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Iterative Regularization Methods for a Discrete Inverse Problem in MRI

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ABSTRACT

We propose and investigate efficient numerical methods for inverse problems related to Magnetic Resonance Imaging (MRI). Our goal is to extend the recent convergence results for the Landweber-Kaczmarz method obtained in [7], in order to derive a convergent iterative regularization method for an inverse problem in MRI.

RESUMEN

Nosotros investigamos y proponemos métodos numéricos eficientes para problemas inversos relacionados con resonancia Magnética de Imagen (MRI). Nuestro objetivo es extender resultados recientes de convergencia para el método de Landweber-Kaczmarz obtenido en [7], a fin de obtener un método de regularización iterativo convergente para un problema inverso en MRI.

Key words and phrases: *Magnetic Resonance Imaging, MRI, Tomography, Medical Imaging, Inverse Problems.*

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1 Introduction

Magnetic Resonance Imaging, also known as MR–Imaging or simply MRI, is a noninvasive technique used in medical imaging to visualize body structures and functions, providing detailed images in arbitrary planes. Unlike *X-Ray Tomography* it does not use ionizing radiation, but uses a powerful magnetic field to align the magnetization of hydrogen atoms in the body. Radio waves are used to systematically alter the alignment of this magnetization, causing the hydrogen atoms to produce a rotating magnetic field detectable by the scanner.

More specifically, when a subject is in the scanner, the hydrogen nuclei (i.e., protons, found in abundance in the human body as water) align with the strong magnetic field. A radio wave at the correct frequency for the protons to absorb energy pushes some of the protons out of alignment. The protons then snap back to alignment, producing a detectable rotating magnetic field as they do so. Since protons in different areas of the body (e.g., fat and muscle) realign at different speeds, the different structures of the body can be revealed.

The image to be identified in MRI corresponds to a complex valued function $\mathcal{P}: [0,1]\times[0,1]\to\mathbb{C}$ and the image acquisition process is performed by so-called *receivers*. Due to the physical nature of the acquisition process, the information gained by the receivers does not correspond to the unknown image, but instead, to $\mathcal{P}$ multiplied by receiver dependent *sensitivity kernels*. In real life applications, the sensitivity kernels are not precisely known and have to be identified together with $\mathcal{P}$. This corresponds to a version of the blind deconvolution problem that has been investigated by many authors. See for example [2, 12, 14]

Our main goal in this article is to investigate efficient iterative methods of Kaczmarz type for the identification problem related to MRI. We extend the convergence results for the *loping Landweber-Kaczmarz* method in [7] and derive a convergent iterative regularization method for this inverse problem.

This article is outlined as follows. In Section 2 the description of a discrete mathematical model for Magnetic Resonance Imaging is presented. In Section 3 we derive the corresponding inverse problem for MRI. In Section 4 we investigate efficient iterative regularization methods for this inverse problem. Using a particular hypothesis on the sensitivity kernels, we are able to derive convergence and stability results for the proposed iterative methods.

2 The direct problem

In what follows we present a discrete model for MRI. In our approach, we follow the notation introduced in [1]. The image to be identified is considered to be a discrete function

$$\mathcal{P} : \{1, \dots, p_{\text{hor}}\} \times \{1, \dots, p_{\text{ver}}\} =: \mathbb{B} \to \mathbb{C}\,,$$

where p_{hor}, $p_{\text{ver}} \in \mathbb{N}_0$ are known. Therefore, the number of degrees of freedom related to this parameter is $p_{\text{num}} := p_{\text{hor}} \times p_{\text{ver}}$ (typical values are $p_{\text{hor}} = p_{\text{ver}} = 256$; $p_{\text{num}} = 65536$).

As mentioned above, the image acquisition process is performed by several *receivers*, denoted here by $\mathcal{R}_j$, $j = 0, \dots, r_{\text{num}} - 1$, where $r_{\text{num}} \in \mathbb{N}_0$ is given (typically one faces the situation where $r_{\text{num}} << p_{\text{num}}$). Due to the physical nature of the acquisition process, the information gained by the receivers does not correspond to the unknown image, but instead, to $\mathcal{P}$ multiplied by receiver dependent *sensitivity kernels*

$$\mathcal{S}_j \;=\; \mathcal{S}(\mathcal{R}_j) : \mathbb{B} \to \mathbb{C}\,,\; j = 0, \dots, r_{\text{num}} - 1\,.$$

In real life applications, the sensitivity kernels $\mathcal{S}_j$ are not precisely known and have to be identified together with $\mathcal{P}$. This fact justifies the following ansatz:

(A1) The sensitivity kernels $\mathcal{S}_j$ can be written as linear combination of the given basis functions $\mathcal{B}_n : \mathbb{B} \to \mathbb{C}$, for $n = 1, \dots, b_{\text{num}}$, and $b_{\text{num}} \in \mathbb{N}_0$.

In other words, we assume the existence of coefficients $b_{j,n} \in \mathbb{C}$ such that

$$\mathcal{S}_j(m) \;=\; \sum_{n=1}^{b_{\text{num}}} b_{j,n}\, \mathcal{B}_n(m)\,,\; m \in \mathbb{B}\,,\; j = 0, \dots, r_{\text{num}} - 1\,. \tag{2.1}$$

In the sequel we make use the abbreviated notations $\mathbf{b}_j := (b_{j,n})_{n=1}^{b_{\text{num}}}$ and $(\mathbf{b}_j) := (\mathbf{b}_j)_{j=0}^{r_{\text{num}}-1}$. Notice that the coefficient vectors $\mathbf{b}_j = \mathbf{b}(\mathcal{R}_j)$ are receiver dependent.

The measured data for the inverse problem is given in a subset of the Fourier space of the image $\mathcal{P}$, i.e. there exists a known subset $\mathbb{M} \subset \mathbb{B}$ (consisting of p_{proj} elements) such that the receiver dependent measurement $\mathcal{M}_j = \mathcal{M}(R_j)$ satisfies

$$\mathcal{M}_j \;:=\; \mathbf{P}[\mathcal{F}(\mathcal{P} \times \mathcal{S}_j)]\,,\; j = 0, \dots, r_{\text{num}} - 1\,.$$

where $\mathcal{F}$ is the *Discrete Fourier Transform* (DFT) operator defined by

$$\begin{aligned} \mathcal{F} : \{f \mid f : \mathbb{B} \to \mathbb{C}\} \;&\to\; \{\hat{f} \mid \hat{f} : \mathbb{B} \to \mathbb{C}\} \\ f \;&\mapsto\; (\mathcal{F}(f))(m) := \sum_{n=0}^{p_{\text{num}}-1} f(n)\, \exp\Big(-\frac{2\pi i}{p_{\text{num}}} nm \Big)\,, \end{aligned}$$

and $\mathbf{P}$ is the operator defined by

$$\begin{aligned} \mathbf{P} : \big\{ f \mid f : \mathbb{B} \to \mathbb{C} \big\} \quad &\to \quad \big\{ g \mid g : \mathbb{M} \to \mathbb{C} \big\} =: Y \\ f \quad &\mapsto \quad (\mathbf{P}[f])(m) := f(m) \,, \ m \in \mathbb{M} \,. \end{aligned}$$

Notice that, due to ansatz (A1) and the linearity of $\mathcal{F}$ and $\mathbf{P}$, the measured data $\mathcal{M}_j \in Y$ can be written in the form

$$\mathcal{M}_j \ = \ \sum_{n=1}^{b_{\rm num}} b_{j,n} \, \mathbf{P}[\mathcal{F}(\mathcal{P} \times \mathcal{B}_n)] \,, \ j = 0, \dots, r_{\rm num} - 1 \,. \tag{2.2}$$

Remark 2.1. *The numerical evaluation of the DFT requires naively $O(p_{\rm num}^2)$ arithmetical operations. However, in practice the DFT must be replaced by the* Fast Fourier Transform *(FFT), which can be computed by the Cooley-Tukey algorithm*[1] *and requires only $O(p_{\rm num} \log(p_{\rm num}))$ operations.*

3 The inverse problem

Next we use the discrete model discussed in the previous section as a starting point to formulate an inverse problem for MRI.

The unknown parameters to be identified are the discrete image function $\mathcal{P}$ and the sensitivity kernels $\mathcal{S}_j$. Due to the ansatz (A1), the parameter space X consists of pairs of the form $(\mathcal{P}, (\mathbf{b}_j))$, i.e.

$$X \ := \ \big\{ (\mathcal{P}, (\mathbf{b}_j)) \, ; \ \mathcal{P} \in \mathbb{C}^{p_{\rm num}} \,, \ (\mathbf{b}_j) \in (\mathbb{C}^{b_{\rm num}})^{r_{\rm num}} \big\} \,.$$

It is immediate to observe that X can be identified with $\mathbb{C}^{(p_{\rm num} + b_{\rm num} \times r_{\rm num})}$, while Y can be identified with $\mathbb{C}^{p_{proj}}$.

The *parameter to output* operators $F_i : X \to Y$ are defined by

$$F_i : (\mathcal{P}, (\mathbf{b}_j)) \mapsto \ \sum_{n=1}^{b_{\rm num}} b_{i,n} \, \mathbf{P}[\mathcal{F}(\mathcal{P} \times \mathcal{B}_n)] \,, \ i = 0, \dots, r_{\rm num} - 1 \,. \tag{3.1}$$

Due to the experimental nature of the data acquisition process, we shall assume that the data $\mathcal{M}_i$ in (2.2) is not exactly known. Instead, we have only approximate measured data $\mathcal{M}_i^\delta \in Y$ satisfying

$$\| \mathcal{M}_i^\delta - \mathcal{M}_i \| \le \delta_i \,, \tag{3.2}$$

with $\delta_i > 0$ (noise level). Therefore, the inverse problem for MRI can be written in the form of the following system of nonlinear equations

$$F_i(\mathcal{P}, (\mathbf{b}_j)) \ = \ \mathcal{M}_i^\delta \,, \ i = 0, \dots, r_{\rm num} - 1 \,. \tag{3.3}$$

It is worth noticing that the nonlinear operators F_i's are continuously Fréchet differentiable, and the derivatives are locally Lipschitz continuous.

[1]The FFT algorithm was published independently by J.W. Cooley and J.W. Tukey in 1965. However, this algorithm was already known to C.F. Gauss around 1805.

4 Iterative regularization

In this section we analyze efficient iterative methods for obtaining stable solutions of the inverse problem in (3.3).

4.1 An image identification problem

Our first goal is to consider a simplified version of problem (3.3). We assume that the sensitivity kernels $\mathcal{S}_j$ are known, and we have to deal with the problem of determining only the image $\mathcal{P}$. This assumption can be justified by the fact that, in practice, one has very good approximations for the sensitivity kernels, while the image $\mathcal{P}$ is completely unknown.

In this particular case, the inverse problem reduces to

$$\tilde{F}_i(\mathcal{P}) \ = \ \mathcal{M}_i^\delta\,, \ i = 0, \ldots, r_{\mathrm{num}} - 1\,, \tag{4.1}$$

where $\tilde{F}_i(\mathcal{P}) = F_i(\mathcal{P}, (\mathbf{b}_j))$, the coefficients $(\mathbf{b}_j)$ being known. This is a much simpler problem, since $\tilde{F}_i : \tilde{X} \to Y$ are linear and bounded operators, defined at $\tilde{X} := \{f \mid f : \mathbb{B} \to \mathbb{C}\}$.

We follow the approaches in [7, 5] and derive two iterative regularization methods of Kaczmarz type for problem (4.1). Both iterations can be written in the form

$$\mathcal{P}_{k+1}^\delta = \mathcal{P}_k^\delta - \omega_k \alpha_k s_k\,, \tag{4.2}$$

where

$$s_k := \tilde{F}_{[k]}(\mathcal{P}_k^\delta)^*(\tilde{F}_{[k]}(\mathcal{P}_k^\delta) - \mathcal{M}_i^\delta)\,, \tag{4.3}$$

$$\omega_k := \begin{cases} 1 & \|\tilde{F}_{[k]}(\mathcal{P}_k^\delta) - \mathcal{M}_i^\delta\| > \tau\delta_{[k]} \\ 0 & \text{otherwise} \end{cases}\,. \tag{4.4}$$

Here $\tau > 2$ is an appropriately chosen constant, $[k] := (k \mod r_{\mathrm{num}}) \in \{0, \ldots, r_{\mathrm{num}} - 1\}$ (a group of r_{num} subsequent steps, starting at some multiple k of r_{num}, is called a *cycle*), $\mathcal{P}_0^\delta = \mathcal{P}_0 \in \tilde{X}$ is an initial guess, possibly incorporating some *a priori* knowledge about the exact image, and $\alpha_k \geq 0$ are relaxation parameters.

Distinct choices for the relaxation parameters α_k lead to the definition of the two iterative methods:

1) If α_k is defined by
$$\alpha_k := \begin{cases} \|s_k\|^2 / \|\tilde{F}_{[k]}(\mathcal{P}_k^\delta) s_k\|^2 & \omega_k = 1 \\ 1 & \omega_k = 0 \end{cases}\,, \tag{4.5}$$
the iteration (4.2) corresponds to the *loping Steepest-Descent Kaczmarz method* (lSDK) [5].

2) If $\alpha_k \equiv 1$, the iteration (4.2) corresponds to the *loping Landweber-Kaczmarz method* (lLK) [7].

The iterations should be terminated when, for the first time, all $\mathcal{P}_k$ are equal within a cycle. That is, we stop the iteration at the index k_*^δ, which is the smallest multiple of $r_{\rm num}$ such that

$$\mathcal{P}_{k_*^\delta} = \mathcal{P}_{k_*^\delta+1} = \cdots = \mathcal{P}_{k_*^\delta+r_{\rm num}-1}\,. \tag{4.6}$$

Convergence analysis of the lSDK method

From (3.1) follows that the operators $\tilde{F}_i$ are linear and bounded. Therefore, there exist $M > 0$ such that

$$\|\tilde{F}_i\| \le M\,,\ i = 0, \dots, r_{\rm num} - 1\,. \tag{4.7}$$

Since the operators $\tilde{F}_i$ are linear, the *local tangential cone condition* is trivially satisfied (see (4.16) below). Thus, the constant τ in (4.4) can be chosen such that $\tau > 2$. Moreover, we assume the existence of

$$\mathcal{P}^* \in B_{\rho/2}(\mathcal{P}_0) \ \text{ such that } \ \tilde{F}_i(\mathcal{P}^*) = \mathcal{M}_i\,,\ i = 0, \dots, r_{\rm num} - 1\,, \tag{4.8}$$

where $\rho > 0$ and $(\mathcal{M}_i)_{i=0}^{r_{\rm num}-1} \in Y^{r_{\rm num}}$ corresponds to exact data satisfying (3.2).

In the sequel we summarize several properties of the lSDK iteration. For a complete proof of the results we refer the reader to [5, Section 2].

Lemma 4.1. *Let the coefficients α_k be defined as in* (4.5)*, the assumption* (4.8) *be satisfied for some $\mathcal{P}^* \in \tilde{X}$, and the stopping index k_*^δ be defined as in* (4.6)*. Then, the following assertions hold:*

1) *There exists a constant $\underline{\alpha} > 0$ such that $\alpha_k > \underline{\alpha}$, for $k = 0, \dots, k_*^\delta$.*

2) *Let $\delta_i > 0$ be defined as in* (3.2)*. Then the stopping index k_*^δ defined in* (4.6) *is finite.*

3) *$\mathcal{P}_k^\delta \in B_{\rho/2}(\mathcal{P}_0)$ for all $k \le k_*^\delta$.*

4) *The following monotony property is satisfied:*

$$\|\mathcal{P}_{k+1}^\delta - \mathcal{P}^*\|^2 \ \le \ \|\mathcal{P}_k^\delta - \mathcal{P}^*\|^2\,,\ k = 0, 1, \dots, k_*^\delta\,, \tag{4.9}$$

$$\|\mathcal{P}_{k+1}^\delta - \mathcal{P}^*\|^2 \ = \ \|\mathcal{P}_k^\delta - \mathcal{P}^*\|^2\,,\ k > k_*^\delta\,. \tag{4.10}$$

Moreover, in the case of noisy data (i.e. $\delta_i > 0$) we have

$$\|\tilde{F}_i(\mathcal{P}_{k_*^\delta}^\delta) - \mathcal{M}_i^\delta\| \le \tau\delta_i\,,\ i = 0, \dots, r_{\rm num} - 1\,. \tag{4.11}$$

Next we prove that the lSDK method is a convergent regularization method in the sense of [3].

Theorem 4.2 (Convergence)**.** *Let α_k be defined as in* (4.5)*, the assumption* (4.8) *be satisfied for some $\mathcal{P}^* \in \tilde{X}$, and the data be exact, i.e. $\mathcal{M}_i^\delta = \mathcal{M}_i$ in* (3.2)*. Then, the sequence $\mathcal{P}_k^\delta$ defined in* (4.2) *converges to a solution of* (4.1) *as $k \to \infty$.*

Proof. Notice that, since the data is exact, we have $\omega_k = 1$ for all $k > 0$. The proof follows from [5, Theorem 3.5]. □

Theorem 4.3 (Stability). *Let the coefficients α_k be defined as in (4.5), and the assumption (4.8) be satisfied for some $\mathcal{P}^* \in \tilde{X}$. Moreover, let the sequence $\{(\delta_{1,m}, \ldots, \delta_{r_{\text{num}},m})\}_{m\in\mathbb{N}}$ (or simply $\{\delta_{\mathbf{m}}\}_{m\in\mathbb{N}}$) be such that $\lim_{m\to\infty}(\max_i \delta_{i,m}) = 0$, and let $\mathcal{M}_i^{\delta_{\mathbf{m}}}$ be a corresponding sequence of noisy data satisfying (3.2) (i.e. $\|\mathcal{M}_i^{\delta_{\mathbf{m}}} - \mathcal{M}_i\| \le \delta_{i,m}$, $i = 0, \ldots, r_{\text{num}} - 1$, $m \in \mathbb{N}$). For each $m \in \mathbb{N}$, let k_*^m be the stopping index defined in (4.6). Then, the lSDK iterates $\mathcal{P}_{k_*^m}^{\delta_{\mathbf{m}}}$ converge to a solution of (4.1) as $m \to \infty$.*

Proof. The proof follows from [5, Theorem 3.6]. □

Convergence analysis of the lLK method

The convergence analysis results for the lLK iteration are analog to the ones presented in Theorems 4.2 and 4.3 for the lSDK method. In the sequel we summarize the main results that we could extend to the lLK iteration.

Theorem 4.4 (Convergence Analysis). *Let $\alpha_k \equiv 1$, the assumption (4.8) be satisfied for some $\mathcal{P}^* \in \tilde{X}$, the operators $\tilde{F}_i$ satisfy (4.7) with $M = 1$, and the stopping index k_*^δ be defined as in (4.6). Then, the following assertions hold:*

1) *Let $\delta_i > 0$ in (3.2). Then the stopping index k_*^δ defined in (4.6) is finite.*

2) *$\mathcal{P}_k^\delta \in B_{\rho/2}(\mathcal{P}_0)$ for all $k \le k_*^\delta$.*

3) *The monotony property in (4.9), (4.10) is satisfied. Moreover, in the case of noisy data, (4.11) holds true.*

4) *For exact data, i.e. $\delta_i = 0$ in (3.2), the sequence $\mathcal{P}_k^\delta$ defined in (4.2) converges to a solution of (4.1) as $k \to \infty$.*

5) *Let the sequence $\{\delta_{\mathbf{m}}\}_{m\in\mathbb{N}}$, the corresponding sequence of noisy data $\mathcal{M}_i^{\delta_{\mathbf{m}}}$, and the stopping indexes k_*^m be defined as in Theorem 4.3. Then, the lLK iterates $\mathcal{P}_{k_*^m}^{\delta_{\mathbf{m}}}$ converge to a solution of (4.1) as $m \to \infty$.*

Proof. The proof follows from corresponding results for the lLK iteration for systems of nonlinear equations in [7]. □

Notice that the assumption $M = 1$ in Theorem 4.4 is nonrestrictive. Indeed, since the operators $\tilde{F}_i$ are linear, it is enough to scale the equations in (4.1) with appropriate multiplicative constants.

4.2 Identification of image and sensitivity

Our next goal is to consider the problem of determining both the image $\mathcal{P}$ as well as the sensitivity kernels $\mathcal{S}_j$ in (3.3). The lLK and lSDK iterations can be extended to the nonlinear system in a straightforward way

$$(\mathcal{P}^\delta_{k+1}, (\mathbf{b}_j)^\delta_{k+1}) = (\mathcal{P}^\delta_k, (\mathbf{b}_j)^\delta_k) - \omega_k \alpha_k s_k \,, \tag{4.12}$$

where

$$s_k := F'_{[k]}(\mathcal{P}^\delta_k, (\mathbf{b}_j)^\delta_k)^* (F_{[k]}(\mathcal{P}^\delta_k, (\mathbf{b}_j)^\delta_k) - \mathcal{M}^\delta_i) \,, \tag{4.13}$$

$$\omega_k := \begin{cases} 1 & \|F_{[k]}(\mathcal{P}^\delta_k, (\mathbf{b}_j)^\delta_k) - \mathcal{M}^\delta_i\| > \tau \delta_{[k]} \\ 0 & \text{otherwise} \end{cases} . \tag{4.14}$$

In the lLK iteration we choose $\alpha_k \equiv 1$, and in the lSDK iteration we choose

$$\alpha_k := \begin{cases} \|s_k\|^2 / \|F'_{[k]}(\mathcal{P}^\delta_k, (\mathbf{b}_j)^\delta_k) s_k\|^2 & \omega_k = 1 \\ 1 & \omega_k = 0 \end{cases} . \tag{4.15}$$

In order to extend the convergence results in [7, 5] for these iterations, we basically have to prove two facts:

1) Assumption (14) in [7].

2) The *local tangential cone condition* [7, Eq. (15)], i.e. the existence of $(\mathcal{P}_0, (\mathbf{b}_j)_0) \in X$ and $\eta < 1/2$ such that

$$\|F_i(\mathcal{P}, (\mathbf{b}_j)) - F_i(\bar{\mathcal{P}}, (\bar{\mathbf{b}_j})) - F'_i(\mathcal{P}, (\mathbf{b}_j))[(\mathcal{P}, (\mathbf{b}_j)) - (\bar{\mathcal{P}}, (\bar{\mathbf{b}_j}))]\|_Y \leq \eta \|F_i(\mathcal{P}, (\mathbf{b}_j)) - F_i(\bar{\mathcal{P}}, (\bar{\mathbf{b}_j}))\|_Y \,, \tag{4.16}$$

for all $(\mathcal{P}, (\mathbf{b}_j))$, $(\bar{\mathcal{P}}, (\bar{\mathbf{b}_j})) \in B_\rho(\mathcal{P}_0, (\mathbf{b}_j)_0)$, and all $i = 1, \ldots, r_{\text{num}}$.

The first one represents no problem. Indeed, the Fréchet derivatives of the operators F_i are locally Lipschitz continuous. Thus, for any $(\mathcal{P}_0, (\mathbf{b}_j)_0) \in X$ and any $\rho > 0$ we have $\|F'_i(\mathcal{P}, (\mathbf{b}_j))\| \leq M = M_{\rho, \mathcal{P}_0, (\mathbf{b}_j)_0}$ for all $(\mathcal{P}, (\mathbf{b}_j))$ in the ball $B_\rho(\mathcal{P}_0, (\mathbf{b}_j)_0) \subset X$.

The local tangential cone condition however, does not hold. Indeed, the operators F_i are second order polynomials of the variables $b_{j,n}$ and $\mathcal{P}$. Therefore, it is enough to verify whether the real function $f(x, y) = xy$ satisfies

$$|f(x, y) - f(\bar{x}, \bar{y}) - f'(x, y)((x - \bar{x}, y - \bar{y}))| \leq \eta |f(x, y) - f(\bar{x}, \bar{y})| \,,$$

in some vicinity of a point $(x_0, y_0) \in \mathbb{R}^2$ containing a local minimizer of f. This, however, is not the case.

Therefore, the techniques used to prove convergence of the lLK and lSDK iterations in [7, 5] cannot be extended to the nonlinear system (3.3).

It is worth noticing that the local tangential cone condition is a standard assumption in the convergence analysis of adjoint type methods (Landweber, steepest descent, Levenberg-Marquardt, asymptotical regularization) for nonlinear inverse problems [3, 4, 6, 9, 10, 11, 13]. Thus, none of the classical convergence proofs for these iterative methods can be extended to system (3.3) in a straightforward way.

Motivated by the promising numerical results and efficient performance of the lLK and lSDK iterations for problems known not to satisfy the local tangential cone condition (see [7, 8, 5]), we intend to use iteration (4.12) for computing approximate solutions of system (3.3). This numerical investigation will be performed in a forthcoming article.

5 Conclusions

We presented the description of a discrete mathematical model for Magnetic Resonance Imaging and derived the corresponding inverse problem for MRI.

We investigate efficient iterative regularization methods for this inverse problem. An iterative method of Kaczmarz type for obtaining approximate solutions for the inverse problem is proposed.

Using a particular assumption on the sensitivity kernels, we are able to prove convergence and stability results for the proposed iterative methods.

The convergence analysis presented in this article extends the results for the *loping Landweber-Kaczmarz* method in [7]. Moreover, we prove that our method is a convergent iterative regularization method in the sense of [3].

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References

[1] F. Bauer and S. Kannengiesser, *An alternative approach to the image reconstruction for parallel data acquisition in MRI,* Mathematical Methods in the Applied Sciences, **30** (2007), 1437–1451

[2] M. Bertero and P. Boccacci, *Introduction to inverse problems in imaging*, Bristol: IoP, Institute of Physics Publishing, 1998.

[3] H. Engl, M. Hanke and A. Neubauer, *Regularization of Inverse Problems*, Kluwer, Dordrecht, 1996.

[4] H. Engl and A. Leitão, *A Mann iterative regularization method for elliptic Cauchy problems*, Numer. Funct. Anal. Optim., **22** (2001), 861–864.

[5] A. De Cezaro, M. Haltmeier, A. Leitão and O. Scherzer, *On Steepest-Descent Kaczmarz methods for regularizing nonlinear systems of ill-posed equations*, Applied Mathematics and Computation, to appear, 2008.

[6] P. Deuflhard, H.W. Engl and O. Scherzer, *A convergence analysis of iterative methods for the solution of nonlinear ill–posed problems under affinely invariant conditions*, Inverse Problems, **14** (1998), 1081–1106.

[7] M. Haltmeier, A. Leitão and O. Scherzer, *Kaczmarz methods for regularizing nonlinear ill-posed equations. I. Convergence analysis*, Inverse Probl. Imaging, **1** (2007), 289–298.

[8] M. Haltmeier, R. Kowar, A. Leitão and O. Scherzer, *Kaczmarz methods for regularizing nonlinear ill-posed equations. II. Applications*, Inverse Probl. Imaging, **1** (2007), 507–523.

[9] M. Hanke, A. Neubauer and O. Scherzer, *A convergence analysis of Landweber iteration for nonlinear ill-posed problems*, Numerische Mathematik, **72** (1995), 21–37.

[10] B. Kaltenbacher, A. Neubauer and O. Scherzer, *Iterative Regularization Methods for Nonlinear Ill–Posed Problems*, Springer Verlag, 2008, to appear.

[11] R. Kowar and O. Scherzer, *Convergence analysis of a Landweber-Kaczmarz method for solving nonlinear ill-posed problems*, Ill-posed and inverse problems, 253–270, VSP, Zeist, 2002.

[12] F. Natterer and F. Wuebbeling, *Mathematical methods in image reconstruction*, SIAM Monographs on Mathematical Modeling and Computation, **5**, Philadelphia, PA, 2007.

[13] U. Tautenhahn, *On the asymptotical regularization of nonlinear ill-posed problems*, Inverse Problems, **10** (1994), 1405–1418.

[14] J. P. Zubelli, R. Marabini, C. Sorzano, and G. Herman, *Three-dimensional reconstruction by Chanine's method from electron microscopic projections corrupted by instrumental aberrations*, Inverse Probl. 19, **4** (2003), 933–949.