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The Dynamic Evolution of Industrial Clusters

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ABSTRACT

A mathematical model and its implementation are presented to analyze the evolution of industrial clusters. The static model is a combination of basic ideas of oligopoly and oligopsony theory as well as fundamental economic results describing the effects of technology development. The dynamic model is based on gradient adjustment in which each decision variable is adjusted in proportion to the corresponding partial derivative of the profit function of the firm controlling this variable. The complexity of the model makes it analytically intractable, therefore computer simulation is used. A sensitivity analysis is performed to examine how the trajectories of the main characteristics of the firms depend on model parameters, and how the entire cluster develops in time.

RESUMEN

Un modelo matemático y su implementación son presentados para analizar la evolución de grupos industriales. El modelo estático es una combinación de ideas básicas de oligopolios y teoria oligoposonia bien como resultados de economia fundamental describiendo los efectos de desarrollo tecnológico. El modelo dinámico es basado en el gradiente de ajustamiento en el cual toda decisión variable es ajustada en proporsión a la correspondiente derivada parcial de la función de utilidad de la firma controlando esta variable. La complejidad del modelo hace este analiticamente muy dificil, por lo tanto es usada simulación computacional. Un análisis de sensibilidad es realizado para examinar como las trajecgtorias de la característica principal de las firmas dependem del modelo de parametros, y como el grupo entero se desarrolla con el tiempo.

Key words and phrases: *Dynamic model, industrial clusters, simulation.*

Math. Subj. Class.: *91B26, 91B70.*

Introduction

The classical oligopoly theory dates back to the work of Cournot [3]. It examines an industry in which several firms produce identical product or offer identical service to a homogeneous market. Since then a significant number of researchers focused on the different extensions and generalizations of Cournot's classical model. A comprehensive summary of the earlier works is given in [5] and multi-product models with some applications are discussed in [6]. In the early stages, oligopolies were considered as noncooperative games in which the firms are the players, their output levels are the strategies, and the profit functions are the payoffs. The existence and uniqueness of the equilibrium was first the main issue, under certain monotonicity and convexity assumptions the existence and uniqueness of equilibrium was proved. This important result was later extended to more realistic model variants including single product models with product differentiation, multiproduct oligopolies, labor-managed and rent-seeking games. The main focus of the studies in oligopoly theory has later turned into dynamic extensions. Models were developed with discrete and continuous time scales and the resulting difference and differential equation systems were investigated. The main issue was the asymptotical stability of the equilibrium, conditions were derived to guarantee that the output trajectories converge to the equilibrium in the long run. Most models were linear, where local and global stability are equivalent and very little attention was given to nonlinear dynamics until the late 80s. In developing dynamic models there are usually two alternative concepts. In the case of best response dynamics it is assumed that each firm adjusts its output into the direction toward its best response. This approach requires the knowledge of the best response functions of the firms, which needs the solution of usually nonlinear optimization problems based on global information on the payoff functions. In the case of gradient adjustments it is assumed that the firms adjust their outputs in proportion to their marginal profits. This idea has a lot of sense, since in the case of positive (negative) gradient value the firm's interest is to increase (decrease) output level. This concept requires only local information about the payoff functions, so it is much more realistic than the use of best response dynamics. A comprehensive summary of the recent developments in this area can be found in [7] and [1].

Most studies in oligopoly theory considered only the market as a link between the firms; the unit price was always a function of the total output level of the industry due to the demand-supply balance. However in realistic economies the firms are linked together in much more complicated ways. First, they use common supply of energy, raw material, labor, capital etc., and therefore they also compete on this secondary market in addition to the market of their product. This idea was elaborated in the studies of oligopsonies [10]. In multiproduct oligopolies on the other hand, the firms might buy and use the products of other firms, so a network of firms develops. Network oligopolies were introduced and some elementary results were reported in [9].

In most models analytic results could be derived under only very special conditions, which are not the case in realistic economies. Instead of investigating very limited cases theoretically, it is much more important and useful to use computer simulation under realistic conditions and examine the evolution of more advanced production systems such as the industrial clusters.

The industry of a certain region consists of several types of firms. Some produce final products which are sold directly to the market, while others (maybe in addition to final products) produce material, parts, components what other firms buy and build in their final products. Therefore there is a complicated input-output relation between the firms. They get their work force, energy and capital from the same secondary markets, where they also compete with each other. The R&D investment of any firm spills over to others who can also benefit from this innovation. In the literature discussing industrial clusters the authors focus on mainly descriptive and statistical issues and very little attention is given to the evolution of the clusters in time [8]. In this paper we will examine the way how an industrial cluster develops in time and how important economic characteristics depend on model parameters. Our simulation methodology is similar to agent-based techniques. Some initial attempts combining industrial clusters with agent-based simulation were presented in [2], [4], [11].

This paper develops as follows. In Section 2 the mathematical model will be outlined, and the simulation methodology and numerical results will be presented in Section 3. Final conclusions will be drawn in Section 4.

1 The Mathematical Model

For the sake of simplicity we assume that there are two types of firms in the cluster: suppliers and producers. Producers sell their products to an open market, while suppliers sell their products to the producers. There are altogether total of m suppliers and n producers in the system. They are linked together in the open market, in the energy and labor markets and by innovation spillovers. In addition, the system has to satisfy the product-balance conditions.

For any supplier i, we denote its output by s_i, the price of its product by p_i^s, its labor usage by L_i^s and its profit by φ_i^s. For any producer j, we denote its production level by z_j, the price of its product by p_j^p, its labor usage by L_j^p, its innovation development by I_j and the total cumulative innovation level by $\widetilde{I}_j$, the impact of innovation level on sale price by $F(\widetilde{I}_j)$, the cost function of innovation by $D_j(I_j)$. The profit of this producer is φ_j^p. The price function of labor in the whole cluster is denoted by p^L, which depends on the total demand of labor.

The profit of a supplier can be obtained as

$$\varphi_i^s = s_i \cdot p_i^s(s_1, \ldots, s_m) - L_i^s(s_i)p^L \left(\sum_{i=1}^{m} L_i^s(s_i) + \sum_{j=1}^{n} L_j^p(z_j, \widetilde{I}_j) \right), \tag{1.1}$$

which is the difference of its revenue and labor cost. We set all other costs to zero. The profit function of the producers is the following:

$$\varphi_j^p = z_j \cdot p_j^p(z_1, \ldots, z_n) F_j(\widetilde{I}_j) - L_j^p(z_j, \widetilde{I}_j) p^L \left(\sum_{i=1}^{m} L_i^s(s_i) + \sum_{j=1}^{n} L_j^p(z_j, \widetilde{I}_j) \right) - \sum_{i=1}^{m} x_{ij} p_i^s(s_1, \ldots, s_m) - D_j(I_j), \tag{1.2}$$

where x_{ij} is the amount of the product of supplier i purchased by producer j. The total output of supplier i is therefore

$$s_i = \sum_{j=1}^{n} x_{ij}. \tag{1.3}$$

In our study we select special function forms. We assume that

$$p_i^s(s_1, \dots, s_m) = A_i - B_i s_i - \sum_{l \neq i} b_{il} s_l, \tag{1.4}$$

that is, the price of any supply decreases if the output by any supplier increases;

$$L_i^s(s_i) = \gamma_i + \delta_i s_i, \tag{1.5}$$

that is, more production requires more labor. The output of producer j is given by the linear function

$$z_j = \sum_{i=1}^{m} a_{ij} x_{ij} + a_{0j}. \tag{1.6}$$

The prices of the final products are also linear:

$$p_j^p = \overline{A}_j - \overline{B}_j z_j - \sum_{l \neq j} \overline{b}_{jl} z_l, \tag{1.7}$$

which assumes that the final products are substitutes. The innovation development and spillover are modeled as

$$\widetilde{I}_j(t+1) = \widetilde{I}_j(t) + I_j + \sum_{l \neq j} k_{jl} I_l, \tag{1.8}$$

that is, each producer invests in innovation and each of them can utilize the innovation development of the competitors. The price of any final product is affected by the innovation dependent factor

$$F_j\left(\widetilde{I}_j\right) = 1 + \left(F_j^{max} - 1\right) \cdot \left(1 - e^{-\omega_j \widetilde{I}_j}\right). \tag{1.9}$$

In this function form we model the fact that with higher technological level better final products are produced, so their prices become higher. If $\widetilde{I}_j = 0$, then this factor equals 1, it increases in $\widetilde{I}_j$ and converges to a maximum value F_j^{max} as $\widetilde{I}_j$ tends to infinity. The need of labor of producer j depends on its production and innovation levels

$$L_j^p(z_j, \widetilde{I}_j) = (\overline{\gamma}_j + \overline{\delta}_j z_j) \cdot e^{-\overline{\overline{\omega}}_j \widetilde{I}_j}, \tag{1.10}$$

that is, innovation decreases the labor need of producers. The innovation development cost is also assumed to be linear

$$D_j(I_j) = u_j + v_j I_j \tag{1.11}$$

and finally the price of labor is a linear function of the total labor usage:

$$p^L = c - d \cdot \left(\sum_i L_i^s + \sum_j L_j^p\right). \tag{1.12}$$

In this decreasing function form we model the fact that in higher labor force the ratio of skilled workers becomes less, so the average wage decreases.

In this model the decision variables of the suppliers are their output levels s_i, while those of the producers are the x_{ij} flows from the suppliers to them. We assume in our model that extra supplies can be sold outside the cluster for the same price, and in the case of supply shortages they can be purchased from sources outside the cluster. We also assume for the sake of simplicity that the firms increase their innovation levels at each time period by a constant innovation step.

2 Simulation and Analysis of the Results

The evolution of the cluster in time is simulated by using gradient adjustments. We start with a set of initial values of all decision variables, and then we adjust them in the following way. We compute first the derivatives of the payoff functions with respect to all decision variables. Then the value of each variable is adjusted in proportion to the value of the partial derivative of the profit function of the associated firm. This step is then repeated at each time period $t \geq 1$.

This dynamism can be generated for a long period of time with many different selected values of model parameters, so we can gain insight into the dependence of the firms' outputs on certain important model characteristics.

In this paper, we consider the simple situation when $m = 3$ and $n = 2$. In the following simulation experiments, a sensitivity analysis is performed, in which we will alter the value of one of the parameters in a given range, while the values of all other parameters are kept constant. In this way we can see how the output trajectories depend on this altered parameter. We will examine first the effect of the maximum prices A_i, $\overline{A}_j$ and c defined in equations (1.4), (1.7), and (1.12). We will also alter the value of parameter $\overline{b}_{jl}$ in price function (1.7), and also the innovation development step I_j. The values of the other parameters are fixed as follows. For suppliers $i = 1$, 2, 3, we have $B_i = 1$, $b_{il} = 0.1$ for $l \neq i$ (to represents the low level interaction between the suppliers in their prices), furthermore $\gamma_i = 10$ and $\delta_i = 0.4$. For producers $j = 1$, 2, we have $a_{0j} = 50$, $a_{ij} = 0.3$, $\overline{B}_j = 1$, $k_{jl} = 0.1$ for $l \neq j$, $F_j^{max} = 2$, $\overline{\gamma}_j = 50$, $\overline{\delta}_j = 0.3$ or 0.5, $u_j = 50$, $v_j = 0.1$, $\omega_j = 0.1$, and $\overline{\overline{\omega}}_j = 0.05$. For the labor market, we have $d = 0.6$. Inside an experiment group only one parameter value is varied, all others are left constant.

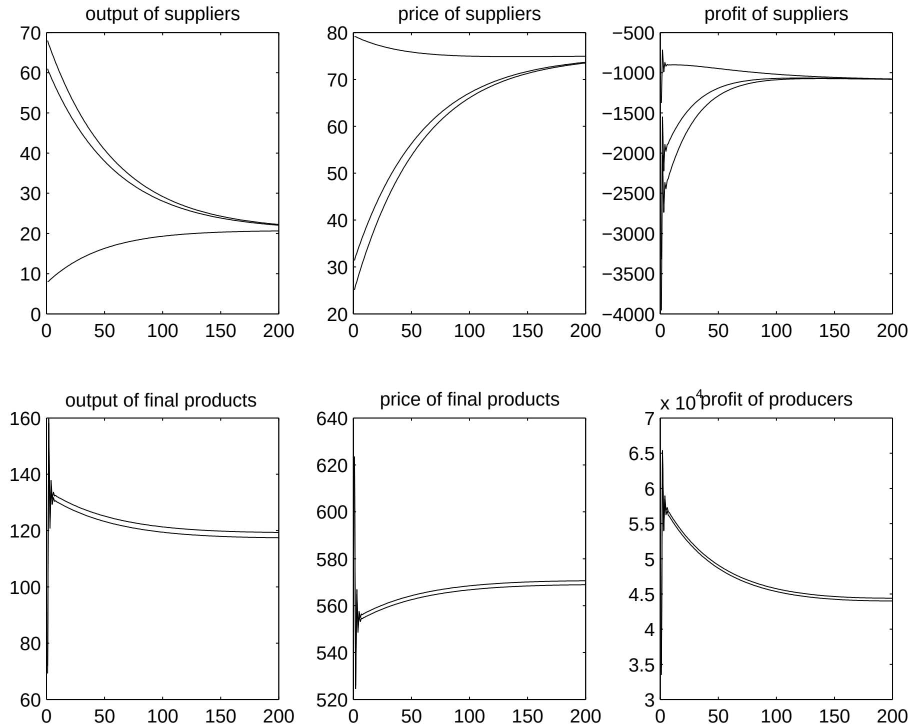
At the beginning of the simulation process, the initial values $x_{ij}(0)$ are generated randomly using uniform distribution from the interval $[0, 40]$. The values of z_j and s_i are calculated according to equations (1.3) and (1.6). The same set of the $x_{ij}(0)$ values are selected in the same simulation group for comparison purposes. The initial value of innovation of all producers is chosen as 1.

The numerical results can be summarized in detail as follows.

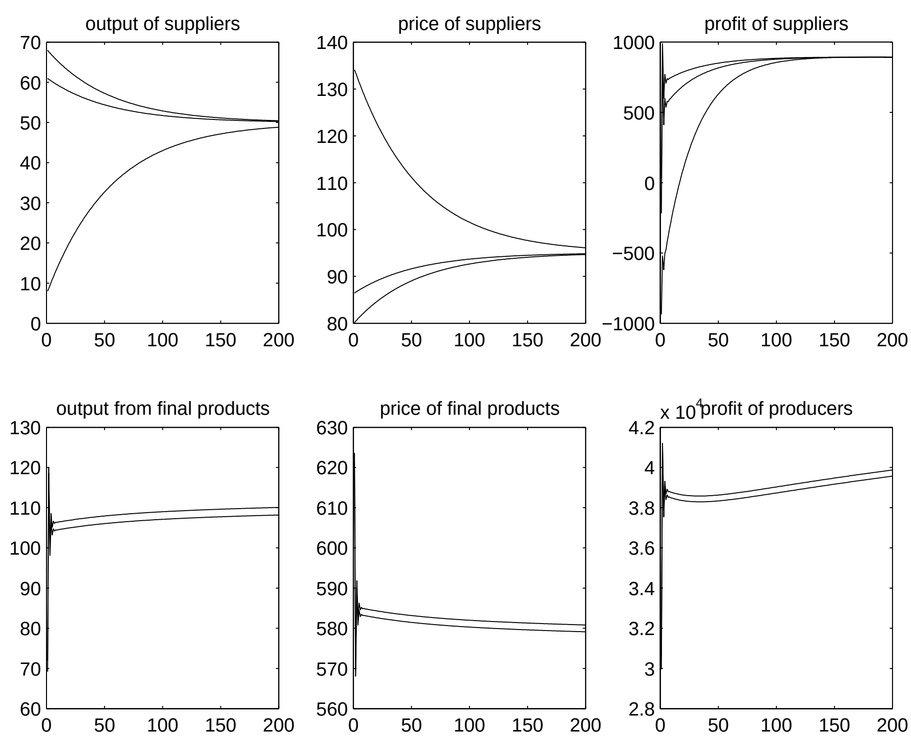
2.1 Effect of changing A_i

In this group of experiments, we have $\overline{\delta}_j = 0.5$, $\overline{A}_j = 700$, $c = 300$, $\overline{b}_{jl} = 0.1$, innovation increment is selected as 0.001 and the value of variable A_i changes from 100 to 400. The generated $x_{ij}(0)$ values in this group are $x_{11}(0) = 32$, $x_{12}(0) = 36$, $x_{21}(0) = 36$, $x_{22}(0) = 25$, $x_{31}(0) = 5$ and $x_{32}(0) = 3$. Figure 1 shows the behavior patterns of the suppliers and producers. Since the initial values for the three suppliers and those of the two producers are different, they have different trajectories.

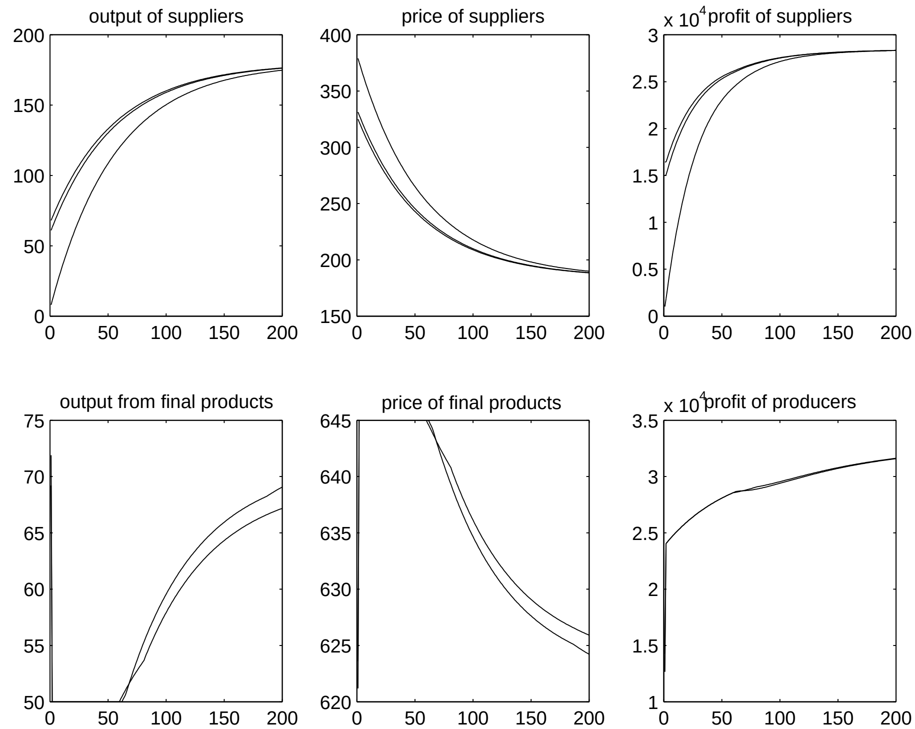
In this group of simulation, when $A_i \leq 135$ (see Figure 1(a)), the profits of the suppliers always start



(a) with $A_i = 100$



(b) with $A_i = 155$



(c) with $A_i = 400$

Figure 1: Results with varying values of A_i

from a negative value and converge to a negative value with relatively small absolute value. Depending on the initial values $x_{ij}(0)$, this limit could be a little different. In such cases government subsidies are needed. The suppliers' productivities decrease (or increase) from the initial values and then converge. The labor of the suppliers is a linear function of the supplier's productivity; hence it has the same pattern as that of their productivity. The prices of the supplies increase or decrease, but also converge. There are some oscillations at the first few iterations in the labor price, and this results in similar oscillations of the profits of the suppliers at the beginning. To react to the higher price of supplies and higher labor price, the producers cut their production levels and labor usage. Even though their price increases, their profit is still decreasing. Some oscillations can be observed in the producers' behaviors: in the productivity, in the price, in the profit, and in the labor usage. Since the labor usage of the producers is approximately a linear function of their output, its pattern is very similar to that of the producer's output. Therefore, we will not discuss the labor usages of the producers and suppliers in later analysis.

Interval (135, 165) for A_i is a pattern transition domain (see Figure 1(b)). Depending on the actual initial values $x_{ij}(0)$, this transition domain could be slightly different. In this range, patterns transfer gradually. When $A_i > 135$, the profit of the suppliers start converging to a positive value. The long term behavior of the output and the profit of producers changes from convex trajectory to concave trajectory; similarly, that of the price of final products changes from a concave trajectory to a convex one; and the trajectory of the labor price transfers from concave into convex.

For $A_i > 165$ (see Figure 1(c)), except the profit of the suppliers, all other trajectories reverse their shape from that observed with $A_i < 135$. Output, profit and labor usage of suppliers increase before they converge, while the final product prices decrease before converging. When A_i becomes larger, the limit values increase accordingly. The labor, output and profit of the producers increase fast at the beginning and then slow down, while their product price and labor price decrease fast at the beginning and then decrease slowly. When A_i is increased, the price of supplies also increases. If $A_i > 300$, then as the reaction to high supply price, the producers decide not to purchase from local suppliers. Producers have constant output level of 50 at the first few iterations, until supply prices and labor usage decrease to a certain value when the producers start having positive marginal profits. Then the producers increase their output levels and purchase from the suppliers. This can be observed in the case of $A_i = 300$, when z_j drops down to 50 and labor usage of the producers decrease down to around 71 and stays there for a few more iterations.

The price function of the supplies not only affects the supplier's own behavior, but also the behavior of producers. A low maximum price implies that the suppliers and also the producers lose money, and it results in job reductions. This situation won't change if the suppliers would increase their prices later on. A high maximum supply price could increase their labor usage, the output and the profit of the suppliers, however the producers' output and labor need could even decrease. This situation can lead to the departure of skilled workers who can find jobs in other regions and this would lead to the increase of the price of labor, so oscillation and hence instability could occur in the system. The other consequences of decreasing labor force, such as offering incentives to people to move into the cluster, hiring immigrants etc. are not considered in this paper. We will return to these issues in a future paper.

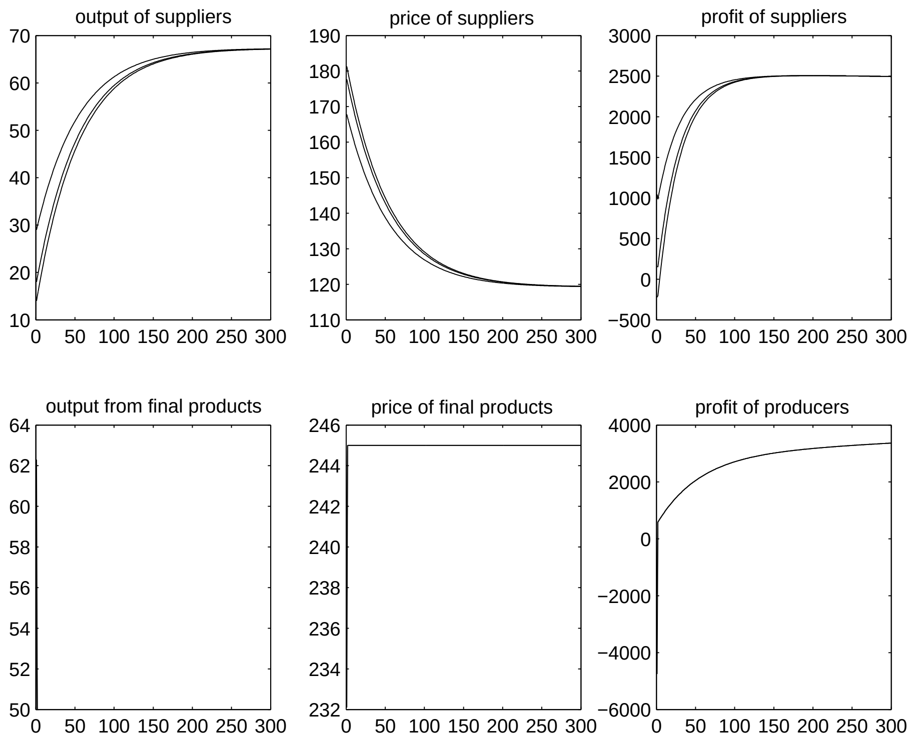
2.2 Effect of changing $\overline{A}_j$

In this group of simulation, we select $\overline{\delta}_j = 0.5$, $A_i = 200$, $c = 300$, $\overline{b}_{jl} = 0.1$ and innovation increment 0.001. The value of $\overline{A}_j$ changes from 300 to 1000 (Figure 2). The generated initial $x_{ij}(0)$ values in this group are $x_{11}(0) = 38$, $x_{12}(0) = 19$, $x_{21}(0) = 9$, $x_{22}(0) = 35$, $x_{31}(0) = 24$ and $x_{32}(0) = 30$.

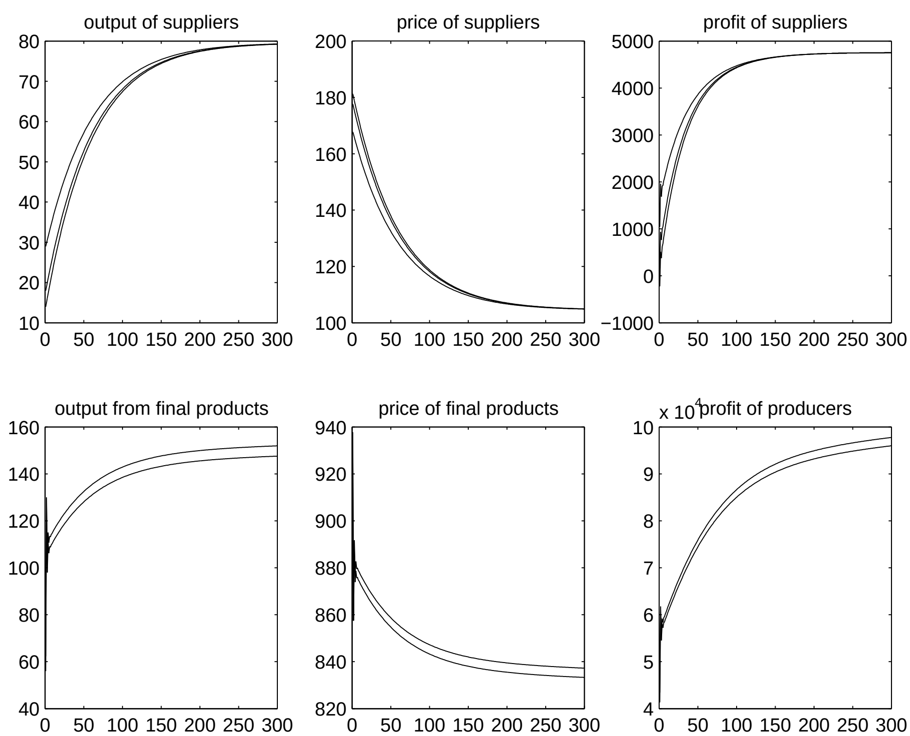
All figures show convergent trajectories indicating stability of the cluster in this selected range of parameter values. The changing value of $\overline{A}_j$ does not alter the patterns of the suppliers' behavior, only the limit values vary. For example, the limit value of the output of the suppliers changes from around 67 for $\overline{A}_j = 300$ to around 80 as $\overline{A}_j = 1000$; the limit of the profit of suppliers changes from around 2496 to around 5000. As the result of convergent supply trajectories, the labor need of suppliers also converges and the limit changes from around 37 to around 42.

In the case of $\overline{A}_j = 300$ (see Figure 2(a)), with $z_1(0) = 71.3$ and $z_2(0) = 75.2$, both trajectories drop down to 50 at the first iteration. The prices of final products are initially 221.18 and 217.67, but both increase to 245 and then keep this price. The initial profits of the producers are -4133.6 and -5794.8, but both increase to the same value of 2164.4 and then increase gradually later on.

The impact of $\overline{A}_j$ on the producers' behavior is relatively strong. In the above group of simulations, when $\overline{A}_j$ is less than 460, the outputs (and therefore the labor) of the producers drop down from their initial values to relatively small values in the first iteration, then keep them for a certain number of iterations, and then increase gradually before they converge. Accordingly, the profit and price of the producers have a sudden increase in the first iteration, then change slowly afterwards. When $\overline{A}_j$ becomes larger, the tendency of the increase in the profit, labor and productivity and that of the decrease in the price of labor become



(a) with $\overline{A}_j = 300$



(b) with $\overline{A}_j = 1000$

Figure 2: Results with varying value of $\overline{A}_j$

stronger. We can also observe that oscillations start to emerge gradually in the behavior of the producers, in the profit of the suppliers and in the price of labor. When $\overline{A}_j$ varies from 460 to 1000 (see Figure 2(b)), the behavior of the producers remains similar, however the productivity of the producers at iteration 300 changes from 59.958 and 61.537 to 159.26 and 161.97; the profit of the producers at iteration 300 changes from 12168 and 12048 to 101770 and 102820; and the price of final products increases from 393.89 and 392.47 to 824.54 and 822.1, respectively.

Similarly to the previous case, the value of the maximum final product price impacts both producers' and suppliers' behavior, however with much stronger impact on the producers. Lower $\overline{A}_j$ value prohibits the producers from earning more profit; too high $\overline{A}_j$ value generates higher profits and leads to the expansion of the final product segment. If higher profit encourages more producers to join the cluster, then the situation might change. We will consider the impact of entry in our later study. The complex impact of labor decrease will be also considered later.

2.3 Effect of changing c

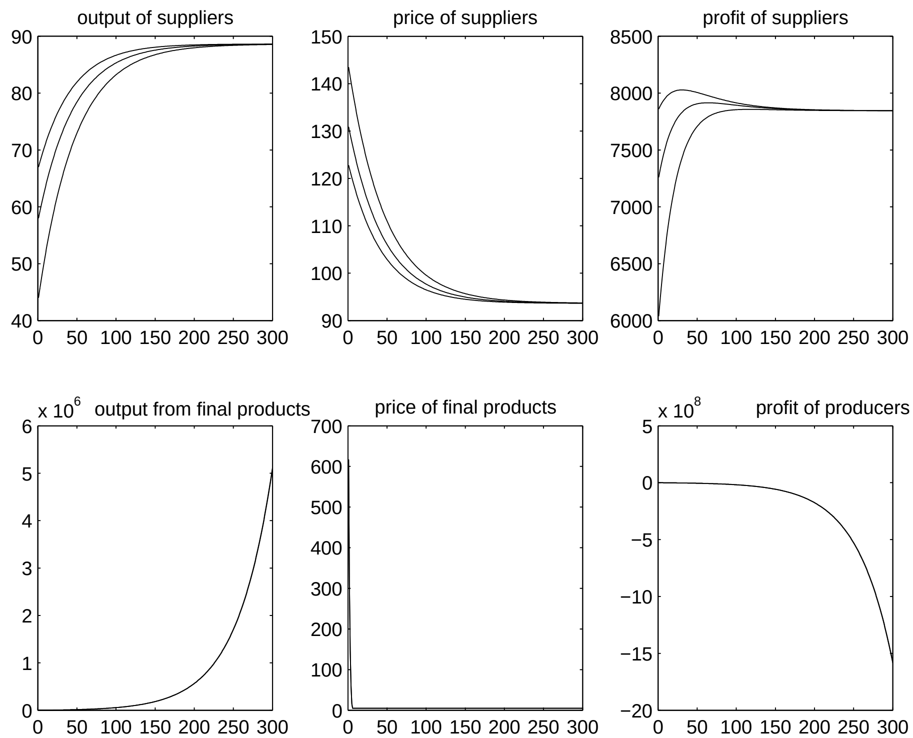
In this group of simulations, we select $\overline{\delta}_j = 0.3$, $A_i = 200$, $\overline{A}_j = 700$, $\overline{b}_{jl} = 0.1$, innovation increment 0.001 and the value of c changes from 100 to 500. The generated initial $x_{ij}(0)$ values in this group are $x_{11}(0) = 19$, $x_{12}(0) = 25$, $x_{21}(0) = 35$, $x_{22}(0) = 32$, $x_{31}(0) = 32$ and $x_{32}(0) = 36$.

With changing value of c, we can observe a pattern change in the behavior of the suppliers and the producers and also in the labor market. For $c < 155$ (see Figure 3(a)), the supplier's productivity, the labor and the profit all increase and then converge, however the final product price decreases and then converges. However, the price of the labor keeps a constant level of 10 (see Figure 3(f)), and the productivity and the labor of the producers increase exponentially. The final product price decreases to 0 in the first few iterations. The profit of the producers decreases from 0 to a negative value rapidly showing that in this case some government intervention is required.

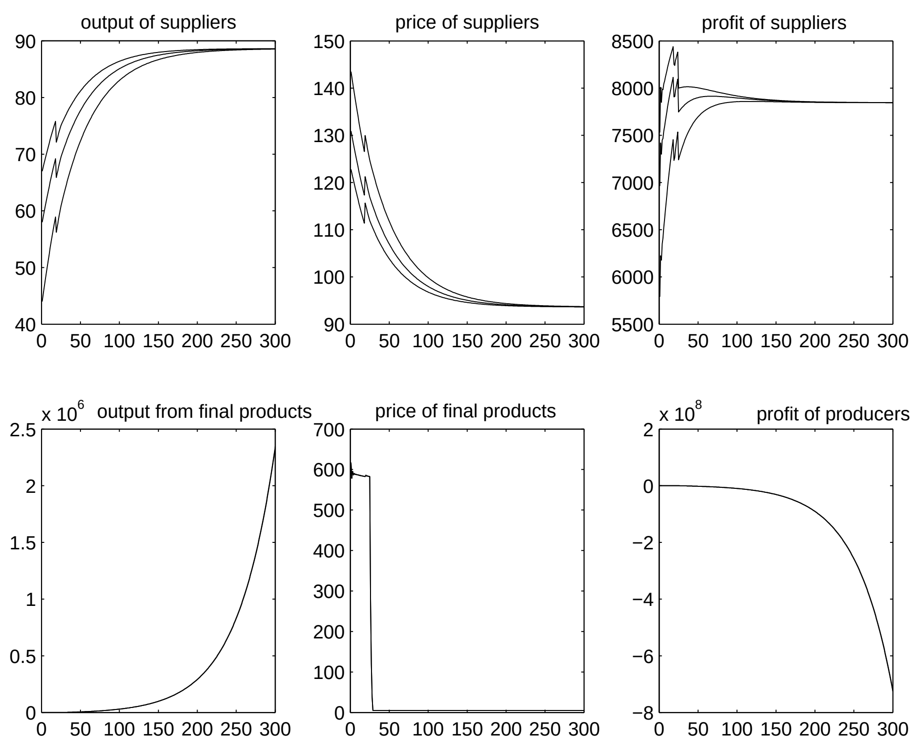
Starting from $c = 155$ (see Figure 3(b)), oscillations emerge in the first few iterations in the suppliers' behavior and these oscillations shift gradually to the later iterations when the value of c becomes larger. The final product price remains at around 600 during these initial iterations, and the price of labor drops to 0 then increases suddenly to 10 after a few oscillations between 0 and 4. The patterns of the other characteristics do not change. At $c = 180$ (Figure 3(c)), there are oscillations in all characteristics of the suppliers with similar main tendencies as observed before, while the patterns of the productivity, profit and the labor of the producers change significantly. They increase first, and then converge to an oscillating pattern, the profit of the producers starts now at a positive value and then converges to a higher value with an oscillating pattern. The price of labor decreases gradually until iteration 110 then starts oscillating between 0 and 4.

As the value of c increases to 190, these oscillations disappear, except at the beginning in the producers' behavior, in the profit of suppliers and in the price of labor. Until $c < 420$, the patterns remain similar but the limiting values have small changes. The case of $c = 200$ is shown in Figure 3(d).

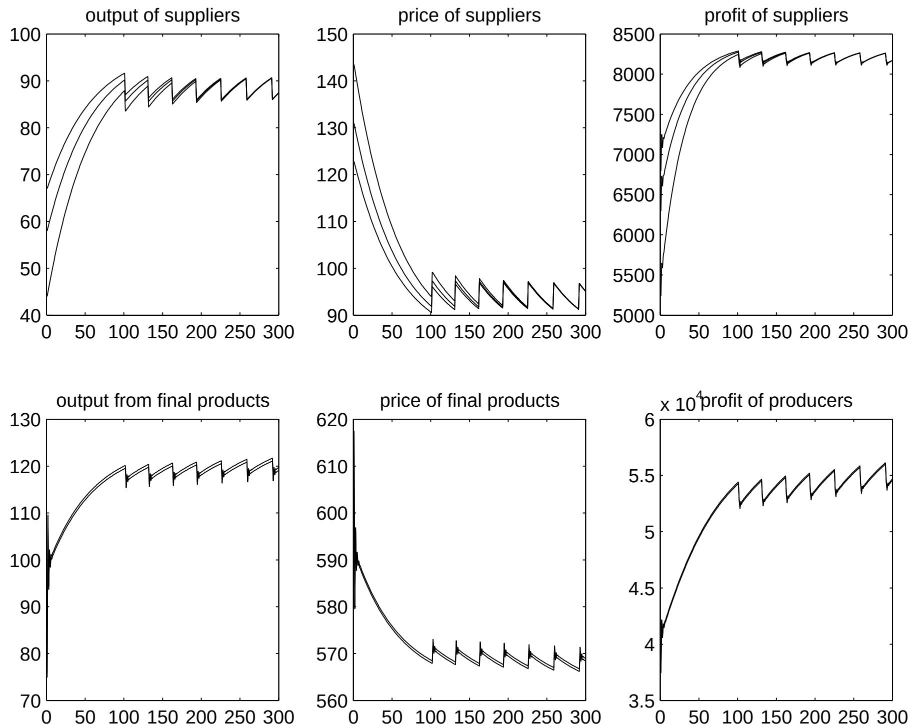
When $c = 350$, the profit of the producer looks like a linear function of time. The patterns of other behavioral trajectories are the same as in previous cases. When c becomes larger, all figures gradually start reversing their shapes, with the difference that the profit of the suppliers now starts from negative values and then converge to an identical negative limit.



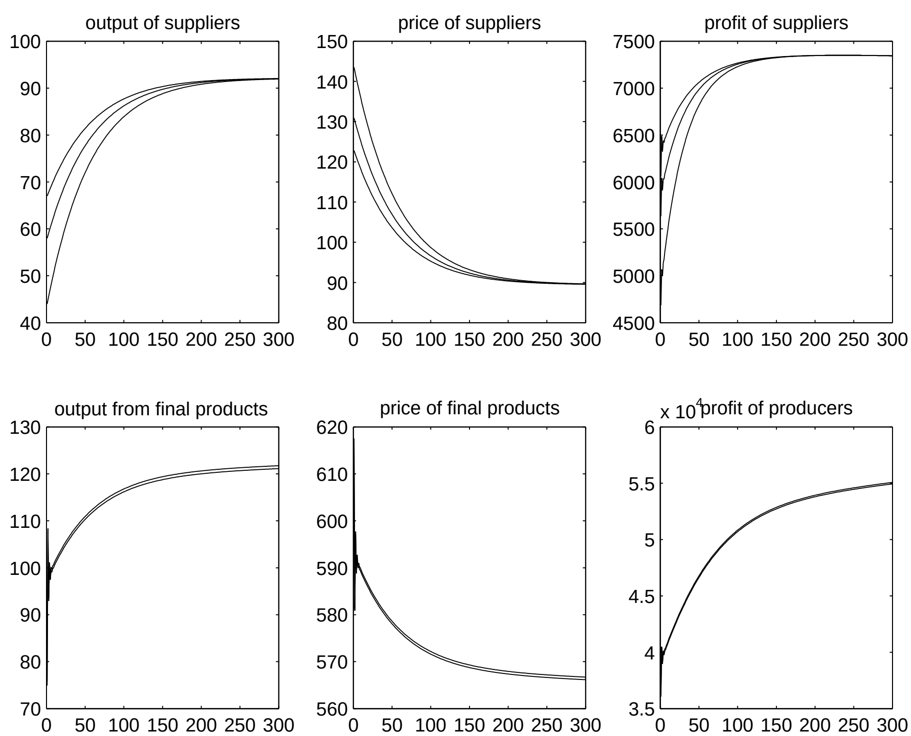
(a) with $c = 100$



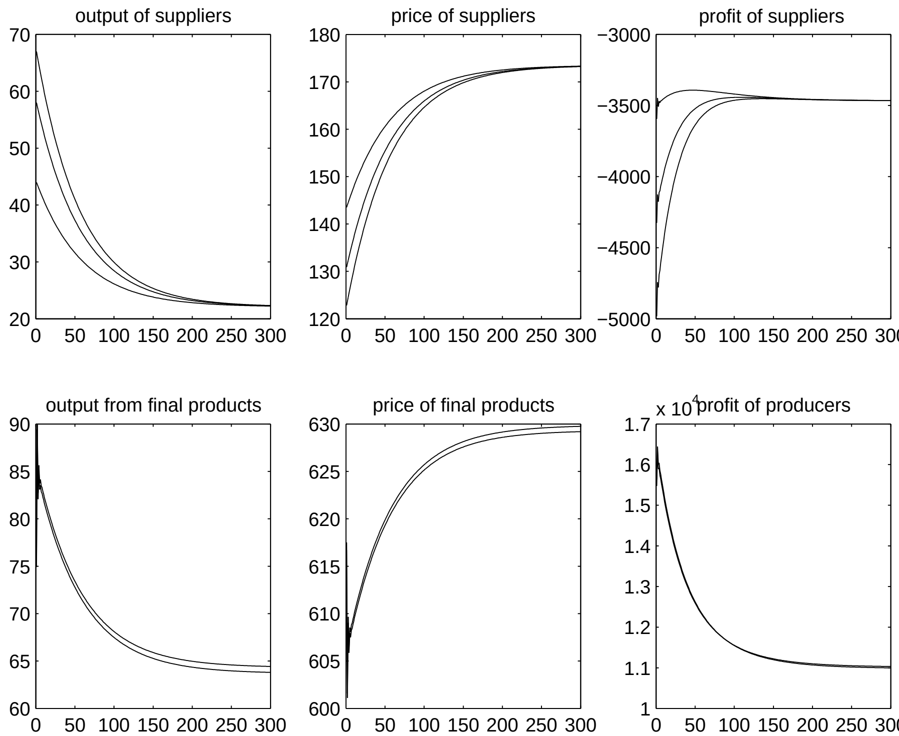
(b) with $c = 160$



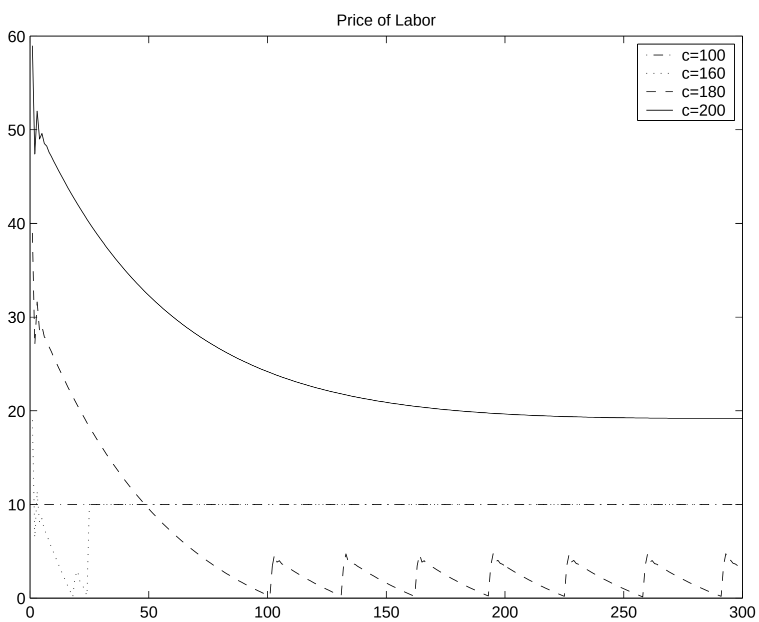
(c) with $c = 180$



(d) with $c = 200$



(e) with $c = 500$



(f) patterns of price of labor when c changed

Figure 3: Results with varying value of c

Different values of c can generate numerous different patterns of the suppliers and the producers, especially in the range between 150 and 190. The short-term behavior of the firms is very sensitive to the value of c. This instability could be controlled by the local government in stabilizing the system with higher profit for the firms which will then attract other firms to join the cluster, and simultaneously by keeping relatively higher labor price to attract skilled workers.

2.4 Effects of changing $\overline{b}_{jl}$

This group of simulations studies how the similarity of the final products affects the behavior of the firms in the cluster. In the simulation, we select $\overline{\delta}_j = 0.3$, $A_i = 200$, $\overline{A}_j = 700$, $c = 300$ and innovation increment 0.001. The value of $\overline{b}_{jl}$ changes from 0.1 to 1. The generated $x_{ij}(0)$ values in this group are $x_{11}(0) = 33$, $x_{12}(0) = 28$, $x_{21}(0) = 22$, $x_{22}(0) = 21$, $x_{31}(0) = 14$ and $x_{32}(0) = 17$.

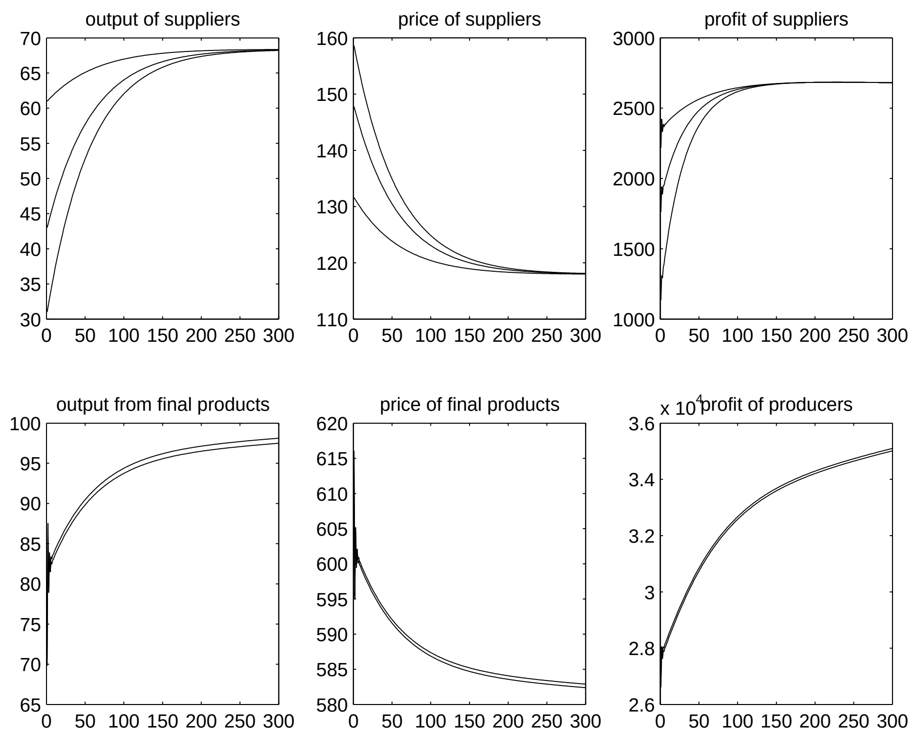
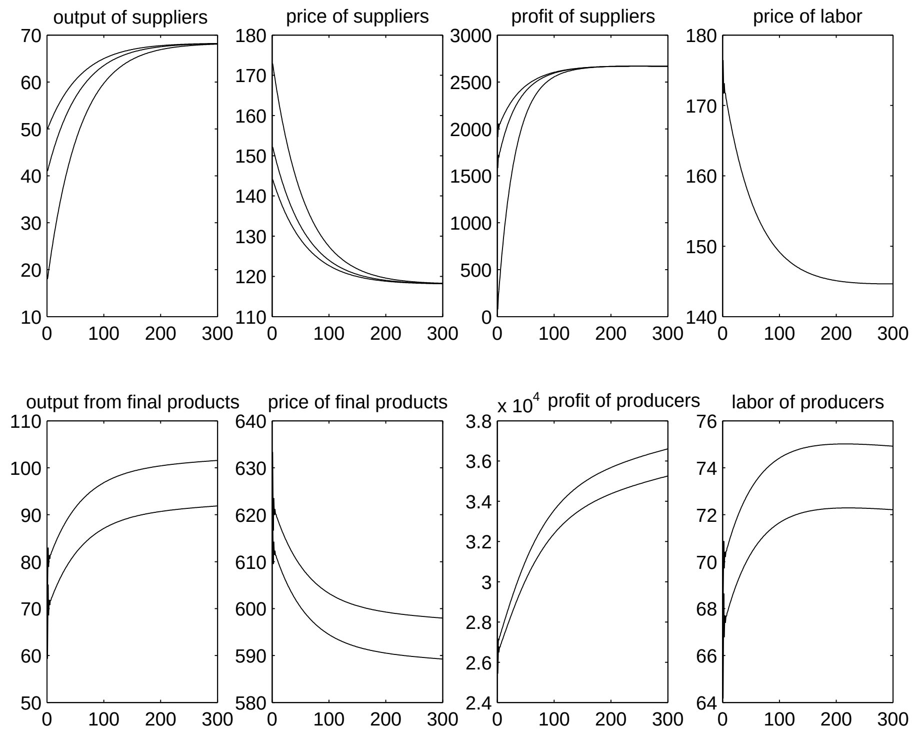


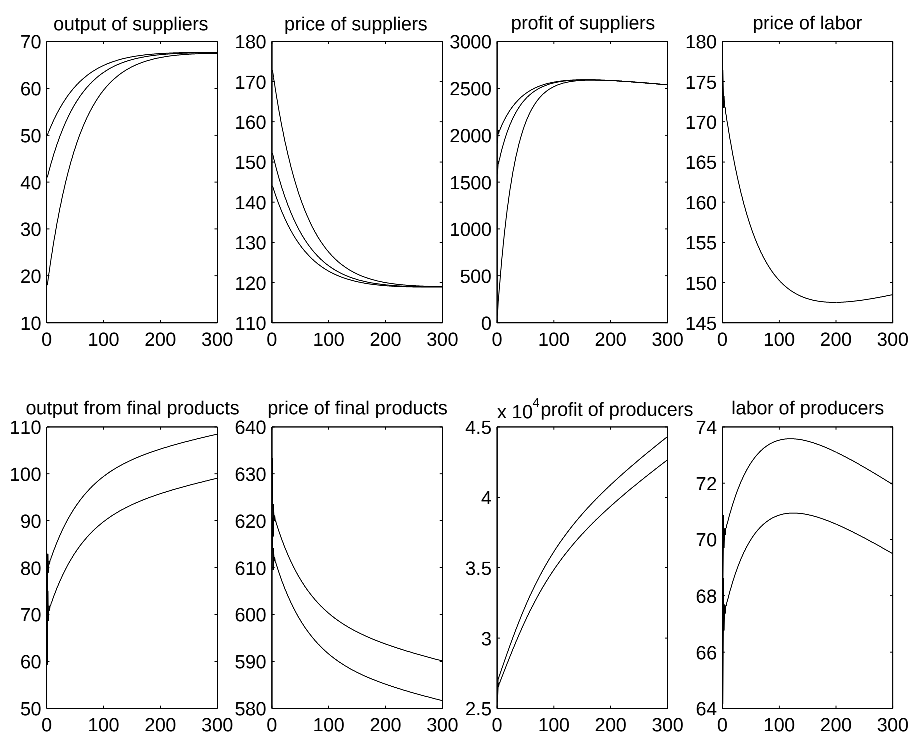
Figure 4: Results with value of $\overline{b}_{jl} = 0.2$

Regardless of the value of $\overline{b}_{jl}$, the patterns are the same for all characteristics. There is a very minor affect on the suppliers' behavior. A typical case ($\overline{b}_{jl} = 0.2$) is shown in Figure 4. With changing the value of $\overline{b}_{jl}$, the labor price at iteration 300 increases from 144 to 148; the labor of producers decreases from 74 to around 70, and the profit of producers from 35101 and 35014 to 26673 and 26556, respectively. The final product price decreases from around 582 to around 527, and the productivity of the producers from 98 to around 86.

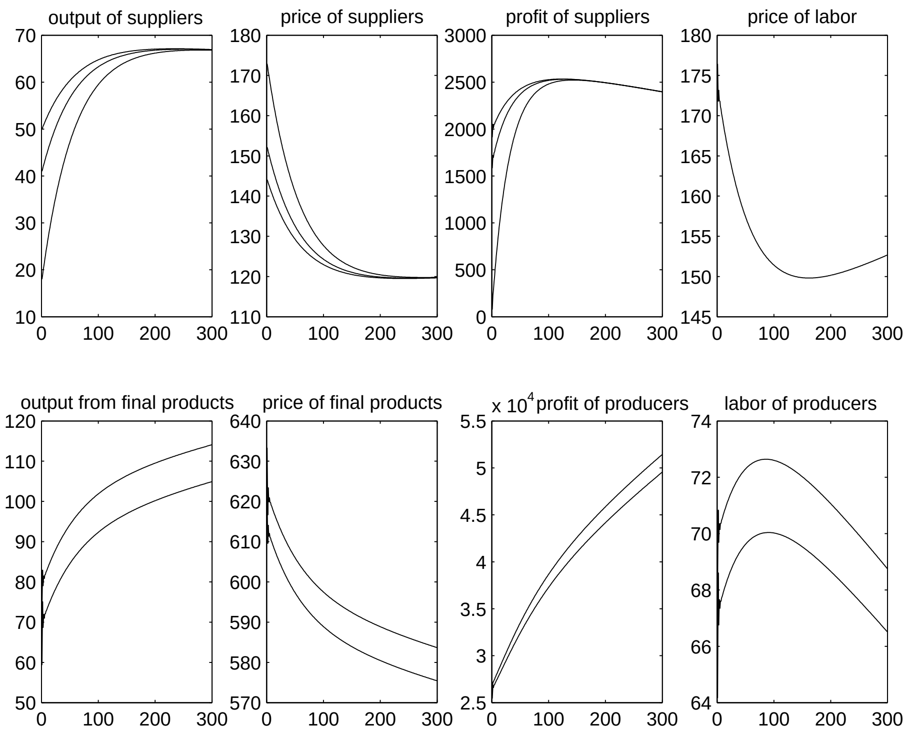
If the final products are more similar to each other, then there is more competition among the producers, they have less profit and less productivity, and in addition, the price in the labor market slightly increases.



(a) with $I_j = 0.001$



(b) with $I_j = 0.005$



(c) with $I_j = 0.009$

Figure 5: Results with varying value of I_j

2.5 Effects of innovation step I_j

In the simulation, the innovation step, I_j, changes from 0.001 to 0.01. We select the other parameters as $\overline{\delta}_j = 0.3$, $A_i = 200$, $\overline{A}_j = 700$, $c = 300$ and $\overline{b}_{jl} = 0.1$ (Figure 5). The generated $x_{ij}(0)$ values in this group are $x_{11}(0) = 11$, $x_{12}(0) = 39$, $x_{21}(0) = 18$, $x_{22}(0) = 23$, $x_{31}(0) = 2$ and $x_{32}(0) = 16$.

The changes in the innovation step I_j affect the limit values of the variables and alter the shape of the trajectories slightly. It is interesting to notice that the price of labor follows a convex trajectory with its minimum occurring earlier as I_j increases. In contrast, the labor usage of the producers is concave, its maximum shifts to earlier periods as well with increasing value of I_j. The profit of the producers gradually changes into a linear shape in time.

The limit values of the price, labor and productivity of the suppliers are not affected much. The output of the producers at time period 300 increases with I_j, and so the price of the final product decreases if I_j increases. The profit of the producers at iteration 300 also shows an increasing tendency with I_j.

3 Conclusions

This paper first introduced a static model of describing the mechanism of industrial clusters including suppliers and final product producers. The competition of these firms was modeled by using the major concepts of the theory of oligopolies and oligopsonies as well as basic economic principles of technology development. For mathematical simplicity we assumed that supply surplus can be sold outside the cluster and in the case of supply shortages the firms are able to purchase their needed supply from outside sources. We also assumed a constant increase in the innovation level of the firms.

In the dynamic extension we used gradient adjustment, in which the firms adjust the values of their decision variables in proportion to the corresponding partial derivatives of their profit functions. Instead of analytic methods computer simulation was used to see the evolution and dynamic development of the firms and therefore those of the entire cluster.

With this simplified model we were able to gain insight into the dependence of the limit values and the shapes of the trajectories of important characteristics of the firms on model parameters such as maximum prices, similarity of final products, and the innovation step.

In our future research we plan to add more realistic features to the model by considering innovation as decision variables, including input-output balances between the firms, entry of firms, attracting and changing the structure of the labor force, and the effect of import and export to mention only a few.

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References

[1] G.I. Bischi, C. Chiarella, M. Kopel and F. Szidarovszky, *Nonlinear Oligopolies: Stability and Bifurcations*, Springer-Verlag, Berlin/New York (in press), 2008.

[2] T. Brenner, *Simulating the Evolution of Localised Industrial Clusters-An Identification of the Basic Mechanism*, Journal of Artificial Societies and Social Simulation, **4(3)**(2001), <http://www.soc.surrey.ac.uk/JASSS/4/3/4.html>.

[3] A. Cournot, *Recherches sur les Principes Mathématiques de la Théorie de Richesses*, Hachett, Paris (English translation, 1960. Researches into the Mathematical Principles of the Theory of Wealth. Kelley, New York), 1838.

[4] F. Giardini, G.D. Tosto and R. Conte, *A Model for Simulating Reputation Dynamics in Industrial Districts*, Simulation Modelling Practice and Theory, **16**(2008), 231–241.

[5] K. Okuguchi, *Expectations and Stability in Oligopoly Models*, Springer-Verlag, Berlin/New York, 1976.

[6] *K. Okuguchi and F. Szidarovszky*, *The Theory of Oligopoly with Multi-product Firms*, Springer-Verlag, Berlin/New York, 1999.

[7] T. Puu and I. Sushko, *Oligopoly Dynamics*, Springer-Verlag, Berlin/New York, 2002.

[8] G.M.P. Swann, M. Prevezer and D. Stout, *The Dynamics of Industrial Clustering*, Oxford University Press, New York, 1998.

[9] F. Szidarovszky, *Network Oligopolies*, Pure Mathematics and Applications, **8(1)**(1997), 117–123.

[10] F. Szidarovszky and K. Okuguchi, *Dynamic Analysis of Oligopsony under Adaptive Expectations*, Southwest Journal of Pure and Applied Mathematics, **2**(2001), 53–60.

[11] J. Zhang, *Growing Silicon Valley on a Landscape: An Agent-based Approach to High-tech Industrial Clusters*, Journal of Evolutionary Economics, **13**(2003), 529–548.