CHEMICAL ENGINEERING TRANSACTIONS

VOL. 45, 2015

A publication of AIDIC
The Italian Association of Chemical Engineering
www.aidic.it/cet

Guest Editors: Petar Sabev Varbanov, Jiří Jaromír Klemeš, Sharifah Rafidah Wan Alwi, Jun Yow Yong, Xia Liu

**ISBN** 978-88-95608-36-5; **ISSN** 2283-9216 DOI: 10.3303/CET1545160

# Mixed Convection Flow over a Permeable Stretching Wedge in the Presence of Heat Generation/Absorption, Viscous Dissipation, Radiation and Ohmic Heating

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The analysis of magnetohydrodynamic mixed convection boundary layer flow over a permeable stretching wedge in the presence of heat generation, viscous dissipation, thermal radiation and ohmic heating has been presented. The governing equations are nondimensionlised using the similarity transformations and the resulting coupled non-linear ordinary differential equations have been solved by using the Homotopy Analysis Method (HAM). The effects of pertinent parameters such as magnetic field parameter, velocity ratio parameter, radiation parameter, heat generation/absorption parameter and Eckert number on the velocity and temperature fields are presented. It is found that the momentum and thermal boundary layer thicknesses decrease with the increase of Eckert number in the presence of suction and heat absorption when the wedge stretches slower than the free stream flow. The present results for skin-friction are compared with the previously published results for the limiting case and an excellent agreement is found between the results.

## 1. Introduction

The study of convective heat transfer along a wedge shape bodies received the attention of many researchers because of their use in engineering and technology. For example transportation, aerospace engineering, packed bed reactors, crude oil extraction, polymer and chemical industry (Saoulio and Bencheikh Lehocine, 2014), nanotechnology and biological science etc. Kafoussias and Nanousis (1997) studied the magnetohydrodynamic (MHD) viscous, incompressible, and electrically conducting laminar boundary-layer flow over a wedge with suction or injection. They concluded that the flow field is influenced appreciably by the applied magnetic field. Yih (1998) numerically investigated the effect of constant suction/blowing on steady two-dimensional laminar forced convection flow past a porous wedge with uniform heat flux using Keller-box method. Kuo (2005) performed heat transfer analysis for the Falkner-Skan wedge flow and the resulting nonlinear boundary-layer equations were solved using the differential transformation method. Hayat et al. (2011) described the mixed convection flow of a non-Newtonian power law fluid past a wedge in a saturated porous medium and they obtained analytical solution by using the homotopy analysis method (HAM). Hsiao (2011) studied the MHD mixed convection flow of a second-grade viscoelastic fluid past a porous wedge and similarity solution was obtained with second-order accurate finite difference method. Pal and Hiranmoy (2013) investigated the influence of thermophoresis and Soret–Dufour effects on magnetohydrodynamic heat and mass transfer over a non-isothermal wedge in the presence of thermal radiation, temperature dependent viscosity, and viscous dissipation. They obtained the similarity solution numerically using the fifth-order Runge-Kutta-Fehlberg method with shooting technique. Their study revealed that the skin-friction coefficient and the local Sherwood number increases with increasing values of thermal radiation parameter in the presence of heat generation/absorption. Ganapathirao et al. (2015) studied the effects of chemical reaction, heat and mass

Please cite this article as: Marneni N., Ashraf M., 2015, Mixed convection flow over a permeable stretching wedge in the presence of heat generation/absorption, viscous dissipation, radiation and ohmic heating, Chemical Engineering Transactions, 45, 955-960 DOI:10.3303/CET1545160

transfer on an unsteady mixed convection flow over a vertical porous wedge in the presence of heat generation/absorption using an implicit finite difference with quasi-linearization technique.

All the above mentioned papers are discussed the boundary-layer flow over a fixed wedge. The study of boundary-layer flow over a moving or stretching surface is important in many engineering processes such as production of polymeric sheets, wire drawing, roof shingles, insulating materials, linoleum, drawing of plastic films and fine-fibre mats etc. However, very limited number of studies was dealing with the boundary-layer flow over a stretching wedge. Ishak et al. (2007) investigated numerically the steady two-dimensional laminar boundary-layer flow of a viscous incompressible fluid past a moving wedge with suction or injection using Keller-box method. They observed that the suction and large angle of the wedge delays flow separation. Su et al. (2012) investigated the MHD mixed convection flow over a permeable stretching wedge in the presence of thermal radiation and ohmic heating using differential transformation and basis function method (DTM-BF). This method is based on Taylor series expansion and usually it is used to solve the differential equations for small values of parameters. Also the DTM gives the divergent solutions when the domain is unbounded or variables go to infinity. In the present paper, homotopy analysis method (HAM) is employed to study the MHD mixed convection flow over a non-isothermal permeable stretching wedge in the presence of heat generation/absorption, viscous dissipation, thermal radiation and ohmic heating. There are no limitations on HAM solution and it is valid for all ranges of parameter values. The influence of different flow parameters on the velocity and temperature variations are discussed graphically.

## 2. Formulation of the problem

Consider the steady two dimensional MHD mixed convection flow and heat transfer over a permeable stretching wedge. The $x$ - axis is taken along the wedge and $y$ - axis is normal to the wedge. The wedge is stretching along $x$ - axis with velocity $U_w$ and the free stream velocity away from the wedge is $U$. The suction/injection velocity across the wedge surface is $v_w$ . A uniform magnetic field of strength $B$ is applied in the $y$ - axis direction and the induced magnetic field is neglected. The temperature of the wedge surface is $T_w$ and the free stream temperature far away from the wedge is $T_\infty$ . The associated governing equations in the presence of heat generation or absorption, thermal radiation, viscous dissipation and ohmic heating are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+g\beta_0(T-T_\infty)\sin\frac{\Omega}{2}-\frac{\sigma B^2}{\rho}(u-U) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=-\frac{u}{\rho C_p}\left(\rho U\frac{dU}{dx}+\sigma B^2 U\right)+\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{Q_0}{\rho C_P}(T-T_\infty)-\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}+\frac{\nu}{C_P}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B^2}{\rho C_P}u^2 \tag{3}$$

Subject to the following boundary conditions

$$u(x,0)=U_w,\ v(x,0)=v_w,\ T(x,0)=T_w. \tag{4}$$

$$u(x,\infty)=U,\ T(x,\infty)=T_\infty. \tag{5}$$

Where

$$U_w=RU,\ U=U_0x^m,\ v_w=-C(\nu U_0)^{\frac{1}{2}}\left(\frac{m+1}{2}\right)x^{\frac{m-1}{2}},\ T_w=T_\infty+bx^{2m} \tag{6}$$

In the above expressions $u$ and $v$ are the velocity components in $x$ and $y$ directions, respectively, $\rho$ is the fluid density, $p$ is the pressure, $\sigma$ is the electrical conductivity and $B=B(x)=B_0x^{(m-1)/2}$ is the component of the total magnetic field $B$ in the $y$-direction, $T$ is the temperature, $\beta_0$ is the coefficient of thermal expansion, $g$ is the gravity field, $\nu$ the kinematic viscosity, $\Omega=\beta\pi$ is the wedge angle, $\beta=2m/(m+1)$ is the wedge angle parameter ( $\beta=0$ and $\beta=1$ corresponds to the horizontal and vertical wall cases, respectively), $C_p$ the specific heat of the fluid, $k$ is the thermal conductivity, $Q_0>0$ is the heat absorption or $Q_0 < 0$ is the heat generation coefficient, $R = U_w/U$ is the velocity ratio, $U_0$ is a constant, $m$ is the Falkner-Skan power-law parameter which is related to the wedge angle $\beta$ by $m = \beta/(2-\beta)$ , $v_w$ is the suction/injection velocity, $C$ is the suction/injection parameter and $b$ is a constant.

The radiative heat flux $q_r$ using the Rosseland approximation is given by

$$q_r = \frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{7}$$

where $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ is the absorption coefficient. It is assumed that the temperature difference within the flow field is small so that $T^4$ can be expanded in Taylor's series about $T_\infty$ and neglecting higher order terms gives $T^4 = 4T_\infty^3 T - 3T_\infty^4$. Under these assumptions Eq(7) reduces to

$$q_r = \frac{-16T_\infty^3\sigma^*}{3k^*}\frac{\partial T}{\partial y} \tag{8}$$

We introduce the following non-dimensional variables to obtain the similarity solution

$$\eta = \sqrt{U}\,(\nu x)^{-(1/2)}\,y,\ \psi = \sqrt{\nu x U}\,f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \xi = \frac{\sigma B_0^2}{\rho U}x, \tag{9}$$

where the stream function $\psi$ is defined as

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{10}$$

so that the continuity Eq(1) is satisfied. The functions $f' = u/U$ and $\theta$ are the dimensionless velocity and temperatures, respectively, $\xi$ is the local magnetic parameter. In view of Eq(8) and Eq(9), the Eq(2) and Eq(3) reduces to a coupled system of non-linear ordinary differential equations

$$f''' + \frac{1+m}{2}ff'' + m\left[1-(f')^2\right] - M(f'-1) + \left(\lambda Sin\frac{\Omega}{2}\right)\theta = 0 \tag{11}$$

$$\frac{1}{\Pr}(1+Nr)\theta'' + \frac{1+m}{2}f\theta' - (2m\,f' + \xi Q)\theta + Ec\left(-(m+M)f' + (f'')^2 + M(f')^2\right) = 0 \tag{12}$$

The boundary conditions, Eq(4) and Eq(5) takes the following form

$$f(0) = C,\ f'(0) = R,\ f'(\infty) = 1. \tag{13}$$

$$\theta(0) = 1,\ \theta(\infty) = 0. \tag{14}$$

Here primes signify the differentiation with respect to $\eta$ . $Gr_x$ , $\mathrm{Re}_x$ , $Ec$ , $\Pr$ are the local Grashof number, local Reynolds number, Eckert number and Prandtl number $M$ is the magnetic parameter, $Nr$ is the thermal radiation parameter, $\lambda$ is the mixed convection parameter, $Q$ is the dimensionless heat generation or absorption parameter and these are defined as

$$\left.\begin{aligned} &Gr_x = \frac{g\beta_0(T_w - T_\infty)x^3}{\nu^2}, \mathrm{Re}_x = \frac{Ux}{\nu}, Ec = \frac{U^2}{C_p(T_w - T_\infty)}, \Pr = \frac{\mu C_p}{k}, M = \frac{\sigma B_0^2}{\rho U_0}, \lambda = \frac{Gr_x}{\mathrm{Re}_x^2}, \\ &Nr = \frac{16T_\infty^3\sigma^*}{3k^* k}, Q = \frac{Q_0}{\sigma C_p B_0^2}. \end{aligned}\right\} \tag{15}$$

The physical quantities of engineering interest are the skin friction coefficient $C_f$ and the Nusselt number $Nu$ which are defined by

$$C_f = \frac{\tau_w}{\rho U^2} \text{ and } Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \text{ where } \tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{16}$$

With the help of similarity variables in Eq(9), we have

$$\mathrm{Re}_x^{1/2} C_f = f''(0) \text{ and } \mathrm{Re}_x^{-1/2} Nu_x = -\theta'(0). \tag{17}$$

### 3. Homotopy analysis solution

Homotopy analysis is used to solve the coupled system of differential equations i.e. Eq(11) and Eq(12) for velocity and temperature fields subject to the boundary conditions in Eq(13) and Eq(14). For that, the initial guesses and auxiliary linear operators are defined as follows:

$$f_o(\eta) = C + \eta - (1-R)(1-\exp(-\eta)), \quad \theta_o(\eta) = \exp(-\eta) \tag{18}$$

$$L_f = \frac{d^3 f}{d\eta^3} + \frac{d^2 f}{d\eta^2}, \quad L_\theta = \frac{d^2\theta}{d\eta^2} + \frac{d\theta}{d\eta} \tag{19}$$

with the following properties

$$L_f[C_1 + C_2\eta + C_3\exp(-\eta)] = 0, \quad L_\theta[C_4 + C_5\exp(-\eta)] = 0 \tag{20}$$

in which $C_i$ ($i = 1$ to 5) are the arbitrary constants. Following the homotopy analysis method outlined in Liao (2004) and solving the zero-order and $l$ th – order systems we obtain

$$f(\eta) = f_0(\eta) + \sum_{l=1}^{\infty} f_l(\eta), \quad \theta(\eta) = \theta_0(\eta) + \sum_{l=1}^{\infty} \theta_l(\eta) \tag{21}$$

$$f_l(\eta) = f_l^*(\eta) + C_1 + C_2\eta + C_3\exp(-\eta), \quad \theta_l(\eta) = \theta_l^*(\eta) + C_4 + C_5\exp(-\eta) \tag{22}$$

$$C_2 = C_4 = 0, \quad C_3 = \left.\frac{\partial f_l^*(\eta)}{\partial\eta}\right|_{\eta=0}, \quad C_1 = -C_3 - f_l^*(0), \quad C_5 = -\theta_l^*(0) \tag{23}$$

with $f_l^*(\eta)$ and $\theta_l^*(\eta)$ as the special solutions of Eq(11) and Eq(12).

### 4. Results and discussion

The convergence of HAM solutions for the functions $f(\eta)$ and $\theta(\eta)$ strongly depends on the convergence control parameters $\hbar_f$ and $\hbar_\theta$. In order to determine the range of convergence region, $\hbar$ - curves are plotted as shown in the Figure 1. The $\hbar_f$ -curve for $f''(0)$ and $\hbar_\theta$ -curve for $\theta'(0)$ are plotted at 20th-order approximations when $M = 1$, $m = 0.2$, $R = 0.2$, $C = 1$, $\Pr = 1$, $Nr = 0.2$, $Ec = 0.2$, $\xi = 2$, $Q = 1$, and $\lambda = 0.8$. The admissible range of $\hbar_f$ for the stated value of the parameters is $-1.7 \le \hbar_f \le -0.2$ and for $\hbar_\theta$ is $-1.5 \le \hbar_\theta \le -0.15$. Here, it is noted that the series solutions in Eq(21) converges for all chosen values of parameters when $\hbar_f = \hbar_\theta = \hbar = -0.5$. In Table 1 the results for the skin-friction coefficient $f''(0)$ obtained by HAM are compared with the results obtained by Yih (1998), Ishak et al. (2007) and Su et al. (2012) for the limiting cases. The results are found to be in excellent agreement which indicates the accuracy of homotopy analysis method.

*Table 1: Comparison of the values of $f''(0)$ for different values of C when $\lambda = 0, m = 1, M = 0$ and $R = 0$*

| C | Numerical | DTM-BF | | HAM |
|---|---|---|---|---|
| | Yih (1998) | Ishak et al. (2007) | Su et al. (2012) | Present results |
| -1.0 | 0.756575091 | 0.7566 | 0.75658 | 0.7565754 |
| -0.5 | 0.969229466 | 0.9692 | 0.96923 | 0.9692304 |
| 0.0 | 1.232587947 | 1.2326 | 1.23259 | 1.2325870 |
| 0.5 | 1.541750966 | 1.5418 | 1.54175 | 1.5417480 |
| 1.0 | 1.889313587 | 1.8893 | 1.88931 | 1.8893020 |

Computations have been carried out by the homotopy analysis method for different values of system parameters such as velocity ratio $R$, magnetic field $M$, thermal radiation $Nr$, heat generation or absorption $Q$ and Eckert number $Ec$. It is important to note that the velocity ratio parameter $R>1$ means that the wedge stretches faster than that of the free stream flow and $R<1$ means that the wedge slower than that of the free stream flow. The suction/injection parameter $C>0$ represents the suction and $C<0$ represents the injection through the wedge surface. Figure 2 depicts the effect of $M$ on the velocity profiles for both $R>1$ and $R<1$ cases. It is observed that the momentum boundary-layer thickness decreases with increasing for $R>1$ while it increases with increasing $M$ for $R<1$. Figure 3 shows the effect of $M$ on the temperature profiles when $R>1$ and $R<1$. It is observed that the thermal boundary-layer thickness reduces with increasing value of $M$ for $R<1$ whereas the thermal boundary layer thickness increases with $M$ for $R>1$. The influence of thermal radiation on the fluid velocity is shown in Figure 4. It can be seen that the fluid velocity increases with the increase of thermal radiation parameter $Nr$ when $R<1$. Thus the radiation effect is to increase the momentum boundary-layer thickness in the presence of suction and heat absorption when $R<1$.

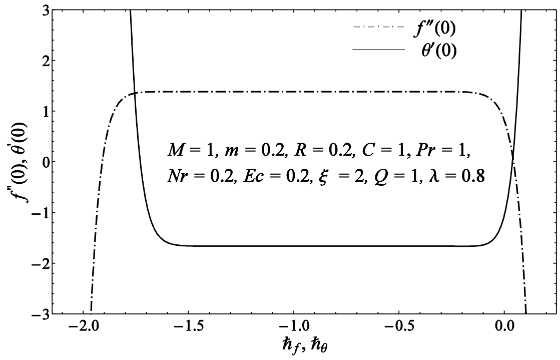


Figure 1: $\hbar$-curve for $f''(0)$ and $\theta'(0)$

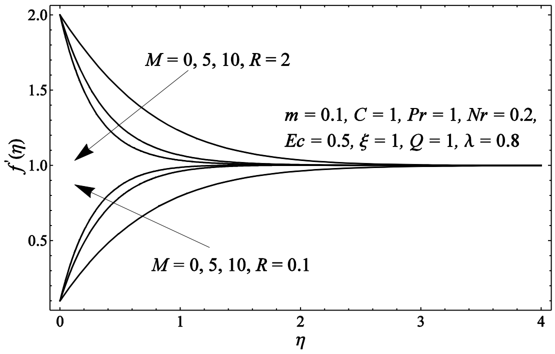


Figure 2: Velocity variation with $M$

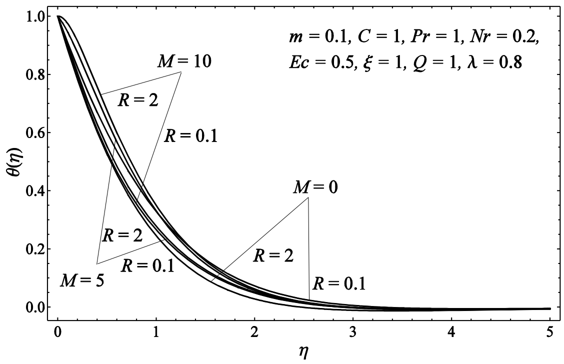


Figure 3: Temperature variation with $M$

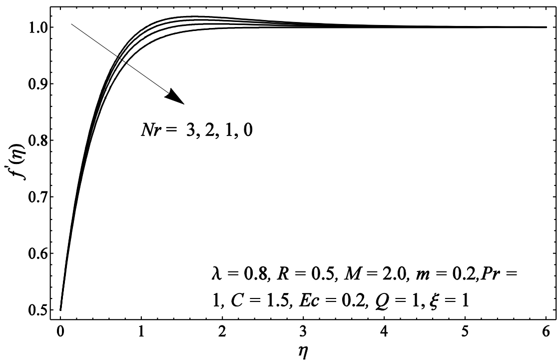


Figure 4: Velocity variation with $Nr$.

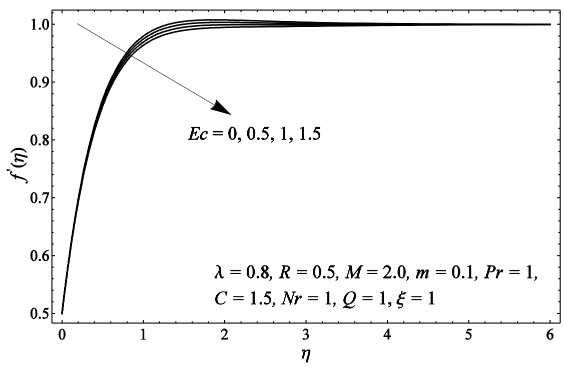


Figure 5: Velocity variation with $Ec$

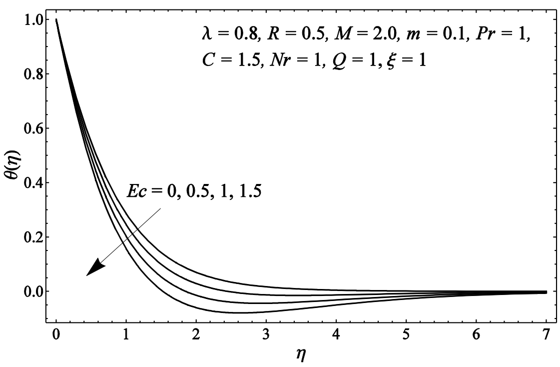


Figure 6: Temperature variation with $Ec$

Figures 5 and 6 shows the effect of $Ec$ on velocity and temperature. It is observed that both momentum and thermal boundary-layer thicknesses decreases with increasing Eckert number $Ec$ when $R<1$. The effects of heat generation or absorption on fluid velocity and temperature are shown in Figures 7 and 8. It is observed that both momentum and thermal boundary-layer thicknesses decreases with increasing heat absorption $(Q > 0)$ whereas an opposite trend is observed with increasing heat generation $(Q < 0)$ in the presence of suction when $R < 1$.

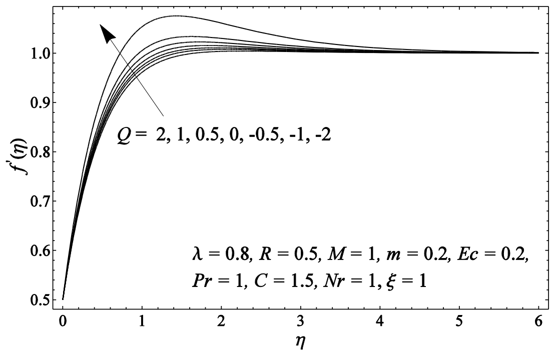


*Figure 7: Velocity variation with* $Q$

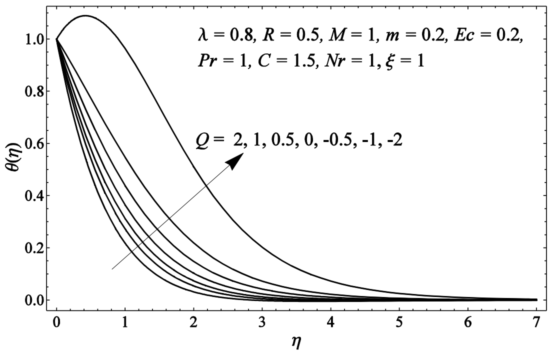


*Figure 8: Temperature variation with* $Q$

## 5. Conclusions

The MHD mixed convection flow over a permeable stretching wedge in the presence of heat generation, viscous dissipation, thermal radiation and ohmic heating has been studied analytically using the homotopy analysis method (HAM). The present results are validated by comparing with the previously published results for limiting cases and the comparison show an excellent agreement between the results. The effects of magnetic field parameter, thermal radiation parameter, Eckert number and heat generation/absorption parameter on the velocity and temperature profiles have been discussed. In the case of suction, it is found that both the momentum and thermal boundary-layer thicknesses increases with increasing heat generation while an opposite trend is found with increasing heat absorption when the porous wedge stretches slower than that of the free stream flow. The present study limited to steady state only, it can be further extended to investigate the unsteady mixed convection flow due to combined heat and mass transfer.

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