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Robust Optimisation for Integrated Planning, Scheduling and Dynamic Optimisation of Continuous Processes under Uncertainty

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In this work, a robust optimisation approach has been developed to handle the uncertainty in integrated planning, scheduling and dynamic optimisation for continuous manufacturing processes. We first propose an integrated optimisation model for multi-product continuous processes. A robust optimisation approach is then introduced to address the process uncertainty. This robust optimisation formulation confines the uncertain coefficients to symmetrical sets and uses an uncertainty budget to control the degree of conservatism for the solution. We then use the flexible recipe method to enhance computational efficiency. A case study demonstrates that the robust production strategy leads to higher profits than the deterministic production strategy for worst-case circumstances under the same uncertainty assumptions.

1. Introduction

Recently, increasing attention has been focused on simultaneous optimisation of multiple different decision levels of continuous manufacturing processes (Zamarripa et al., 2013), since integrated models usually lead to better overall performance (Corsano et al., 2013). Because of the large scale and computational complexity of the integrated model (Nie et al., 2012), most previous work only concentrate on two adjacent levels, such as planning and scheduling (Erdirik-Dogan and Grossmann, 2008), scheduling and control (Chu et al, 2015), and scheduling and dynamic optimisation (Chu and You, 2014a). In this work, we integrate all three levels together and formulate an integrated planning, scheduling and dynamic optimisation model of continuous processes (Tlacuahuac et al., 2014).

Uncertainty is a common and critical concern in operations of continuous processes, since many process parameters are subject to changes (Chu and You, 2014b). We apply a robust optimisation approach (Ben-Tal et al., 2009) to address uncertainties in the integrated planning, scheduling and dynamic optimisation of continuous processes. In this approach, symmetric uncertainty sets are used to feature uncertainty ranges and an uncertainty budget is exploited to control conservatism of the solution (Bertsimas and Sim, 2003). The robust optimisation approach balances the performance and conservatism by adjusting the value of the uncertainty budget. The robust production strategy results higher profit under the worst-case circumstances when the system is subject to uncertainty. Finally, the flexible recipe method is applied to improve computational efficiency (Chu and You, 2012).

2. Model formulation

In this work, we focus on a multi-product continuous manufacturing process that produces several products throughout the entire planning horizon (Shi et al., 2015). As shown in Figure 1, the integrated problem has 3 different levels, the planning problem, the scheduling problem, and the dynamic optimisation problem. The entire planning horizon is divided into a set of planning periods according to order demand from customers. The order demand for each product is given at the end of each planning period. The planning problem is dealing with production planning, inventory level, etc. In each planning period, there is one scheduling problem. Each planning period, which is also the scheduling horizon, is

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then divided into a set of scheduling slots. The number of scheduling slots in each scheduling horizon is the same as the number of the total products. The scheduling problem is solved to determine the production sequence, starting time and end time of one scheduling slot, etc. Each scheduling slot includes a production period and a transition period. The dynamic optimisation problem is solved to determine the optimal transition trajectory.

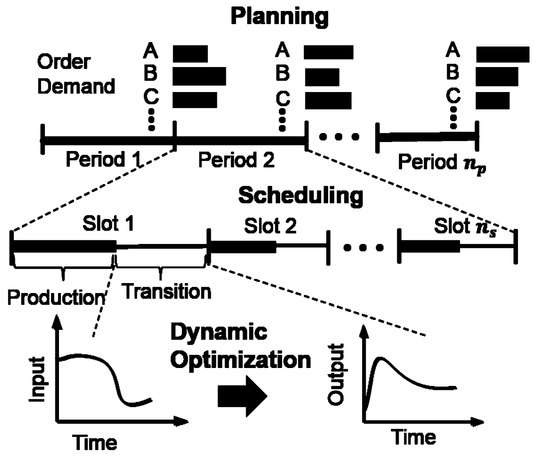

Figure 1: Structure of the integrated planning, scheduling, and dynamic optimisation problem

We propose a full-space, deterministic mixed integer nonlinear programming (MINLP) model which integrates the three level problems together as follows.

$$\max \quad Profit = \sum_i \sum_t \mathrm{p}_{\mathrm{it}} \cdot S_{it} - \sum_t INVC_t - \sum_l \sum_t CT_{lt} - \sum_i \sum_t \mathrm{cop}_{\mathrm{it}} \cdot G_{it} \tag{1}$$

s.t. Planning model & Scheduling model
Dynamic optimisation model

The objective is to maximise the total profit over the planning horizon. The total profit is the revenue minus inventory cost, transition cost, and production cost. cop_{it} is the operation cost of product i in period t. G_{it} is the production amount of product i in period t. The planning model is used to determine the production assignment and the inventory level to ensure fulfilment of the order demands. Note that most existing work use linear estimation approaches to approximate inventory cost (Erdirik-Dogan and Grossmann, 2008), which well estimates the non-convex inventory profile. In each scheduling horizon, the scheduling optimisation model is solved to obtain the production sequence, and the starting and end times of each scheduling slot. Symmetry breaking constraints are applied to simplify the solution space (Yue and You, 2013). The dynamic model usually contains a set of differential equations that feature the chemical reaction, a set of algebraic equations which express the heat and mass balance. Dynamic optimisation is performed to determine the optimal transition trajectory in each scheduling slot. The planning and scheduling problems provide the dynamic problem with initial and final values of the state variables, while the dynamic problem returns the optimal transition cost and transition time to the planning and scheduling problems.

3. Robust optimisation

To overcome uncertainty in production processes, we introduce a robust optimisation approach to the integrated optimisation problem (Bertsimas and Sim, 2003). This robust approach avoids the non-convexity and complexity of nonlinear formulation. It takes advantage of symmetric uncertainty sets to represent the uncertainty coefficient and uses a budget parameter to control the degree of conservatism.

In this work, we assume that the production cost cop_{it} in Eq(1) is subject to uncertainty. The production costs of different products, given as fixed parameters in the nominal problem, are now allowed to vary within the symmetric bounds, as shown in Eq(1). cop_{it}^{u} denotes the uncertain production cost, which is the sum of the nominal cost cop_{it} and the uncertain cost change ΔCOP_{it} . The uncertain production cost is confined by the fixed symmetric bound $dcop_{it}$.

$$cop_{it}^{u} = \left(cop_{it} + \Delta COP_{it}\right) / \left[cop_{it} \text{ - } dcop_{it}, cop_{it} + dcop_{it}\right] \tag{1}$$

In Eq(2), the nominal profit is divided into 2 parts. P_1 is the revenue minus inventory and transition costs. P_2 is revenue minus production costs.

$$Profit = P_1 + P_2 = \left(\sum_i \sum_t \mathrm{p_{it}} \cdot S_{it} - \sum_t INVC_t - \sum_l \sum_t CT_{lt}\right) + \left(-\sum_i \sum_t \mathrm{cop_{it}} \cdot G_{it}\right) \tag{2}$$

In Eq(3), the uncertain production cost cop_{it}^{u} is substituted into P_2.

$$P_3 = -\sum_i \sum_t cop_{it}^{u} \cdot G_{it} = -\sum_i \sum_t \mathrm{cop_{it}} \cdot G_{it} - \sum_i \sum_t \Delta COP_{it} \cdot G_{it} = P_2 + P_4 \tag{3}$$

Since the nominal problem is a maximisation problem, the worst case should be a minimisation problem, as shown in Eq(4). U is the set of indices of the uncertain production costs. S is a subset of U. The number of elements in S is bounded by a budget parameter Γ , which is an integer parameter to define the level of conservatism. The value of this parameter can be selected from 0 to the number of uncertain parameters. The readers are referred to Bertsimas and Sim (2003) for further details about this parameter. The sensitivity analysis of choosing different values of Γ are discussed in Section 5 and Figure 3.
The aim of the robust optimisation problem is to find the optimal solution under the worst-case circumstances.

$$\min_{\{S|S\in U, |S|\le\Gamma\}} \left(-\sum_{(i,t)\in S} \Delta COP_{it} \cdot G_{it}\right) \tag{4}$$

Eq(4) is then formulated into a more general form, shown in the following robust counterpart formulation. Z_{it} is used to denote how much ΔCOP_{it} changes.

$$\min \quad P_4 = -\sum_{(i,t)\in U} dcop_{it} \cdot \left|G_{it}\right| \cdot Z_{it} \tag{5}$$

$$\text{s.t} \quad \sum_{(i,t)\in U} Z_{it} \le \Gamma \tag{6}$$

$$0 \le Z_{it} \le 1, \quad \forall (i,t) \in U \tag{7}$$

The dual problem of the above robust counterpart problem is then formulated as shown in Eq(9)-(11).

$$\max \quad P_5 = -DCU \cdot \Gamma - \sum_{(i,t)\in U} DCL_{it} \tag{8}$$

$$\text{s.t} \quad DCU + DCL_{it} \ge dcop_{it}, \quad \forall (i,t) \in U \tag{9}$$

$$DCU, DCL_{it} \ge 0, \quad \forall (i,t) \in U \tag{10}$$

Now, we add the above robust dual formulation into the deterministic MINLP and formulate a full space robust problem. However, this full space problem is computationally demanding. The flexible recipe method is then applied to improve the computational efficiency.

4. Flexible recipe method

The flexible recipe method is based on the structure of the full space MINLP problem (Chu and You, 2013a). As mentioned in Section 2, the planning and scheduling problems provide the dynamic optimisation problems with initial values and final values of the state variables. If the decision variables in the planning and scheduling problems are treated as given parameters, the transition sequence in each scheduling slot is then fixed. In this case, the dynamic optimisation problem for one scheduling slot can be solved independently from that of another scheduling slot. The flexible recipe method links the planning and scheduling problem with a set of candidate flexible recipe collections instead of detailed dynamic optimisation problems. The flexible recipe provides a surrogate model for the dynamic models. While it is only an approximation of the full dynamic system, the flexible recipe approach can greatly improve the computational efficiency of the solving process.
There are four steps to build the candidate flexible recipe collections (Chu and You, 2012). First, all potential transition processes should be considered. Here we consider one example that manufactures

three products (A, B, and C). There are six potential transition processes for this three-product case, A->B, A->C, B->C,B->A, C->A, and C->B. Similarly, for one case with N products, there are totally N(N-1) potential transition processes. Second, the minimum transition time for each potential transition process is obtained by solving the dynamic optimisation problem with the objective to minimise the transition time. Third, for each possible transition process we should determine the location of a set of transition times. In this work, the intervals between two adjacent transition time points have the equal length. Finally, at each transition time, the dynamic optimisation problem with the objective to minimise the transition cost is solved to obtain the corresponding minimum transition cost. Thus, a set of discrete transition time and cost pairs are obtained, and they are regarded as the candidate flexible recipe collections.

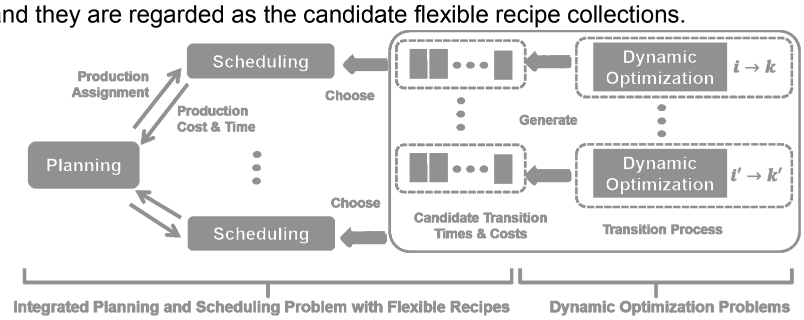

Figure 2: Structure of the flexible recipe method

The structure of the flexible recipe method for the integrated problem is shown in Figure 2. The full space MINLP problem is decomposed into an integrated planning and scheduling problem with flexible recipes and a set of dynamic optimisation problems. All of the dynamic optimisation problems are solved offline using the collocation method to generate the candidate flexible recipe collections (Biegler, 2007). The candidate transition times and costs are linked to the planning and scheduling problem. The integrated planning and scheduling problem with flexible recipes is a mixed integer linear programming (MILP) problem, which is easier to solve than the full space MINLP problem. The deterministic flexible recipe problem is denoted as problem (DF) and it is shown in Eq(12).

(DF) max $Profit = P_1 + P_2$ (12)
s.t. Planning model & Scheduling model
Candidate flexible recipe collections

The robust flexible recipe problem is denoted as problem (RF), shown in Eq. (13).

(RF) max $Profit = P_1 + P_2 + P_5$ (13)
s.t. Planning model & Scheduling model
Candidate flexible recipe collections
Robust constraints (9) and (10)

5. Case study

A case study for a methyl methacrylate (MMA) polymerisation process is used to demonstrate the proposed modelling and solution methods. The dynamic process is a nonlinear free radical polymerisation which uses azobisisobutyronitrile as initiator and toluene as the solvent (Congalidis et al., 1989). The process dynamics can be modelled by the following differential equations:

$$dC_m/dt = -\left(\mathrm{k_p}+\mathrm{k_{fm}}\right)\sqrt{\left(2\mathrm{f}^*\mathrm{k_I}\right)/\left(\mathrm{k_{T_d}}+\mathrm{k_{T_c}}\right)}C_m\sqrt{C_I} + F\left(C_{m_{in}} + C_m\right)/V \tag{11}$$

$$dC_I/dt = -\mathrm{k_I}C_I + \left(F_I C_{I_{in}} - FC_I\right)/V \tag{12}$$

$$dD_0/dt = \left(0.5\mathrm{k_{T_c}}+\mathrm{k_{T_d}}\right)C_I\left(2\mathrm{f}^*\mathrm{k_I}\right)/\left(\mathrm{k_{T_d}}+\mathrm{k_{T_c}}\right) + \mathrm{k_{fm}}\sqrt{\left(2\mathrm{f}^*\mathrm{k_I}\right)/\left(\mathrm{k_{T_d}}+\mathrm{k_{T_c}}\right)}C_m\sqrt{C_I} - FD_0/V \tag{13}$$

$$dD_1/dt = M_m(\mathrm{k_p}+\mathrm{k_{fm}})\sqrt{\left(2\mathrm{f}^*\mathrm{k_I}\right)/\left(\mathrm{k_{T_d}}+\mathrm{k_{T_c}}\right)}C_m\sqrt{C_I} - \mathrm{k_p}\,FD_1/V \tag{14}$$

In this case study, three products (A, B, and C) are produced. The entire planning horizon is divided into three planning periods. The time length of each planning period is 24 h. The objective is to maximise the total profit over the planning horizon. In the (RF) problem, we assume that all production costs can be subject to uncertainty. The symmetric bound of the uncertain set $dcop_{it}$ is plus/minus half of the nominal production cost cop_{it}. This range is determined according to the historical data. The "symmetric bound" is for the bounds that plus/minus the same range of a parameter.

For the flexible recipe method, there are total six potential transition processes. We take the transition process from product A to product B as an example. The dynamic problem of determining the minimum transition time from A to B contains 3,380 continuous variables and 3,563 constraints. CONOPT 3 solves this nonlinear optimisation problem in 0.6 s and returns 0.44 h as the minimum transition time. We then solve the dynamic optimisation problems to minimise the transition cost under different specifications of the transition time. Note that the dynamic optimisation problems are solved offline and each dynamic optimisation problem returns a transition time and cost pair. The flexible recipe candidate collections are built with all the transition time and cost pairs.

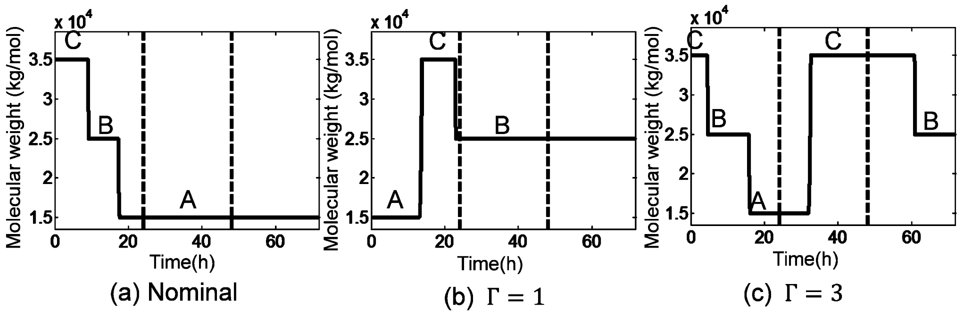

Figure 3: Results of case study for the (a) nominal problem; (b) Problem (RF) with a budget parameter of 1; (c) (RF) with a budget parameter of 3

Figure 3(a) shows the production sequence of the (DF) problem. In this case study, all the production costs are given parameters and not subject to any changes during the production process. The optimal sequence is C->B->A. The (DF) problem is an MILP problem, which contains 900 binary variables, 1,048 continuous variables, and 261 constraints. This MILP problem is solved with CPLEX 12 in 0.20 s. The optimal total profit is \$ 8,171.

The optimal production sequence of the (RF) problem for Γ =1, shown in Figure 3(b), which is A->C->B. In this case study, only one production cost is subject to uncertainty during the production process. The (RF) problem is still an MILP problem. There are 900 binary variables, 1,059 continuous variables, and 271 constraints in this problem. CPLEX 12 solves this problem in 0.31 s and returns \$ 5,590 as the optimal robust profit. The robust optimisation is for the worst case circumstance, which means that if we use the robust production strategy, during the production process even there is one production cost deviating from its nominal value but confined by the symmetric bound during the production process, the final profit of the process should be larger than \$ 5,590. If we still adopt the optimal production strategy which is solved for the deterministic problem as shown in Figure 3(a), once there is an uncertain production cost, the profit may drop from \$ 8,171 for the deterministic case study to \$ 5,015 for the worst uncertain case, which is only 90 % of the robust profit of \$ 5,590.

For the (RF) problem with Γ =3, Figure 3(c) shows the optimal sequence, C->B->A->C->B. In this case study, we assume that three production costs are subject to uncertainty during the production process. This problem is solved with CPLEX 12 in 0.41s and the optimal robust profit is \$ 2,814. Since we only change the value of a parameter in the (RF) problem, it has the same problem size with the previous robust problem in which Γ =1. If there are 3 uncertain production costs during the production process, and we still apply the production strategy for the deterministic situation as shown in Figure 3(a), the final profit drops from \$ 8,171 to \$ 1,013 for the worst case, which is only 36 % of the robust profit of \$ 2,814.

As we can see, the (RF) problems have the same number of binary variables as the (DF) problem. The (RF) problems only introduce a set of continuous variables and linear constraints into the model formulation, that is, converting the deterministic problem into a robust problem does not change the linearity of the model formulation. On the other hand, unlink scenario-based stochastic programming formulation that applies a number of scenarios increasing exponentially with the number of uncertain parameters (Chu and You, 2013b), the robust formulation only slightly enlarge the problem size of the deterministic problem. For the case study in this work, the problem size increases 1.0 % regarding the number of the continuous variables, and 3.8 % regarding the number of the constraints. As for the final total profit, the deterministic production strategy suffers significant profit drop if uncertainties occur during the production process. The robust problem provides a better production strategy by taking uncertainties into its model formulation, and results in better final profit as demonstrated in the case studies.

6. Conclusions

In this work, we developed a robust optimisation approach to deal with uncertainties in integrated operations of continuous processes. We first formulated an MINLP model for the integration of planning, scheduling and dynamic optimisation of multi-product continuous processes. The robust optimisation problem was then introduced to handle process uncertainties. Last, the flexible recipe method was exploited to enhance the computational efficiency. The result showed that the robust optimisation approach was able to control the degree of conservatism by using a budget parameter. The robust production strategy led to higher profits than the deterministic production strategy when uncertainty occurred under the worst-case circumstances.

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