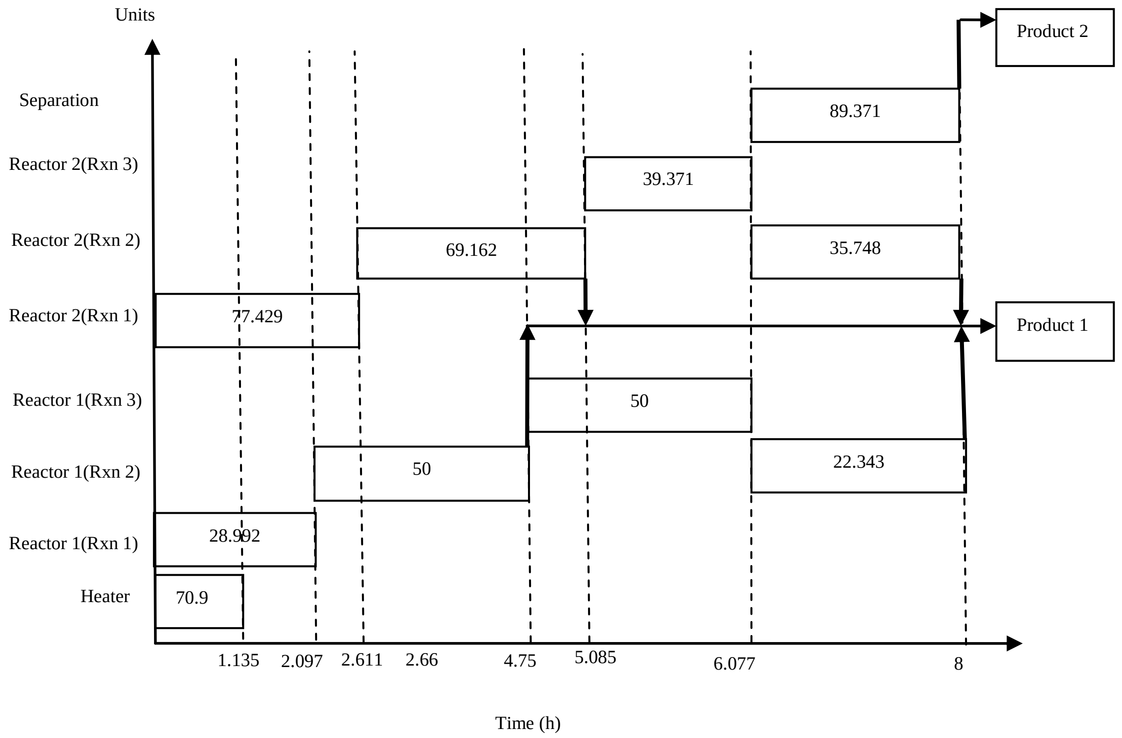

Figure 2: Production schedule for Example 1 (Majozi, 2010)

Table 2: Energy requirement of streams with shifted temperatures for a given schedule (Figure 2)

Task	Unit	Start time (h)	End time (h)	Temperature (°C)		Mass flow (kg)	Cp (kJ/kg°C)	MCp (kJ/°C)
				Inlet	Outlet			
Heating	HR	0.00	1.13	55	75	70.90	2.50	177.25
Rxn 1	RR1	0.00	2.09	95	65	28.99	3.50	101.47
Rxn 1	RR2	0.00	2.61	95	65	77.42	3.50	270.97
Rxn 2	RR1	2.09	4.75	75	105	50.00	3.20	160.00
Rxn 2	RR2	2.61	5.08	75	105	69.16	3.20	221.31
Rxn 3	RR1	4.75	6.07	105	135	50.00	2.60	130.00
Rxn 3	RR2	5.08	6.07	105	135	39.37	2.60	102.36
Rxn 2	RR1	6.07	8.00	75	105	22.34	3.20	78.19
Rxn 2	RR2	6.07	8.00	75	105	35.74	3.20	125.09
Separation	SR	6.07	8.00	125	50	89.37	2.80	250.24

The minimum temperature difference was assumed as 10 °C. The energy requirements of streams with shifted temperatures for a given schedule (Figure 2) are listed in Table 2. Using the proposed model, the hot utility and cold utility requirements were calculated to be 13 MJ and 15.4 MJ. Potential reductions of 52 % in hot utility and 48 % in cold utility was achieved. The LP model had 800 constraints and 500 continuous variables. The solution time is negligible (fraction of seconds). One of the possible networks to achieve this target is shown in Figure 3.

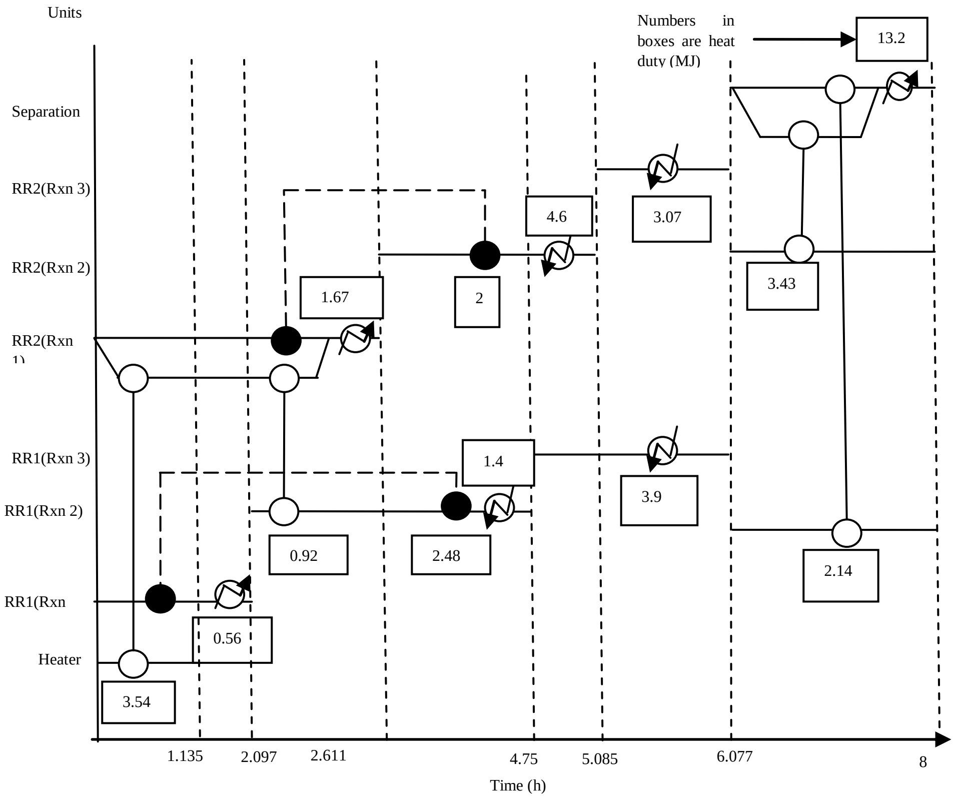

Figure 3: Possible HEN for Example 3 (heat exchangers with dotted line indicate heat exchange through intermediate fluid; the numbers in boxes represent heat duty in MJ)

4.2 Example 2: Four streams heat exchanger network

The stream data for the example (Kemp and Deakin, 1989) are given in Table 3. The minimum allowable temperature difference for heat exchange is given to be 10 ºC. It should be noted that without heat integration hot utility and cold utility requirement are 470 kWh and 510 kWh. Applying the proposed model hot utility and cold utility requirement are calculated to be 134 kWh and 174 kWh. A reduction of 71 % in hot utility and 65 % in cold utility can be observed. The results matches with Kemp and Deakin (1989). The LP model has 129 constraints and 80 continuous variables and the solution time is negligible.

Table 3: Stream data for Example 2

Stream name	Supply temp. (ºC)	Target temp. (ºC)	MC_P (kW/ºC)	Start time (h)	End time (h)
C1	80	140	8	0	0.5
H1	170	60	4	0.25	1
C2	20	135	10	0.5	0.7
H2	150	30	3	0.3	0.8

5. Conclusions

A mathematical model has been proposed to calculate the utility targets for a batch process. The model was developed based on the source-demand classification of process streams. Instead of using the conventional classification of hot and cold streams, the model simultaneously treats each stream as a source at its shifted supply temperature and as a demand at its shifted target temperature. The developed model can be used to calculate utility targets for any batch process with a fixed schedule. The developed model is linear and hence, guarantees a global optimal solution. Applications of the developed mathematical formulation on illustrative examples demonstrate some significant energy saving potentials. The relatively simple and linear formulation may allow the extension of the formulation to flexible schedule batch process that may result in simpler formulations. The developed formulation employs direct heat integration between different time intervals. The formulation can be extended to include indirect heat integration. Research is currently in progress to address such issues.

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References

Bandyopadhyay S., Sahu G.C., 2010. Modified Problem Table Algorithm for Energy Targeting. Ind. Eng. Chem. Res. 49, 11557–11563.

Du J., Yang P., Li J.L., Liu L.L., Meng Q.W., 2013, Heat Exchanger Network Synthesis for Batch Processes by Involving Heat Storages, 35, 943-948 DOI:10.3303/CET1335157

Fernández I., Renedo C.J., Pérez S.F., Ortiz A., Mañana M., 2012. A review: Energy recovery in batch processes. Renew. Sustain. Energy Rev. 16, 2260–2277.

Kemp I.C., Deakin A.W., 1989. The cascade analysis for energy and process integration of batch processes. II: Network design and process scheduling. Chem. Eng. Res. Des. 67, 510–516.

Kondili E., Pantelides C.C., Sargent R.W.H., 1993. A general algorithm for short-term Scheduling of batch operations – I. MILP formulation. Comput. Chem. Eng. 17, 211–227.

Linnhoff B., Flower J.R., 1978. Synthesis of heat exchanger networks: I. Systematic generation of energy optimal networks. AIChE J. 24, 633–642.

Majozi T., 2010. Batch Chemical Process Integration, 2010. Springer, Netherlands.

Stamp J., Majozi T., 2011. Optimum heat storage design for heat integrated multipurpose batch plants. Energy 36, 5119–5131.

Rossi F., Manenti F., Kozin K.A., Goryunov A.G., 2014, Defeating the sustainability challenge in batch processes through low-cost utilities usage reduction, Chemical Engineering Transactions, 39, 697-702 DOI:10.3303/CET1439117

Wang Y., Wei Y., Feng X., Chu K.H., 2014. Synthesis of heat exchanger networks featuring batch streams. Appl. Energy 114, 30–44.