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Yamabe Solitons with potential vector field as torse forming

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ABSTRACT

The Riemannian manifolds whose metric is Yamabe soliton with potential vector field as torse forming admitting Riemannian connection, semisymmetric metric connection and projective semisymmetric connection have been studied. An example is constructed to verify the theorem concerning Riemannian connection.

RESUMEN

Se estudian las variedades Riemannianas cuya métrica es un solitón de Yamabe con vector de potencial que forma un virol (superficie desarrollable) con respecto a conexiones Riemanniana, semisimétrica métrica y proyectiva semisimétrica. Se construye un ejemplo explícito para verificar las hipótesis del teorema en el caso de la conexión Riemanniana.

Keywords and Phrases: Yamabe soliton, torse forming vector field, torqued vector field, semisymmetric metric connection, projective semisymmetric connection.

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1 Introduction

The curvature tensor, Ricci tensor and scalar curvature of a Riemannian manifold M of dimension n equipped with Riemannian metric g with respect to Levi-Civita connection ∇ are denoted by R, S and r respectively. Hamilton ([5], [6]) introduced the notion of Yamabe flow, which is an evolution equation for metrics on M as follows:

$$\frac{\partial}{\partial t} g = -rg.$$

When $n = 2$, the Yamabe flow is equivalent to the Ricci flow. However, for $n > 2$, they do not agree.

A Yamabe soliton on M is, a special solution of the Yamabe flow, a triplet (g, V, σ) such that

$$\frac{1}{2}\pounds_V g = (r - \sigma)g, \tag{1.1}$$

where $\pounds_V$ is the Lie derivative in the direction of $V \in \chi(M)$ and σ is a constant. The nature of such soliton depends on the behaviour of σ. The Yamabe soliton is said to be shrinking, steady and expanding according as $\sigma < 0$, $= 0$ and > 0 respectively. If $\sigma \in C^\infty(M)$ then the metric satisfying (1.1) is called almost Yamabe soliton [1]. For $n = 2$ such soliton is equivalent with Ricci soliton, but for $n > 2$, they do not. Yamabe solitons have been studied by several authors such as [5], [6], [9], [10] and references there in.

As a generalization of concircular, concurrent and parallel vector field, Yano [14] introduced the torse-forming vector field. A nowhere vanishing vector field τ is said to be a torse-forming on M if

$$\nabla_X \tau = fX + \gamma(X)\tau, \tag{1.2}$$

where $f \in C^\infty(M)$ and γ is an 1-form.

If the 1-form γ in (1.2) vanishes identically, then τ is concircular [13]. Concircular vector fields also known as geodesis vector fields since integral curves of such vector fields are geodesis. Recently, Chen [2] studied Ricci solitons with concircular vector field. If $f = 1$ and $\gamma = 0$ then τ is concurrent [16]. The vector field τ is recurrent if it satisfies (1.2) with $f = 0$. Also if $f = \gamma = 0$, the vector field τ in (1.2) is parallel vector field.

As a consequence of torse forming vector field, recently Chen [3] introduced a new vector field, called torqued vector field. If the vector field τ satisfies (1.2) with $\gamma(\tau) = 0$ then τ is called torqued vector field. Here, f is known as the torqued function and the 1-form is the torqued form of τ.

In this paper we have studied Yamabe solitons, whose potential vector field is torse forming, on Riemannian manifolds with respect to Riemannian connection (RC), semisymmetric metric connection (SSMC) and projective semisymmetric connection (PSSC) and prove the following:

Theorem 1.1. *Let (g, τ, σ) be a Yamabe soliton on M with respect to RC ∇. Then the following holds:*

τ	*condition of existence*	*conditions of shrinking, steady and expanding*
torse-forming	$r - f - \frac{1}{n}\gamma(\tau)=$*constant*	$r - f - \frac{1}{n}\gamma(\tau) \lesseqqgtr 0$
concircular	$r - f=$ *constant*	$r - f \lesseqqgtr 0$
concurrent	$r=$*constant*	$r \lesseqqgtr 1$
recurrent	$r - \frac{1}{n}\gamma(\tau)=$ *constant*	$r - \frac{1}{n}\gamma(\tau) \lesseqqgtr 0$
parallel	$r =$ *constant*	$r \lesseqqgtr 0$
torqued	$r - f =$*constant*	$r - f \lesseqqgtr 0$

Theorem 1.2. *Let (g, τ, σ) be a Yamabe soliton on M with respect to SSMC $\bar{\nabla}$. Then the following holds:*

τ	*condition of existence*	*conditions of shrinking, steady and expanding*
torse-forming	$r - f - 2(n-1)a$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}=$*constant*	$r - f - 2(n-1)a$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0$
concircular	$r - f - (n-1)\{2a + \frac{1}{n}\pi(\tau)\}=$*constant*	$r - f - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
concurrent	$r - 1 - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} = \mathsf{constant}$	$r - 1 - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
recurrent	$r - 2(n-1)a$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}=$ *constant*	$r - 2(n-1)a$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0$
parallel	$r - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} = \mathsf{constant}$	$r - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
torqued	$r - f - (n-1)\{2a + \frac{1}{n}\pi(\tau)\}$ $=$*constant*	$r - f - (n-1)\{2a + \frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$

Theorem 1.3. *Let* (g, τ, σ) *be a Yamabe soliton on* M *with respect to PSSC* $\widetilde{\nabla}$*. Then the following holds:*

τ	*condition of existence*	*conditions of shrinking, steady and expanding*
torse-forming	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}$=*constant*	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0$
concircular	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha$ $+\frac{1}{n}\pi(\tau)\}$ =*constant*	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha$ $+\frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
concurrent	$r + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha$ $+\frac{1}{n}\pi(\tau)\}$ =*constant*	$r - 1 + \mathrm{Tr}\cdot\beta$ $-(n-1)\{\mathrm{Tr}\cdot\alpha + \frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
recurrent	$r + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}$= *constant*	$r + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha$ $-\frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0$
parallel	$r + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha + \frac{1}{n}\pi(\tau)\}$ =*constant*	$r + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha +$ $\frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$
torqued	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha +$ $\frac{1}{n}\pi(\tau)\}$=*constant*	$r - f + \mathrm{Tr}\cdot\beta - (n-1)\{\mathrm{Tr}\cdot\alpha +$ $\frac{1}{n}\pi(\tau)\} \lesseqqgtr 0$

Section 2 consists with preliminaries. The proof of our theorems are given in section 3. In section 4, we have constructed an example to verify Theorem 1.1.

Remark. The conditions of existence of Theorem 1.1, Theorem 1.2 and Theorem 1.3 are only necessary. Finding sufficient conditions for the existence of solitons is a much deeper problem and this is not addressed in the present manuscript.

2 Preliminaries

The relation between the semisymmetric metric connection (SSMC) $\bar{\nabla}$ and ∇ of M is given by ([4], [7], [15])

$$\bar{\nabla}_X Y = \nabla_X Y + \pi(Y)X - g(X,Y)\rho, \tag{2.1}$$

where $\pi(X) = g(X, \rho)$ for all $X \in \chi(M)$. If $\bar{R}$ (resp. $\bar{S}$ and $\bar{r}$) are the curvature tensor (respectively Ricci tensor and scalar curvature) of M with respect to SSMC, then [4]

$$\bar{R}(X,Y)Z = R(X,Y)Z - P(Y,Z)X + P(X,Z)Y - g(Y,Z)LX + g(X,Z)LY, \tag{2.2}$$

$$\bar{S}(Y,Z) = S(Y,Z) - (n-2)P(Y,Z) - a g(Y,Z), \tag{2.3}$$

$$\bar{r} = r - 2(n-1)a, \tag{2.4}$$

where P is a tensor field of type $(0,2)$ given by

$$P(X,Y) = g(LX,Y) = (\nabla_X \pi)(Y) - \pi(X)\pi(Y) + \frac{1}{2}\pi(\rho)g(X,Y)$$

and $a = Tr.P$ for any $X, Y \in \chi(M)$. The relation between projective semisymmetric connection $\widetilde{\nabla}$ and ∇ is [17]

$$\widetilde{\nabla}_X Y = \nabla_X Y + \psi(Y)X + \psi(X)Y + \phi(Y)X - \phi(X)Y, \tag{2.5}$$

where the 1-forms ϕ and ψ are given by $\phi(X) = \frac{1}{2}\pi(X)$ and $\psi(X) = \frac{n-1}{2(n+1)}\pi(X)$. If $\widetilde{R}$ (resp. $\widetilde{r}$ and $\widetilde{S}$) are the curvature tensor, Ricci tensor and scalar curvature of M with respect to $\widetilde{\nabla}$, then ([12], [17])

$$\widetilde{R}(X,Y)Z = R(X,Y)Z + \beta(X,Y)Z + \alpha(X,Z)Y - \alpha(Y,Z)X, \tag{2.6}$$

$$\widetilde{S}(Y,Z) = S(Y,Z) + \beta(Y,Z) - (n-1)\alpha(Y,Z), \tag{2.7}$$

$$\widetilde{r} = r + Tr.\beta - (n-1)Tr.\alpha, \tag{2.8}$$

for all $X, Y, Z \in \chi(M)$, where

$$\beta(X,Y) = \frac{1}{2}[(\nabla_Y \pi)(X) - (\nabla_X \pi)(Y)],$$

$$\alpha(X,Y) = \frac{n-1}{2(n+1)}(\nabla_X \pi)(Y) + \frac{1}{2}(\nabla_Y \pi)(X) - \frac{n^2}{(n+1)^2}\pi(X)\pi(Y).$$

3 Proof of the Theorems

Proof of the Theorem 1.1. Let (g, τ, σ) be a Yamabe soliton on M. Then from (1.1) we get

$$\frac{1}{2}(\pounds_\tau g)(X,Y) = (r - \sigma)g(X,Y). \tag{3.1}$$

Now from (1.2) we have

$$\begin{aligned}(\pounds_\tau g)(X,Y) &= g(\nabla_X \tau, Y) + g(X, \nabla_Y \tau) \\ &= 2fg(X,Y) + \gamma(X)g(\tau,Y) + \gamma(Y)g(\tau,X)\end{aligned} \tag{3.2}$$

for all $X, Y \in \chi(M)$. In view of (3.2), (3.1) yields

$$(r - \sigma - f)g(X,Y) = \frac{1}{2}\{\gamma(X)g(\tau,Y) + \gamma(Y)g(\tau,X)\}. \tag{3.3}$$

Taking contraction of (3.3) over X and Y we get

$$n(r - \sigma - f) = \gamma(\tau). \tag{3.4}$$

This leads to the following:

Proposition 3.1. *Let (g, τ, σ) be a Yamabe soliton on M with respect to RC ∇. If τ is torse-forming then this soliton is shrinking, steady and expanding according as*

$$r - f - \frac{1}{n}\gamma(\tau) \lesseqqgtr 0,$$

provided as $r - f - \frac{1}{n}\gamma(\tau)$ is constant.
From Proposition 3.1, we obtain Theorem 1.1.
Proof of the Theorem 1.2. We now consider (g, τ, σ) is a Yamabe soliton on M with respect to semisymmetric metric connection. Then we have

$$\frac{1}{2}(\bar{\mathcal{L}}_\tau g)(X, Y) = (\bar{r} - \sigma)g(X, Y), \tag{3.5}$$

where $\bar{\mathcal{L}}_\tau$ is the Lie derivative along τ of $\bar{\nabla}$. From (2.1) we get

$$\begin{aligned} (\bar{\mathcal{L}}_\tau g)(X, Y) &= g(\bar{\nabla}_X \tau, Y) + g(X, \bar{\nabla}_Y \tau) \\ &= g(\nabla_X \tau + \pi(\tau)X - g(X, \tau)\rho, Y) \\ &+ g(X, \nabla_Y \tau + \pi(\tau)Y - g(Y, \tau)\rho) \\ &= (\mathcal{L}_\tau g)(X, Y) + 2\pi(\tau)g(X, Y) \\ &- [g(X, \tau)\pi(Y) + g(Y, \tau)\pi(X)]. \end{aligned} \tag{3.6}$$

Using (2.4) and (3.6) in (3.5), we get

$$\begin{aligned} \frac{1}{2}(\mathcal{L}_\tau g)(X, Y) &= (r - \sigma)g(X, Y) - \{2(n-1)a + \pi(\tau)\}g(X, Y) \\ &+ \frac{1}{2}[g(X, \tau)\pi(Y) + g(Y, \tau)\pi(X)]. \end{aligned} \tag{3.7}$$

In view of (3.2), (3.7) yields

$$\begin{aligned} &\{r - \sigma - f - 2(n-1)a - \pi(\tau)\}g(X, Y) \\ &+\frac{1}{2}[\{\pi(Y) - \gamma(Y)\}g(\tau, X) + \{\pi(X) - \gamma(X)\}g(\tau, Y)] = 0. \end{aligned} \tag{3.8}$$

Contracting (3.8) over X and Y, we get

$$n\{r - \sigma - f - 2(n-1)a\} - (n-1)\pi(\tau) - \gamma(\tau) = 0. \tag{3.9}$$

This leads to the following:

Proposition 3.2. *Let (g, τ, σ) be a Yamabe soliton on M with respect to SSMC $\bar{\nabla}$. If τ is torse-forming then this soliton is shrinking, steady and expanding according as*

$$r - f - 2(n-1)a - \frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0,$$

provided $r - f - 2(n-1)a - \frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}$ is constant.
From Proposition 3.2, we obtain Theorem 1.2.

Proof of the Theorem 1.3. We now consider (g, τ, σ) is a Yamabe soliton on M with respect to $\widetilde{\widetilde{\nabla}}$. Then we have

$$\frac{1}{2}(\widetilde{\widetilde{\mathcal{L}}}_\tau g)(X,Y) = (\widetilde{\widetilde{r}} - \sigma)g(X,Y), \tag{3.10}$$

where $\widetilde{\widetilde{\mathcal{L}}}_\tau$ is the Lie derivative along τ of $\widetilde{\widetilde{\nabla}}$. From (2.5) we get

$$\begin{aligned}(\widetilde{\widetilde{\mathcal{L}}}_\tau g)(X,Y) &= g(\widetilde{\widetilde{\nabla}}_X\tau, Y) + g(X, \widetilde{\widetilde{\nabla}}_Y\tau) \\ &= (\mathcal{L}_\tau)(X,Y) + \frac{1}{n+1}\{2n\pi(\tau)g(X,Y) \\ &- \pi(X)g(\tau,Y) - \pi(Y)g(X,\tau)\}.\end{aligned} \tag{3.11}$$

Using (2.8) and (3.11) in (3.10), we get

$$\begin{aligned}\frac{1}{2}(\mathcal{L}_\tau g)(X,Y) &= (r-\sigma)g(X,Y) \\ &+ [\mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha]g(X,Y) \\ &- \frac{1}{2(n+1)}\{2n\pi(\tau)g(X,Y) \\ &- \pi(X)g(Y,\tau) - \pi(Y)g(X,\tau)\}.\end{aligned} \tag{3.12}$$

In view of (3.2), (3.12) yields

$$\begin{aligned}&\{r - \sigma - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha - \frac{n}{n+1}\pi(\tau)\}g(X,Y) \\ &+\frac{1}{2}[\{\frac{\pi(Y)}{n+1} - \gamma(Y)\}g(\tau,X) + \{\frac{\pi(X)}{n+1} - \gamma(X)\}g(\tau,Y)] = 0.\end{aligned} \tag{3.13}$$

Contracting (3.13) over X and Y, we get

$$n\{r - \sigma - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha\} - (n-1)\pi(\tau) - \gamma(\tau) = 0. \tag{3.14}$$

This leads to the following:

Proposition 3.3. *Let (g, τ, σ) be a Yamabe soliton on M with respect to $\widetilde{\widetilde{\nabla}}$. If τ is torse-forming then this soliton is shrinking, steady and expanding according as*

$$r - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha - \frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\} \lesseqqgtr 0,$$

provided $r - f + \mathrm{Tr}\cdot\beta - (n-1)\mathrm{Tr}\cdot\alpha - \frac{1}{n}\{(n-1)\pi(\tau) + \gamma(\tau)\}$ is constant.
From Proposition 3.3, we obtain Theorem 1.3.

4 Example

Here we construct an example to verify Theorem 1.1.

Example: Let us consider a 3-dimensional manifold $M = \{(x, y, z) \in \mathbb{R}^3 : z \neq 0\}$. Let $\{e_1, e_2, e_3\}$ be a linearly independent global frame on M given by

$$e_1 = z^2 \frac{\partial}{\partial x},\ e_2 = z^2 \frac{\partial}{\partial y},\ e_3 = \frac{\partial}{\partial z}.$$

Let g be the Riemannian metric defined by $g(e_i, e_j) = \begin{cases} 1, & \text{i=j} \\ 0, & \text{i} \neq \text{j} \end{cases}$.

These vector field and such metric is used in ([8], [11]). Using Koszul formula, we have [11]

$$\begin{aligned}
&\nabla_{e_1} e_1 = \frac{2}{z} e_3, \quad \nabla_{e_1} e_2 = 0, \quad \nabla_{e_1} e_3 = -\frac{2}{z} e_1, \\
&\nabla_{e_2} e_1 = 0, \quad \nabla_{e_2} e_2 = \frac{2}{z} e_3, \quad \nabla_{e_2} e_3 = -\frac{2}{z} e_2, \\
&\nabla_{e_3} e_1 = 0, \quad \nabla_{e_3} e_2 = 0, \quad \nabla_{e_3} e_3 = 0.
\end{aligned}$$

The scalar curvature of this manifold is also computed in [11] and it is $r = -\frac{32}{z^2}$.

Since $\{e_1, e_2, e_3\}$ forms a basis, any vector field $X, Y, U \in \chi(M)$ can be written as

$X = a_1 e_1 + b_1 e_2 + c_1 e_3, \quad Y = a_2 e_1 + b_2 e_2 + c_2 e_3, \quad U = a_3 e_1 + b_3 e_2 + c_3 e_3$,

where $a_i, b_i, c_i \in \mathbb{R}^+$ for $i = 1, 2, 3$ such that

$$\frac{a_1 a_2 + b_1 b_2}{c_1} + c_1\left(\frac{b_2}{b_1} - \frac{a_2}{a_1} - 1\right) \neq 0.$$

If we choose the 1-form γ by $\gamma(W) = g(W, \frac{2}{z} e_3)$ for any $W \in \chi(M)$ and considering $f \in C^\infty(M)$ as

$$f = \frac{2}{z} \left\{ \frac{a_1 a_2 + b_1 b_2}{c_1} + c_1 \left(\frac{b_2}{b_1} - \frac{a_2}{a_1} - 1 \right) \right\}.$$

Then the relation

$$\nabla_X Y = fX + \gamma(X) Y \tag{4.1}$$

holds. Consequently Y is a torse-forming vector field. Now from (4.1) we get

$$\begin{aligned}
(\pounds_Y g)(X, U) &= g(\nabla_X Y, U) + g(X, \nabla_U Y) \\
&= 2fg(X, U) + \gamma(X) g(Y, U) + \gamma(U) g(Y, X).
\end{aligned} \tag{4.2}$$

Also we can calculate

$$\begin{cases} g(X, U) = a_1 a_3 + b_1 b_3 + c_1 c_3, \\ g(Y, U) = a_2 a_3 + b_2 b_3 + c_2 c_3, \\ g(Y, X) = a_1 a_2 + b_1 b_2 + c_1 c_2, \end{cases} \tag{4.3}$$

$$\gamma(X) = \frac{2c_1}{z}, \gamma(Y) = \frac{2c_2}{z}, \gamma(U) = \frac{2c_3}{z}. \tag{4.4}$$

In view of (4.3) and (4.4), (4.2) yields

$$\begin{aligned}\frac{1}{2}(\pounds_Y g)(X,U) &= \frac{1}{z}\Big[\Big\{\frac{2(a_1a_2+b_1b_2)}{c_1}+2c_1\Big(\frac{b_2}{b_1}-\frac{a_2}{a_1}-1\Big)\Big\}(a_1a_3+b_1b_3+c_1c_3)\\ &+ c_1(a_2a_3+b_2b_3+c_2c_3)+c_3(a_1a_2+b_1b_2+c_1c_2)\Big]. \qquad (4.5)\end{aligned}$$

Also

$$(r-\sigma)g(X,U)=(-\frac{32}{z^2}-\sigma)(a_1a_3+b_1b_3+c_1c_3). \qquad (4.6)$$

Assuming that $a_1a_3+b_1b_3+c_1c_3\neq 0$ and

$$3c_1(a_2a_3+b_2b_3+c_2c_3)+3c_3(a_1a_2+b_1b_2+c_1c_2)-2c_2(a_1a_3+b_1b_3+c_1c_3)=0,$$

we get (g,Y,σ) is an Yamabe soliton, i.e $\frac{1}{2}(\pounds_Y g)(X,U)=(r-\sigma)g(X,U)$ holds, provided

$$\begin{aligned}\sigma &= -\frac{32}{z^2}-\frac{2}{z}\Big\{\frac{(a_1a_2+b_1b_2)}{c_1}+c_1\Big(\frac{b_2}{b_1}-\frac{a_2}{a_1}-1\Big)\Big\}\\ &- \frac{c_1(a_2a_3+b_2b_3+c_2c_3)+c_3(a_1a_2+b_1b_2+c_1c_2)}{(a_1a_3+b_1b_3+c_1c_3)z}\\ &= r-f-\frac{1}{3}\gamma(Y)\\ &= constant.\end{aligned}$$

Thus the condition of existence of Yamabe soliton (g,Y,σ) on a 3-dimensional Riemannian manifold with potential vector field Y as torse forming in Theorem 1.1 is verified.

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