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A Simulation-Based Optimization Method for Solving the Integrated Supply Chain Network Design and Inventory Control Problem under Uncertainty

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One of the key objectives of an optimised supply chain is to maintain a low operation cost as well as the service quality at a satisfactory level under demand uncertainty. However, the supply chain network design and inventory control optimisation are usually conducted in a sequential manner where the supply chain network is first determined by solving a mixed-integer programing (MIP) problem, and then the supply chain system with the given network design is tested as a “what if” problem in order to evaluate and improve its performance. Over the last decade, simulation modelling is regarded as an efficient tool for evaluating the performance of a real-world supply chain under different conditions and flexible control policies and simulation-based optimisation has been widely studied for solving inventory management problem under various uncertainties. In this work a hybrid computational framework is proposed to solve both the network design problem and the associated inventory control problem simultaneously. By incorporating region-wise metamodeling method to reduce the computation load, a multi-echelon supply chain case with 13 inventory stocking nodes can be solved within 3,621 s with the proposed algorithm. As a comparison, the simulation-based problem is also solved by the genetic algorithm (GA) toolbox in MATLAB, which only returns a 31 % higher cost after 13,844 s.

1. Introduction

Supply chain design and optimization is one of the key elements for manufacturing companies to integrate the production process, distribution facilities and the end customers into a highly efficient entity (Garcia and You, 2015). The decisions toward building a profitable supply chain usually can be categorised as long-term planning problems and short-term controlling ones (You and Grossmann, 2008a). Among these, the strategic planning is usually dedicated to design the optimal supply chain network as well as to determine the best candidates for production processes, manufacturing locations and inventory stocking sites (Rodriguez et al., 2013). After the network design decisions are made, inventory control parameters are then optimised so as to minimise the total operational cost including expenditures from inventory holding, transportation and potential loss from backordering during a given number of periods under unpredictable demand fluctuations (You and Grossmann, 2011a). The network design problem is usually tackled by a deterministic mixed-integer programing where the parameters are simplified to their expected values (You et al., 2011). On the other hand, the inventory control problem can be solved via simulation-based approaches in order to simulate the demand uncertainty therefore to provide a reasonable estimation (Jung et al., 2008). Due to the different natures of the above problems, the optimisation for strategic planning and stochastic inventory control decisions are usually carried out in a sequential approach. Due to the goal of enterprise-wide optimisation, determining the optimal decision for the integrated network design and inventory control optimisation problem becomes important for the manufacturing industry for higher operational efficiency.

In this paper, we propose a hybrid optimisation framework to simultaneously address supply chain network design and inventory control problem. The integrated problem is formulated as a mixed-integer black box optimisation model, and further decomposes the integrated model into a simulation-based optimisation

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upper level problem and a stochastic mixed-integer program master problem. An outer-approximation algorithm is proposed to solve this hybrid optimisation problem. The linearised stochastic program under-estimator determines the discrete variables for the supply chain network design; inventory decisions are obtained through the simulation-based optimisation subproblem with the fixed discrete decisions, which progressively adds cutting hyperplanes to the master problem. To address high-dimension issues induced by a large network, the simulation-based optimisation subproblem is solved via a decomposed surrogate modelling method by utilising the structural information embedded in the supply chain network.

2. Methodology

2.1 Problem statement

A multi-echelon divergent inventory system which employs the base-stock policy is considered. The supply chain includes candidate sites for both the production and distribution purposes. Several customer regions are served at the end of the supply chain whose demands occur periodically at the beginning of each period. The uncertain demands are assumed to follow some known probability distributions (e.g. normal distributions with known mean and standard deviation values). Each upstream inventory may serve multiple downstream inventory stocking nodes and orders at the beginning of each review period to replenish its inventory position to a target base-stock level (You and Grossmann, 2010). Orders can be satisfied immediately if the supplier has sufficient on-hand stocks otherwise the unmet portion are fully backlogged until enough on-hand inventories are available. The fulfilled orders arrive at the downstream nodes after a predetermined deterministic lead-time. All orders placed to the plants are satisfied immediately. However, due to the limited storage and production capacity at the plants, the holding cost is assumed to be nonlinearly correlated with the on-hand inventory level (Yang, 2014). The detailed supply chain operations are programed as a discrete-event simulator (Figure 1) which can autonomously take actions according to the inventory control policy and the parameters settings as the simulation clock moves through the simulation horizon. All costs and changes of the inventory levels can be recorded and the total supply chain operation cost is returned after a sufficiently large number of replications.
The supply chain is dedicated to satisfy the customers' demands for a lowest total operation cost which is the sum of the inventory holding cost, the backorder cost and the transportation cost. Therefore, the objective is to find the best supply chain network structure which stipulates the supplier-customer relationships between each node as well as to determine the optimal target base-stock levels at each inventory storing nodes which can minimise the expected total operational cost.

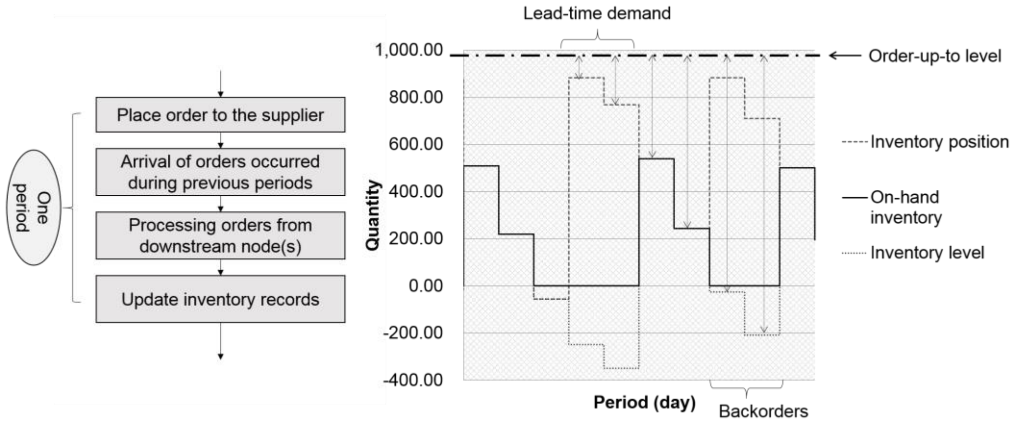

Figure 1: Illustrations of the discrete-event simulation procedures (left) and the base-stock policy (right).

2.2 Mathematical formulation for the integrated problem

In this work, a single-stage model is used to approximate the inventory nodes in the supply chain where a discrete-time demand process is assumed (Graves et al., 2000). However, we are considering a stochastic inventory control problem of which the demands have some known probability distributions (Yue and You, 2013). The demand occurs at each period can be assumed as independently and identically distributed (You and Grossmann, 2008b). Thus, we discretize the distribution of stochastic demand and formulate a stochastic programing model MINLP_M. The single stage model uses the following constraints Eq(2) and Eq(3) to describe the inventory status for the inventory storing node *i* at time period *t* in each scenario *s*, where *bs*, *ih* and *bo* are non-negative variables representing the target inventory position, on-hand inventory level and backorder amount. Eq(4) is a shorthand expression of the total operation cost induced by node *i* in scenario *s*. The nonlinear term $f_{is}^{inv}(y, ih_{is})$ in Eq(4) represents the inventory holding cost, which is a nonlinear response depends on the on-hand inventory level.

$$\text{(MINLP\_M)}\quad \min: \quad C^{ttl} = \frac{1}{N^{scn}} \sum_{s \in S} \sum_{i \in I} C_{is}^{ttl} \tag{1}$$

$$\text{s.t.}\quad bs_{is} - \sum_{j=1}^{t} d_{ijs} \geq ih_{its} \quad \forall i \in I \quad \forall t \in T_i \quad \forall s \in S \tag{2}$$

$$\sum_{j=1}^{t} d_{ijs} - bs_{is} \leq bo_{its} \quad \forall i \in I \quad \forall t \in T_i \quad \forall s \in S \tag{3}$$

$$C_{is}^{ttl} = f_{is}^{inv}(y, ih_{is}) + f_{is}^{bo}(y, bo_{is}) + f_{is}^{trn}(y) \quad \forall i \in I \quad \forall s \in S \tag{4}$$

$$bs_{is}, bo_{its}, ih_{its} \in \mathbb{R}^+,\; y \in \mathbb{Z}^+ \quad \forall i \in I \quad \forall t \in T_i \quad \forall s \in S \tag{5}$$

The set T_i stipulates the specific number of periods for node *i* to finish a full inventory cycle which starts from the arrival of the previous orders after a deterministic delivery time. d_{ijs} is the amount of orders to node *i* at time period *j* in scenario *s*, which is the sum of the demands from its downstream nodes. Since the model includes both continuous decision variables *bs* to set the target inventory positions as well as discrete decision variables *y* to determine the network structure over a number of possible realisation of the demands *d*, the resulting mathematical formulation is a stochastic mixed-integer nonlinear programing (MINLP) which can lead to a significant computational challenge by using the off-the-shelf solvers directly. On the other hand, the above formulation is under the assumption that the demands from each node can be 100 % satisfied during every review period. Also, the backorder cost is set to be greater than the initial inventory holding cost (otherwise all demands would be backlogged so as to achieve a lower total operation cost at the optimal solution). This will eventually result in a slightly higher total operational cost from the inventory simulation than from solving MINLP_M if the service level is maintained at a high level (e.g. > 95 %). Thus, MINLP_M is an under-estimator of the total operation cost of the inventory system.

2.3 Surrogate-based optimization for the simulation problem

Existing publications for supply chain design with inventory control under uncertainty mainly follow the stochastic inventory approach (You and Grossmann, 2011b), which only approximates the inventory dynamics under uncertainty. However, a more accurate estimation of the stochastic inventory systems and the expected total cost under fixed network design and demand uncertainty can be obtained from Monte Carlo simulation, which can be integrated into the network optimization framework by using surrogate modeling. There are a number of previous works on the surrogate-based optimisation to such inventory control problems under a predetermined supply chain network design. These include the surrogate-based global search algorithm (Wan et al., 2005), off-line linearised model formulation method (Jung et al., 2008) and cutting-plane with hypothesis test method (Chu et al., 2015). The surrogate-based optimisation framework provides a shortcut to optimise these black-box responses. These methods first fit the input-output to a response surface model (RSM) thus enabling the gradient-based solvers to find a solution in a relatively shorter time. However, a typical supply chain may consist of a considerable number of inventory carrying facilities, which may suffer from the curse of dimensionality when using the design of experiment (DOE) techniques to fit the surrogate models and the increasing dimensionality will also cause a big challenge during the optimisation phase. Reduced order modelling (ROM) is an important technique to avoid excessive computation budgets to build a full-space metamodel since most of the sampling efforts are placed in the reduced order subspaces rather than in the entire input space. On the other hand, the embedded structural information of a supply chain can be utilised to define the subspaces for building the reduced order surrogates for the performance measurements at different nodes (Gunnerud et al., 2013). In other words, due to the buffering effects by keeping a certain amount of safety stock, the expected cost incurs at a certain inventory stocking node is mostly affected by the control parameters itself and its direct supplier, if any, while less affected by the other distant nodes in the supply chain network. Thus we can fit the reduced order surrogate model separately for each inventory node in the supply chain relating only its regional information and then use the aggregated models as a low-fidelity approximation to the simulation response. As a result, the total number of design points reduces drastically comparing to the full space model if a large-scale inventory network is considered. In this work, the surrogate models are built through the Design and Analysis of Computer Experiment (DACE). Thus, the aggregated kriging estimators give a low-fidelity approximation which facilitates the following optimisation procedure by utilising the gradient-based nonlinear solvers. The low-fidelity kriging models are updated iteratively in accordance to the trust-region method with the DOE techniques (Figure 2 right) and eventually converge to an optimum.

2.4 Solution procedure

The optimisation algorithm follows an outer-approximation (OA) scheme (Figure 2 left), which is initially used to solve MINLP problems (Duran and Grossmann, 1986). The optimization algorithm starts with a feasible solution in which the initial values for the discrete variables stipulate the starting design of the supply chain network. As discussed previously, the upper-level simulation-based optimisation method employs a trust-region method to find the optimum for Sim_Opt.

After solving the upper-level Sim_Opt, the optimal value for the continuous variables is returned when the trust-region radius shrinks to the minimum value. The solution to the continuous variables is used to replace the constraints in Eq(4) which contains nonlinear terms with linear approximations. In this way, MINLP_M is essentially reduced to a stochastic MILP with appended cutting hyperplanes constraints (MILP_M). Also, the linearisation greatly shortens the computation time for solving the approximated lower bound. The updated integer variables are again passed back to the simulator and starts a new iteration to solve Sim_Opt with updated network structure. Different from the classic OA approach in solving MINLPs, the integer cut is not employed in our framework so as to reduce the chance of being trapped in a local minimum since the global optimality for the simulation-based optimisation is not guaranteed. In our work, the termination of the entire framework is reached when the optimality gap stops to diminish since a zero gap is not desirable using the single-stage inventory model as approximation.

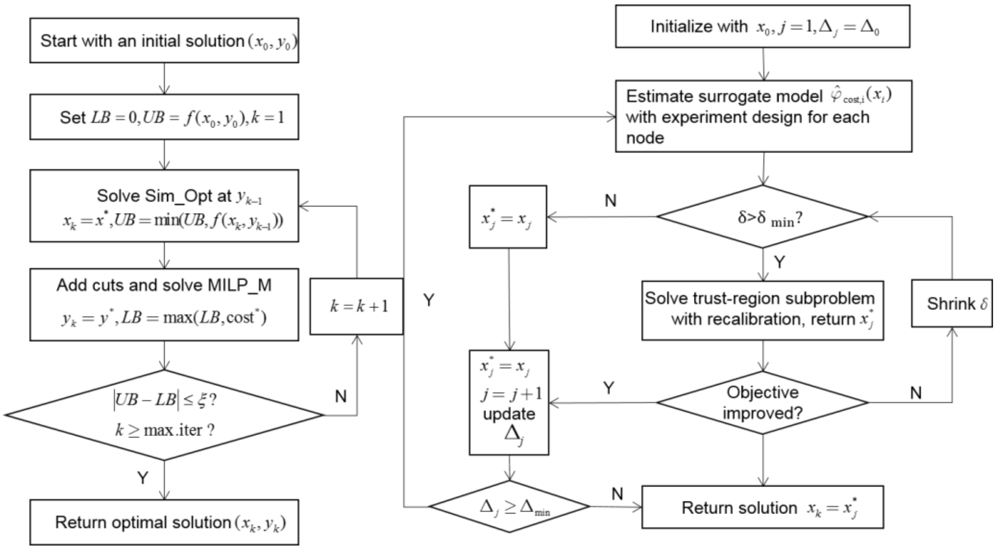

Figure 2: Flow chart of the hybrid optimization framework for solving the integrated problem (left) and flow chart of the trust-region framework for solving the upper-level simulation-based optimization

3. Case study

The hybrid optimisation framework is demonstrated by an integrated supply chain network design and inventory control optimisation problem. The supply chain system has 5 potential production sites to produce the product and 3 candidate locations to set up the warehouses for stocking the product. The company is dedicated to fulfil the demands from 5 different sale regions and each region is served through a distribution center (Figure 3). This is a large scale problem since the three-echelon supply chain network has a total of 13 potential inventory storing nodes and there are 13 base-stock levels to be determined along with to determine the best network design from an enormous number of possible solutions (up to 207,025 possible combinations). The sales regions place orders to their upstream distribution centers every day and the stochastic demand follows the normal distribution. Each inventory facility operates according to the base-stock policy. They monitor their inventory positions at the beginning of each period and order up to the base-stock level through their upstream nodes. If the products stock overnight, a certain amount of holding cost will incur depending on the amount of the on-hand inventory level. In the case study problem, the unit price - on-hand inventory curve is assumed to be convex in order to represent a constraint capacity for holding inventory at a place (Pando et al., 2012). If there is backlogged order exists, a backordering cost will be charged. Each shipment will also incur a transportation cost which depends on its size and the distance between the sender and the receiver.

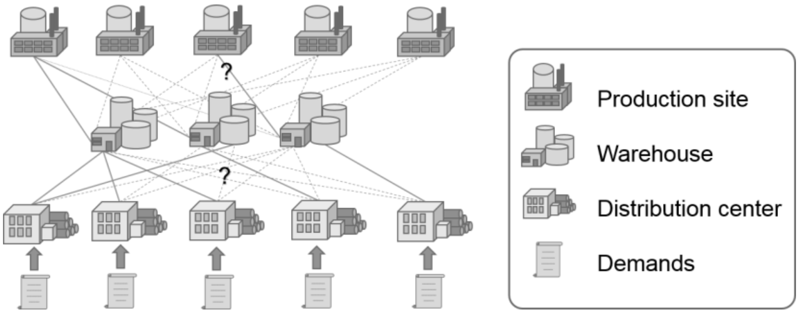

Figure 3: Supply chain network with candidate production sites and links in the case study. Dashed links are candidate links while solid lines are the optimal solution for the network design

The simulation is coded in JAVA and the optimisation framework is developed in MATLAB. The planning horizon for the case study is 100 days, and a 20 days' warm-up period is considered in advance to reach the equilibrium status. For solving the upper-level simulation-based optimisation problem, an increasing number of replications for estimating the DACE surrogate model is used as the trust-region size gradually shrink. After getting the reduced order surrogate model which approximates the input-output relation, the resulting nonlinear programing (NLP) problem is solved in MATLAB by calling the KNITRO libraries. The trust-region framework returns the solution for the upper-level problem when a minimum trust-region size is reached. On the other hand, we sample 50 randomly generated scenarios when discretizing the stochastic MILP under estimator for solving the lower level problem. The OA decomposition algorithm stops when the optimality gap no longer improves by more than 0.1 %.

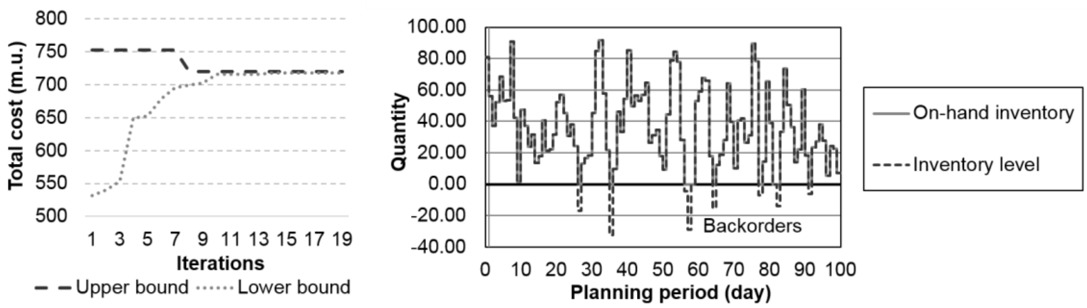

Figure 4: Illustration of the iterative result (left) and the inventory trajectory of distribution centre 1 (right)

The proposed hybrid framework requires 3,621 s in solving the integrated supply chain network design and inventory control problem under demand uncertainty. Over 90 % of the computation time is spent on solving the upper-level simulation-based optimisation problem. The optimal supply chain network design and the iterative result are shown in Figure 3 and Figure 4 (left). The algorithm terminates after 20 iterations as the maximum number of iterations is reached. The minimum operation cost can be inferred from the upper bound 721.1 m.u. (monetary unit) and the gap between the upper bound and lower bound is 5.7 m.u. (0.8 % compare to the total operation cost). In Figure 4 (right), the optimal dynamics of the inventory at distribution center 1 is given. The inventory system performs at a high service level and a great proportion of the stochastic demands are satisfied without being backlogged. On the other hand, the average on-hand inventory level is not overly high at the optimal solution that it still allows a reasonable chance to have stockouts rather than causing an unnecessarily high inventory holding cost.

For comparison purpose, the case study problem is also solved by the genetic algorithm (GA) toolbox in MATLAB. As a pure black-box optimisation, the objective function for the GA is a simulation response of both the continuous input variables for setting the base-stock levels and the integer variables for determining the network structure for the entire supply chain system. Therefore, the output of the function is evaluated through Monte-Carlo simulation provided the network structure and the base-stock level settings. The case study problem is hard to be directly solved by GA, since a total of 21 variables comprise the input space (13 continuous base-stock levels and 8 integer variables for the network design). The GA returns the solution with a total operation cost of 945.9 m.u. after 13,844 s, which is 31 % greater than the solution by using our proposed hybrid framework.

4. Conclusions

This paper presented a hybrid optimisation framework for addressing the integrated supply chain network design and inventory control problem under demand uncertainty. Our proposed approach decomposed the integrated problem into a stochastic programing based master problem and a simulation-based upper-level problem. Furthermore, the simulation-based problem was solved in a surrogate-based trust-region framework and the region-wise metamodeling method provided a multi-fidelity optimisation pathway for determining the optimal base-stock levels with fewer computation efforts. A three-echelon supply chain design and inventory control problem was studied. Comparing to the solution returned by MATLAB GA toolbox for optimising the mixed-integer black-box problem, the solution returned by our proposed algorithm is 23.6 % lower than the solution returned by GA with a considerably shorter computation time. A future research direction is to consider larger-scale problems with faster algorithms, and also consider issues related to energy saving and pollution reduction, in addition to economic optimization.

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