CUBO A Mathematical Journal
Vol.10, N$^{\underline{o}}$01, (33–41). March 2008

On Some What Fuzzy Faintly Semicontinuous Functions

G. Palanichetty and G. Balasubramanian
Ramanujan Institute for Advanced Studies in Mathematics
University of Madras, Department of Mathematics
I.R.T. Polytechnic College, Krishnagiri – 635 108 Tamilnadu
Chepauk, Chennai – 600 005,India
email: gpalanichetty@yahoo.co.in

ABSTRACT

In this paper the concept of some what fuzzy faintly semicontinuous functions, some what fuzzy faintly semiopen functions,weakly some what fuzzy faintly semiopen functions are introduced. Some characterizations and interesting properties of these functions are discussed.

RESUMEN

El concepto de funciones fuzzy débil semicontinuas, funciones fuzzy débil semiabiertas son introducidas. Caracterizaciones e propriedades de este tipo de funciones son discutidas.

Key words and phrases: *Some what fuzzy faintly semicontinuous functions, some what fuzzy faintly semiopen functions, fuzzy semidense set, fuzzy semiseparable space, fuzzy D_S-space.*

Math. Subj. Class.: *54A40.*

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [8]. Based on the concept of fuzzy sets, C.L.Chang [4] introduced and developed the concept of fuzzy topological spaces. Since then various important notions in the classical topology such as continuous functions [4] have been extended to fuzzy topological spaces.

The concept of some what continuous functions was introduced and studied by Karl R. Gentry and Hughes B.Hoyle [5] in general topological spaces and this concept was extended to fuzzy topological spaces in [6].. In [2] the concept of fuzzy faintly α-continuous functions was introduced and studied. Also some characterizations of fuzzy faintly continuous functions are given. The purpose of this paper is to introduce and study the concept of some what fuzzy faintly semicontinuous functions and it is infact a generalization of fuzzy faintly continuous functions introduced and studied in [2]. Section 2 deals with preliminaries. Section 3 deals with some what fuzzy faintly semicontinuous functions and some of its characterizations. Section 4 deals with some what fuzzy faintly semiopen functions and some of its basic properties and characterizations.

2 Preliminaries

In this paper by (X, T) we mean fuzzy topological space in the sense of [4]. Let λ be a fuzzy set. We define closure of $\lambda = \mathrm{cl}\lambda = \wedge\{\mu/\mu \geq \lambda, \mu$ is fuzzy closed $\}$ and Interior of $\lambda = \mathrm{int}\lambda = \vee\{\sigma/\sigma \leq \lambda, \sigma$ is fuzzy open$\}$.In [7] the semiinterior and semiclosure of a fuzzy set λ are defined as S-int$(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu$ is fuzzy semiopen$\}$ and S-cl$(\lambda) = \wedge\{\mu/\mu \geq \lambda, \mu$ is fuzzy semiclosed$\}$ and also shown that S-cl$(1-\lambda) = 1-$S-int(λ) and S-int$(1-\lambda) = 1-$S-cl(λ).

A fuzzy set λ in (X, T) is called proper if $\lambda \neq 0$ and $\lambda \neq 1$.A fuzzy point x_p in X is a fuzzy set in X defined by

$$x_p(y) = \begin{cases} p, p \in (0,1] & \text{for } y = x, y \in X; \\ 0, & \text{for } y \neq x, y \in X. \end{cases}$$

x and p are respectively called the support and the value of x_p. A fuzzy point x_p is said to be quasi-coincident [1] with α, denoted by $x_p q \alpha$, if and only if $p > \alpha'(x)$ or $p + \alpha(x) > 1$ where α' denotes the complement of α.. A fuzzy subset λ in a fuzzy topological space X is said to be a $q-$ neighbourhood for a fuzzy point x_p if and only if there exists a fuzzy open subset β such that $x_p q \beta \leq \lambda$. A fuzzy set λ is called fuzzy θ-**open** [1] if and only if $\mathrm{int}_\theta(\lambda) = \lambda$ where $\mathrm{int}_\theta(\lambda) = \vee\{x_p \in X/$ for some open q- neighborhood β of $x_p, \mathrm{cl}\beta \leq \lambda\}$ and $\mathrm{cl}_\theta(\lambda) = \wedge\{\mu/\mu \geqslant \lambda, \mu$ is θ - closed$\}$. λ is called θ-closed if $\mathrm{cl}_\theta(\lambda) = \lambda$. A fuzzy set λ

in a fuzzy topological space (X,T) is called dense (θ- dense, semidense) if there exists no fuzzy closed set (θ- closed set, semiclosed set) μ such that $\lambda < \mu < 1$.

A fuzzy set λ in (X,T) is called fuzzy semiopen [3] if for some fuzzy open set $v, v \leq \lambda \leq \text{cl}\,(v)$, equivalently λ is called fuzzy semiopen if $\lambda \leq \text{cl int}(\lambda)$.If λ and μ are any two fuzzy sets in X and Y respectively, we define $\lambda \times \mu : X \times Y \to I$ as follows:

$$(\lambda \times \mu)(x,y) = \min(\lambda(x), \mu(y)).$$

A fuzzy topological space X is product related to a fuzzy topological space Y if for any fuzzy set v in X and ξ in Y whenever $\lambda'(=1-\lambda) \ngeq v$ and $\mu'(1-\mu) \ngeq \xi$ imply $\lambda' \times 1 \bigvee 1 \times \mu' \geq v \times \xi$, where λ is a fuzzy open set in X and μ is a fuzzy open set in Y,there exists a fuzzy open set λ_1 and a fuzzy open set $\mu_1$in Y such that $\lambda_1' \geq v$ or $\mu_1' \geq \xi$ and $\lambda_1' \times 1 \bigvee 1 \times \mu_1' = \lambda' \times 1 \bigvee 1 \times \mu'$. If (X,T) and (Y,S) are any two fuzzy topological spaces, we define a product fuzzy topology $T \times S$ on $X \times Y$ to be that fuzzy topology for which $\mathcal{B} = \{\lambda \times \mu / \lambda \in T, \mu \in S\}$ forms a base.

3 Some what fuzzy faintly semicontinuous functions

In [2] the concept of fuzzy faintly continuous function is given as follows :

Definition 3.1 *Let $f : (X,T_1) \to (Y,T_2)$ be a function from the fuzzy topological space (X,T_1) to the fuzzy topological space (Y,T_2). f is called fuzzy faintly continuous if $f^{-1}(\lambda)$ is fuzzy open for every fuzzy $\theta-$ open set λ in Y.*

In [7] the concept of somewhat fuzzy semicontinuous function is introduced as follows.

Definition 3.2 *Let $f : (X,T) \to (Y,S)$ be a function from the fuzzy topological space (X,T) to the fuzzy topological space (Y,S). f is called somewhat fuzzy semicontinuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0$ implies there exists a fuzzy semiopen set μ of X such that $\mu \leq f^{-1}(\lambda)$.*

We are now ready to make the following:

Definition 3.3 *Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function $f : (X,T) \to (Y,S)$ is called some what fuzzy faintly semicontinuous if for every fuzzy θ- open set λ in Y such that $f^{-1}(\lambda) \neq 0$, there exists a fuzzy semiopen set $0 \neq \mu$ in (X,T) such that $\mu \leq f^{-1}(\lambda)$.*

Clearly every fuzzy faintly continuous function [2] is some what fuzzy faintly semicontinuous but the converse is not true as the following example shows.

Example 3.4 *Consider Example 2.3 [2]. Let $X = Y = I = [0,1]$. Let μ_1, μ_2, μ_3 be fuzzy sets on I defined as follows:*

$$\mu_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$\mu_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x + 2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

and

$$\mu_3(x) = \begin{cases} x, & 0 \leq x \leq \frac{1}{4}; \\ 1, & \frac{1}{4} \leq x \leq 1. \end{cases}$$

Clearly $T = \{0, \mu_2, 1\}, S = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ are two fuzzy topologies on I. Let $f : (I,T) \to (I,S)$ be defined as follows $f(x) = x$, for each x in I. Now we can verify $f^{-1}(\mu_3) = \mu_3$ and $f^{-1}(1) = 1$ are fuzzy semiopen subsets of (I,T) and also μ_3 and 1 are the only fuzzy θ-open subsets of (I,S). It is easy to see that μ_3 and 1 are fuzzy semiopen sets in (I,T). Hence **f** ***is somewhat fuzzy faintly semicontinuous;*** *But since $f^{-1}(\mu_3) = \mu_3 \notin T_1$,* **f** ***is not fuzzy faintly continuous***.

Proposition 3.5 *Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f : (X,T) \to (Y,S)$ be a function. Then the following are equivalent.*

(a) f is some what fuzzy faintly semicontinuous.

(b) If λ is a fuzzy θ- closed set of Y such that $f^{-1}(\lambda) \neq 1$, then there exists a proper fuzzy semiclosed set μ of X such that $\mu \geq f^{-1}(\lambda)$.

(c) If λ is a fuzzy semidense set, then $f(\lambda)$ is a fuzzy θ - dense set in Y.

Proof. (a)$\Rightarrow$ (b) Suppose f is some what fuzzy faintly semicontinuous and λ is any fuzzy θ-closed set in Y such that $f^{-1}(\lambda) \neq 1$. Therefore clearly $1 - \lambda$ is fuzzy θ - open and $f^{-1}(1-\lambda) = 1 - f^{-1}(\lambda) \neq 0$. But by (a) there exists a fuzzy semiopen set μ^* in (X,T) such that $\mu^* \neq 0$ and $\mu^* \leq f^{-1}(1-\lambda)$. Therefore $1 - \mu^* \geq 1 - f^{-1}(1-\lambda) = 1 - \left[1 - f^{-1}(\lambda)\right] = f^{-1}(\lambda)$. Put $1 - \mu^* = \mu$. Clearly μ is proper fuzzy semiclosed set and $\mu \geq f^{-1}(\lambda)$. This shows (a) $\Rightarrow$ (b).

(b)$\Rightarrow$ (c) Let λ be a fuzzy semidense set in X and suppose $f(\lambda)$ is not fuzzy θ - dense in Y. Then there exists a fuzzy θ- closed set say μ^* such that

$$f(\lambda) < \mu^* < 1. \tag{A}$$

Now $\mu^* < 1 \Rightarrow f^{-1}(\mu^*) \neq 1$. Then by (b) there exists a proper fuzzy semiclosed set σ in (X,T) such that $\sigma \geq f^{-1}(\mu^*)$. But by (A), $f^{-1}(\mu^*) > f^{-1}[f(\lambda)] \geq \lambda$ that is $\sigma > \lambda$. This implies there exists a proper fuzzy semiclosed set σ such that $\sigma > \lambda$ which is a contradiction, since λ is fuzzy semidense set. This proves (b) $\Rightarrow$ (c).

(c) $\Rightarrow$ (a) Let λ be any fuzzy θ-open set in (Y,S) and suppose $f^{-1}(\lambda) \neq 0$ and hence $\lambda \neq 0$. We want to show that f is some what fuzzy faintly semicontinuous. That is we want to show that there exists a fuzzy semiopen set μ in (X,T) such that $0 \neq \mu \leq f^{-1}(\lambda)$.That is we want to show that S-int$[f^{-1}(\lambda)] \neq 0$.Suppose S-int$[f^{-1}(\lambda)] = 0$.Then

$$\text{S-cl}\left[1 - f^{-1}(\lambda)\right] = 1 - \text{S-int}[f^{-1}(\lambda)] = 1 - 0 = 1.$$

This means $1-f^{-1}(\lambda)$ is fuzzy semidense in X. Now by (c), $f[1-f^{-1}(\lambda)]$ is fuzzy θ-dense in Y. That is $\text{cl}_\theta f[1-f^{-1}(\lambda)] = 1$; but $f[1-f^{-1}(\lambda)] = f[f^{-1}(1-\lambda)] \leq 1-\lambda < 1$ (since $\lambda \neq 0$). Since $1-\lambda$ is fuzzy θ-closed and $f[1-f^{-1}(\lambda)] \leq 1-\lambda$,$\text{cl}_\theta f[1-f^{-1}(\lambda)] \leq 1-\lambda$.That is $1 \leq 1-\lambda \Rightarrow \lambda \leq 0 \Rightarrow \lambda = 0$, which is a contradiction to the fact that $\lambda \neq 0$. Therefore we must have S-int$[f^{-1}(\lambda)] \neq 0$. This proves that f is some what fuzzy faintly semicontinuous. □

Proposition 3.6 *Let (X,T) and (Y,S) be fuzzy topological spaces and $f : (X,T) \to (Y,S)$ be some what fuzzy faintly semicontinuous. Let A be a subset of X such that $\chi_A \wedge \mu \neq 0$ for some $0 \neq \mu$ in (X,T). Let $T/_A$ be the induced fuzzy topology on A. Then $f/A : (A, T/_A) \to (Y,S)$ is some what fuzzy faintly semicontinuous.*

Proof. Suppose λ is a fuzzy θ - open set in (Y,S) such that $f^{-1}(\lambda) \neq 0$. Since f is some what fuzzy faintly semicontinuous, there exists a fuzzy semiopen set μ in (X,T) such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$. But $\mu/A \in T/_A$ and $\mu/A \neq 0$ (since $\chi_A \wedge \mu \neq 0$ for all $\mu \in T$). Now $(f/A)^{-1}(\lambda)(x) = \lambda[(f/A)(x)] = \lambda f(x) > \mu(x) = \mu/A(x)$ for $x \in A$. Also μ is a fuzzy semiopen set in (X,T) implies μ/A is fuzzy semiopen in $(A, T/_A)$ and $\mu/A < (f/A)^{-1}(\lambda)$. This proves that f/A is some what fuzzy faintly semicontinuous. □

Definition 3.7 *Let X be a set, T and T' be any two fuzzy topologies for X. We say that T' is weakly semiequivalent to T if $\lambda \neq 0$ is a fuzzy semiopen set in (X,T), then there is a fuzzy semiopen set μ in (X,T') such that $\mu \neq 0$ and $\mu \leq \lambda$.*

Proposition 3.8 *Let (X,T) and (Y,S) be any two fuzzy topological spaces and suppose that $f : (X,T) \to (Y,S)$ is some what fuzzy faintly semicontinuous. Let T' be a fuzzy topology weakly semiequivalent to T. Then $f : (X,T') \to (Y,S)$ is some what fuzzy faintly semicontinuous.*

Proof. Let λ be a fuzzy θ- open set in S such that $f^{-1}(\lambda) \neq 0$. Since $f : (X,T) \to (Y,S)$ is some what fuzzy faintly semicontinuous , there is a fuzzy semiopen set μ in (X,T) and $\mu \neq 0$

such that $\mu \leq f^{-1}(\lambda)$. But by the hypothesis T' is weakly semiequivalent to T. Therefore there exists a fuzzy semiopen set μ^* such that μ^*in (X,T') and $\mu^* \neq 0$ and $\mu^* < \mu$. But $\mu < f^{-1}(\lambda)$ implies $\mu^* \leq f^{-1}(\lambda)$. This means $f:(X,T') \to (Y,S)$ is some what fuzzy faintly semicontinuous. □

Definition 3.9 *Let Y be a set. Let S and S' be any two fuzzy topologies in Y. We say that S' is weakly θ-equivalent to S if $\lambda \neq 0$ is a fuzzy θ-open set in (Y,S'), then there exists a fuzzy θ-open set μ in (Y,S) such that $\mu \leq \lambda$.*

Proposition 3.10 *Let (X,T) and (Y,S) be any two fuzzy topological spaces and suppose $f:(X,T) \to (Y,S)$ is some what fuzzy faintly semicontinuous. Let S' be a fuzzy topology weakly θ-equivalent to S. Then $f:(X,T) \to (Y,S')$ is some what fuzzy faintly semicontinuous.*

Proof. Let λ be a fuzzy θ-open set in (Y,S') such that $f^{-1}(\lambda) \neq 0$. Since S' is weakly θ-equivalent to S, there exists a fuzzy θ-open set λ^* in (Y,S) such that $0 \neq \lambda^* \leq \lambda$. Now $0 \neq f^{-1}(\lambda^*) \leq f^{-1}(\lambda)$ .Since f is some what fuzzy faintly semicontinuous from (X,T) to (Y,S) there exists a fuzzy semiopen set in (X,T) say μ such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda^*)$. This means $\mu \leq f^{-1}(\lambda)$ and so f is some what fuzzy faintly semicontinuous from (X,T) to (Y,S'). □

Proposition 3.11 *Let (X,T) and (Y,S) be fuzzy topological spaces. Suppose that $f:(X,T) \to (Y,S)$ is some what fuzzy faintly semicontinuous, T' and S' are fuzzy topologies for X and Y respectively such that T' is weakly semiequivalent to T and S' is weakly θ-equivalent to S. Then $f:(X,T') \to (Y,S')$ is some what fuzzy faintly semicontinuous.*

Proof. Proof follows from Propositions 3.8 and 3.10. □

Definition 3.12 *A fuzzy topological space (X,T) is said to be fuzzy separable (semiseparable) if there exists a fuzzy dense (semidense) set λ in (X,T) such that $\lambda \neq 0$ for atmost countably many points of X.*

Proposition 3.13 *If $f:(X,T) \to (Y,S)$ is a some what fuzzy faintly semicontinuous function and if X is fuzzy semiseparable, then Y is fuzzy θ-separable.*

Proof. Since X is fuzzy semiseparable, there exists a fuzzy dense set λ such that $\lambda \neq 0$ for atmost countably many points of X. Also since f is some what fuzzy faintly semicontinuous, it follows by Proposition 3.5 that $f(\lambda)$ is fuzzy θ- dense in (Y,S) and since $\lambda \neq 0$ and λ is semidense for atmost countably many points, it follows that $f(\lambda) \neq 0$ for atmost countably many points. Thus we find that (Y,S) is fuzzy θ-separable. □

4 Some what fuzzy faintly semiopen functions

Definition 4.1 *Let (X,T) and (Y,S) be fuzzy topological spaces. $f:(X,T)\to(Y,S)$ is called some what fuzzy faintly semiopen function if and only if for any fuzzy semiopen set $\lambda,\lambda\neq 0$ in (X,T) implies that there exists a fuzzy θ- open set μ in (Y,S) such that $\mu\neq 0$ and $\mu<f(\lambda)$. That is $int_\theta\,[f(\lambda)]\neq 0$.*

Proposition 4.2 *Let $(X,T),(Y,S)$ and (Z,R) be fuzzy topological spaces. Suppose that $f:(X,T)\to(Y,S)$ and $g:(Y,S)\to(Z,R)$ are some what fuzzy faintly semiopen functions. Then $g\circ f:(X,T)\to(Z,R)$ is some what fuzzy faintly semiopen.*

Proof. Let λ be a fuzzy semiopen set in T. Since f is some what fuzzy faintly semiopen then there exists a fuzzy θ-open set μ in S such that $\mu\leq f(\lambda)$. Now $\mathrm{int}_\theta f(\lambda)\in S$ and since g is some what fuzzy faintly semiopen, then there exists a fuzzy θ-open set γ in (Z,R) such that $\gamma<g\left[\mathrm{int}f\left(\lambda\right)\right]$. But $g\left[\mathrm{int}f\left(\lambda\right)\right]<g\left[f\left(\lambda\right)\right]$. Thus we find that there exists a fuzzy θ-open set γ in (Z,R) such that $\gamma<\left(g\circ f\right)(\lambda)$. This proves $g\circ f$ is some what fuzzy faintly semiopen. □

Proposition 4.3 *Let (X,T) and (Y,S) be fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an onto function. Then the following are equivalent.*

(a) f is some what fuzzy faintly semiopen.

(b) If λ is a fuzzy semiclosed set in X such that $f(\lambda)\neq 1$, then there exists a fuzzy θ-closed set μ in Y such that $\mu\neq 1$ and $\mu>f\left(\lambda\right)$.

Proof. (a) $\Rightarrow$ (b). Let λ be a fuzzy semiclosed set in X such that $f(\lambda)\neq 1$. Then $(1-\lambda)$ is a fuzzy semiopen set such that $f(1-\lambda)=1-f(\lambda)\neq 0$. Since f is some what fuzzy faintly semiopen, there exists a fuzzy θ-open set σ in S such that $\sigma\neq 0$ and $\sigma\leq f(1-\lambda)$. Now $1-\sigma$ is a fuzzy θ-closed set in Y such that $1-\sigma\neq 1$ and $\sigma<f(1-\lambda)$. Put $1-\sigma=\mu$. Then $\mu>1-f(1-\lambda)=f(\lambda)$. This proves (a) $\Rightarrow$ (b).

(b) $\Rightarrow$ (a). Let λ in (X,T) be a fuzzy semiopen set such that $\lambda\neq 0$. Then $1-\lambda$ is fuzzy semiclosed and $1-\lambda\neq 1$, $f(1-\lambda)=1-f(\lambda)\neq 1$.. Hence by (b) there exists a fuzzy θ-closed set μ in Y such that $\mu\neq 1$ and $\mu>f(1-\lambda)=1-f(\lambda)$, that is, $f(\lambda)>1-\mu$ and let $1-\mu=\gamma$ (say). Clearly γ is a fuzzy θ-open set in Y such that $\gamma<f(\lambda)$ and $\gamma\neq 0$. Hence f is some what fuzzy faintly semiopen. This proves (b) $\Rightarrow$ (a). □

Proposition 4.4 *Suppose (X,T) and (Y,S) be fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be any onto function. Then the following are equivalent.*

(a) f is some what fuzzy faintly semiopen

(b) If λ is a fuzzy θ- dense set in Y, then $f^{-1}(\lambda)$ is fuzzy semidense set in X.

Proof. (a) $\Rightarrow$ (b). Assume f is some what fuzzy faintly semiopen. Suppose λ is fuzzy θ-dense set in Y we have to show that $f^{-1}(\lambda)$ is fuzzy semidense in X. Suppose not, then there exists a fuzzy semiclosed set μ in X such that $f^{-1}(\mu) < \mu < 1$. Now $\lambda = ff^{-1}(\lambda) < f(\mu) < f(1)$(Since f is onto). Since f is some what fuzzy faintly semiopen by Proposition 4.3, there exists a fuzzy θ-closed set δ in Y such that $f(\mu) < \delta$.. Thus we find $\lambda < f(\mu) < \delta < 1$, which is a contradiction to our hypothesis that λ is fuzzy θ-dense in X. Hence $f^{-1}(\lambda)$ must be fuzzy semidense set. This proves (a) $\Rightarrow$ (b).

(b) $\Rightarrow$ (a). Assume that $f^{-1}(\lambda)$ is fuzzy semidense in X where λ is fuzzy θ-dense in Y. We want to show that f is some what fuzzy faintly semiopen. Assume that $\lambda \in T$ and $\lambda \neq 0$, be a fuzzy semiopen set in (X,T) we have to show that $\text{int}_\theta f(\lambda) \neq 0$. Suppose not, then $\text{int}_\theta f(\lambda) = 0$ whenever $\lambda \in T$ then $\text{cl}_\theta[1 - f(\lambda)] = 1 - \text{int}_\theta f(\lambda) = 1 - 0 = 1$. That is, $1 - f(\lambda)$ is fuzzy θ-dense in Y. Therefore by assumption $f^{-1}[1 - f(\lambda)]$ is fuzzy semidense in X. Therefore $1 = \text{S-cl}\left[f^{-1}(1 - f(\lambda))\right] = \text{S-cl}[1 - \lambda] = 1 - \lambda$. This shows $\lambda = 0$ which is a contradiction and so $\text{S-int} f(\lambda) \neq 0$.This proves (b) $\Rightarrow$ (a). □

In [6], the concept of fuzzy D-space is defined as follows:

Definition 4.5 *A fuzzy topological space (X,T) is called a fuzzy D-space if every non-zero fuzzy open set λ of (X,T) is dense in X.*

Now we are ready to make the following.

Definition 4.6 *A fuzzy topological space (X,T) is called a **fuzzy D_S- space (D_θ- space)** if every non-zero fuzzy semiopen set λ(non-zero fuzzy θ- open set) in X is semidense (θ-dense) in X.*

Proposition 4.7 *Let $f : (X,T) \to (Y,S)$ be some what fuzzy faintly semicontinuous. Suppose X is a fuzzy D_S- space. Then Y is a fuzzy D_θ- space.*

Proof. Let λ be a non-zero fuzzy θ- open set in Y. We want to show that λ is fuzzy θ- dense in$Y$. Suppose not. Then there exists a fuzzy θ- closed set μ in Y such that $\lambda < \mu < 1$. Therefore $f^{-1}(\lambda) < f^{-1}(\mu) < f^{-1}(1) = 1$.Since $\lambda \neq 0, f^{-1}(\lambda) \neq 0$ and since f is some what fuzzy faintly semicontinuous, $0 \neq \text{S-int} f^{-1}(\lambda) < f^{-1}(\mu) < \text{S-cl} f^{-1}(\mu) < 1$. This is a contradiction to the assumption that (X,T) is a fuzzy D_S- space. Hence Y is a fuzzy D_θ-space. □

Proposition 4.8 *Let X_1, X_2, Y_1 and Y_2 be fuzzy topological spaces such that Y_1 is a product related to Y_2 and X_1 is a product related to X_2. Then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ of some what fuzzy faintly semicontinuous mapping $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is some what fuzzy faintly semicontinuous.*

Proof. Let $\alpha = \vee (\alpha_i \times \alpha_j)$, where α_i's and α_j's are fuzzy θ-open sets of Y_1 and Y_2 respectively be a fuzzy θ-open set of $Y_1 \times Y_2$. Then we have

$$(f_1 \times f_2)^{-1}(a) = \vee \left(f_1^{-1}(\alpha_i) \times f_2^{-1}(\alpha_j) \right)$$

Since f_1 and f_2 are somewhat fuzzy faintly semicontinuous functions, there exists fuzzy semiopen sets σ_i in X_1, fuzzy semiopen set σ_j in X_2 such that $\sigma_i \leq f_1^{-1}(\alpha_i); \sigma_j \leq f_2^{-1}(\alpha_j)$. Therefore $(f_1 \times f_2)^{-1}(\alpha) \geq \vee (\sigma_i \times \sigma_j)$. Since X_1 and X_2 are product related, it follows from [3] that $\vee (\sigma_i \times \sigma_j)$ is fuzzy semiopen. Hence $f_1 \times f_2$ is some what fuzzy faintly semicontinuous. □

Received: December 2005. Revised: October 2007.

References

[1] A. Mukherjee, *Fuzzy faintly continuous functions, Fuzzy Sets and Systems*, **59** (1993), 59–63.

[2] A. Mukherjee and S. Debnath, *On fuzzy faintly semicontinuous functions*, Indian J. pure appl. Math., **34**(2003), 1625–1630.

[3] A.S. Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinunity*, Fuzzy Sets and Systems, **44** (1991), 303–308.

[4] C.L. Chang, *Fuzzy topological spaces*, J.Math. Anal.Appl., **141** (1968), 82–89.

[5] K.R. Gentry and H.B. Holye, *Some what continuous functions*, Czech. Math. J., **21**(1971), 5–12.

[6] G. Thangaraj and G. Balasubramanian, *On some what fuzzy continuous functions*, J. Fuzzy Math., **11** (2003), 1-12.

[7] G. Thangaraj and G. Balasubramanian, *On some what fuzzy semicontinuous functions*, Kybernetika, **37** (2001), 165–170.

[8] L.A. Zadeh, *Fuzzy sets, Information and Control*, **8**(1965), 338–353.