Graphical interpolation of Munsell data

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ABSTRACT

This paper analyzes the problem of converting Munsell color notations (hue, value and chroma) into CIE 1931 Yxy coordinates using the graphical method outlined in 1943 by the Optical Society of America (OSA). Various software programs can now perform this task, running on computers or color control instruments. Nonetheless, the OSA graphical technique remains the reference procedure to transform Munsell notations into CIE coordinates. The ASTM "Standard Practice for Specifying Color by the Munsell System" provides the data and charts needed to perform such conversion. Accordingly, Munsell value can be easily transformed into CIE luminance Y through a search in a data table. However, a guided procedure to graphically convert hue and chroma into CIE chromaticity coordinates is not available yet. To fill this gap, this paper proposes a graphical technique, which can use simple drawing computer programs such as PowerPoint, or more sophisticated tools for Computer Aided Design or Graphic Design, such as SolidWork or Adobe Illustrator. As an example, the proposed procedure is applied to a set of Munsell color chips from the Munsell Book of Colors, with nominal Munsell notations, but unknown CIE coordinates. The CIE Yxy coordinates of the analyzed chips are then digitally obtained by two ad hoc software conversion programs, and by instrumental measurements performed with a commercial spectrophotometer. Finally, a detailed comparison of the graphically and digitally interpolated data and of the instrumental measurements is provided and discussed.

KEYWORDS Munsell Color System, CIE coordinates, Visual interpolation, Color measurement

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1. Introduction

It is a truth universally acknowledged that specifying, matching or communicating colors in an objective way is a difficult task, even for expert practitioners endowed with sophisticated equipment. On the other hand, this unpleasant aspect of colorimetry can shed some light on the long-lasting interest of industry, academia and standard bodies for the Munsell Color System (MCS) and its companion, the Munsell Book of Colors (MBC) (Cochrane 2014).

The MCS was proposed by Albert Munsell in 1905, with the aim of defining a perceptually uniform color space (Newhall 1940) (Newhall et al. 1943). Based on a huge number of visual judgments by human observers, the MCS characterizes colors by three independent parameters - hue, value and chroma, representing tint (e.g. red, yellow, or green), lightness, and difference from neutral grey, respectively (ASTM Standard D 1535-14 2014).

The MBC, conversely, consists of a collection of uniformly distributed reference colors, printed on removable paper chips. Released in 1915 and commercialized in 1929, the MBC was finally updated after 1943 (Cochrane 2014), based on the studies of an ad hoc subcommittee of the Optical Society of America (OSA). The final OSA results, published at first in (Newhall et al. 1943), are now included in the international standard (ASTM Standard D 1535-14 2014).

The MBC chips originate a regular tridimensional grid in the Munsell color space (Newhall et al. 1943). They span 40 hues at constant steps of 2.5, integer values from 1 to 9, and even chroma from 2 towards the Mac Adam limit, i.e. the maximum chroma theoretically achievable by any object color of given hue and value (Wyszecki and Stiles 2000).

Using the MBC, one can obtain the Munsell specification of an object color: to this end, (ASTM Standard D 1535-14 2014) recommends selecting (by direct visual comparison) the perceptually closest MBC chips and linearly interpolating their Munsell notations.

For various reasons (some of which will be briefly discussed in the following), the MCS and the MBC are still adopted today in many color application fields, such as fashion and design, soil analysis, food control, forensic pathology, color education, and many others (Cochrane 2014).

The practical use of the MCS, however, very often brings about the consequent need to convert the Munsell notations into CIE 1931, CIELAB, or RGB coordinates (Wyszecki and Stiles 2000), in particular when the obtained color data require further processing (Paglierani and Valan 2018).

In the literature, the problem of transforming CIE 1931 coordinates into Munsell notations has received greater attention than the opposite conversion, from MCS to CIE – see, for instance, (Rheinboldt and Menard 1960), (Simon and Frost 1987), (Smith et al. 1990), (Mahyar et al. 2008). From the 60’s up to nowadays, in fact, most of the efforts in this field have aimed at converting by digital means instrumental colorimetric data (i.e. the CIE coordinates evaluated by suitably processing measured reflectance spectra) into Munsell notations – which should indeed be the result of a perceptual judgment by a human observer (Newhall 1940), (Wyszecki and Stiles 2000).

As a result, low cost, portable measuring devices with user-friendly interfaces are now available on the market, which can provide not only CIE or CIELAB data, but are also capable to digitally provide perceptual Munsell notations from instrumental spectrophotometric measurements – see, for instance (Konica Minolta 2014).

Digital techniques and color-control instrumentation can be extremely useful when a large amount of colors must be characterized (Centore 2011). On the other hand, their practical use can be problematic in certain color control applications, specifically when the specimen under test is liquid or semi-liquid (e.g. beers, icecream or sauces in food control), non-compact (such as sand, in soil analysis, or hair, in hair dyeing control), or visually complex (e.g. diamonds or other gems) (Munsell Color 2020). In these cases, the use of the MBC chips still prevails, owing to their practical effectiveness and easy applicability.

Furthermore, various industrial or commercial recommendations issued by national or international standard bodies still adopt the MCS and ad hoc versions of the MBC to specify colors; two interesting examples are the Brazilian System for Soil Classification (De Souza 2020) or the United States Standards for Grades of Frozen French-Fried Potatoes, (Munsell Color 2020).

Thus, either for the reasons summarized above or just because they prefer relying on the human color vision rather than on electronic devices, many color professionals still use the MCS and the MBC in their work. To support such practitioners, this paper will focus on the Munsell to CIE conversion problem. Thus, for the sake of brevity, CIE to Munsell conversion techniques will not be considered in the following.

(Newhall et al. 1943) gave a fundamental contribution to the Munsell to CIE conversion problem, by providing the Munsell Renotation Data (MRD) - i.e a collection of reference Munsell notations with the corresponding CIE Yxy coordinates, and outlining a graphical conversion procedure based on specific MRD charts available in (ASTM Standard D 1535-14 2014).

One relevant limitation of the MRD (and, consequently, of the MRD charts) is the fact that the provided reference colors are only expressed as Munsell notations and CIE 1931 Yxy specifications; hence, the MRD charts are available only in the CIE xy chromaticity plane (Newhall et al. 1943). As a consequence, their application can be extended to other color spaces only by means of a successive transformation, from CIE to the target color space. For instance, once a Munsell notation has been graphically turned to CIE coordinates using the MRD charts, the corresponding CIELAB data can only be evaluated by applying sequentially the well-known algebraic expressions for the CIE to CIELAB conversion (Wyszecki and Stiles 2000).

A direct transformation between the MCS and the CIELAB color spaces by means of digital techniques has been analyzed in (Mahyar et al. 2008), where the conversion from CIELAB coordinates to Munsell hue is investigated. Conversely, graphical techniques between CIELAB (or any other color space) and MCS are not known to the authors.

The higher uniformity of the more recent CIELAB space (or of other color spaces) with respect to the CIE 1931 color system system could be exploited to improve the accuracy of the conversion process (Wyszecki and Stiles 2000). In fact, as will be highlighted in the following, the low uniformity exhibited by the CIE 1931 xy chromaticity plane is a critical aspect in the OSA graphical conversion. Nonetheless, this paper will focus only on the use of the MRD charts for the Munsell to CIE 1931 conversion procedure. The analysis of the impact of the low uniformity of the CIE 1931 color space on the conversion process, and its extension to other color spaces, can represent further steps of the research activity presented in this paper.

Another drawback of the OSA graphical procedure is that it can be quite complex and time consuming. For this reason, various software programs have been proposed, which can perform the Munsell to CIE conversion (BabelColor 2018), (Centore 2011), (Paglierani and Valan 2018, Conference Paper). Such programs are usually based on linear, linear and radial (Centore 2011), or spline interpolation techniques (Paglierani and Valan 2018, Conference Paper). Neural network conversion techniques have also been investigated (Kang et al 2009) (Tominaga 1993). To the best of the authors' knowledge, however, these techniques are not used in practical applications yet.

A partial comparison of the results provided by linear, linear and radial, or spline interpolation techniques is given in (Paglierani and Valan 2018, Conference Paper), where the lack of a suitable set of reference colors outside the MRD was highlighted as a major problem in performance analysis activities.

This paper aims to contribute to this topic, by providing common guidelines and a simplified procedure to obtain reference MCS and CIE 1931 data outside the MRD. In fact, as clearly stated in (ASTM Standard D 1535-14 2014), the graphical approach is the reference conversion technique that should be used to determine the accuracy of any computer program performing the Munsell-to-CIE conversion.

In the Munsell to CIE conversion, the Munsell value can be simply transformed into CIE luminance through a search in a data table (ASTM Standard D 1535-14 2014).

On the contrary, graphically transforming Munsell hue and chroma into the CIE chromaticity coordinates xy can be more complex (Centore 2011). Moreover, a detailed guided procedure to systematically perform such task, to the best of the authors' knowledge, is not available yet.

To fill this gap, this paper proposes a graphical Munsell to CIE conversion technique based on the MRD charts and supported by simple and popular computer programs for drawing, e.g. PowerPoint (Microsoft Powerpoint 2020) or LibreOfficeDraw (LibreOffice Draw 2020), or by more complex Computer Aided Design or Graphical Design applications, such as Solid Work (Solidwork 2020) or Adobe Illustrator (Adobe Illustrator 2020).

The paper demonstrates the use of such tools in the Munsell to CIE conversion through practical examples, and verifies its reproducibility by comparing the results achieved by independent operators using different tools.

The proposed procedure is then applied to a set of Munsell color chips included in the MBC - hence, with nominal Munsell notations, but of unknown CIE coordinates (intermediate MBC chips).

In the paper, the CIE Yxy coordinates of the intermediate MBC chips are also obtained by two computer programs, i.e. (Centore 2011) and (Paglierani and Valan 2018, Conference Paper). Moreover, they are directly evaluated from instrumental spectrophotometric measurements obtained with a commercial instrument (Konica Minolta 2014), (Wyszecki and Stiles 2000).

Finally, the CIE coordinates obtained by graphical or digital conversion of Munsell notations and the ones directly evaluated from spectrophotometric measurements are thoroughly compared and discussed.

The procedure presented in this paper can be used by researchers who need to validate a novel digital conversion algorithm, or compare the accuracy performance of different conversion techniques. The proposed procedure can also provide a systematic way for independent researchers to produce reference color data.

Furthermore, this paper can be considered a tutorial for all those color practitioners who wish to acquire a deeper understanding about MCS and MBC.

The structure of the paper is as follows. The next section briefly introduces the MCS, the MBC and the MRD. Section 3 presents and discusses the proposed graphical conversion procedure, and the experimental use of different digital tools. Section 4 summarizes the conversion algorithms proposed in (Centore 2011), (Paglierani and Valan 2018, Conference paper). Finally, the graphical conversion results, the digital conversion results and the obtained measurements are described, compared and discussed.

2. The Munsell Color System

The MCS associates colors to points in a tridimensional space, so that equal perceptual differences between colors imply equal Euclidean distances between the corresponding points (ASTM Standard D 1535-14 2014) (Newhall et al. 1943).

In MCS the color attributes, hue value and chroma, are represented by cylindrical coordinates: hue is the angle about the neutral axis, value is to the distance from the plane containing the black point, and chroma is the radial distance from the neutral axis.

A Munsell notation is a combination of letters and numbers, written in the form *pHV*/*C*, which defines the color of an opaque object with respect to the Munsell hue *pH*, the Munsell value *V*, and Munsell chroma *C*.

In this notation, *p* is a real number in (0,10] and *H* a letter in {*R*,*YR*,*Y*,*GY*,*G*,*BG*,*B*,*PB*,*P*,*RP*} (ASTM Standard D 1535-14 2014). The value *V* is a real number in [0,10]; 0 corresponds to ideal black and 10 to ideal white. The chroma *C* is real and non-negative. Examples of Munsell notations are 8.75G3.8/2 or 6PB3/5.5. In some cases, it may be convenient to use the all-number notation (see fig. 1), in which the hue is a real number in (0,100] or in (0,10] (The Munsell and Kubelka-Munk toolbox 2019).

The MRD are a collection of 2734 Munsell specifications, whose corresponding CIE Yxy coordinates (relative to Illuminant C) are given in (ASTM Standard D 1535-14 2014) and (Newhall et al. 1943); they are available in digital format at (Munsell Renotation Data 2019).

Fig.2 shows the subset of MRD points with Munsell value equal to 5, on the CIE xy plane: one can see the manually drawn curves representing the constant-chroma and constant-hue loci of the MRD (Newhall et al. 1943). The MRD points lie at the intersections between ovoids and radial curves. As an example, the red arc in fig.2 is the constant-chroma arc between the MRD notations 7.5R 5/12 and 10R 5/12.

The MRD and the MRD charts are the basic elements for the Munsell conversion procedures, be they graphical or purely digital. However, before analyzing the different types of procedures, we first need to define the Munsell to CIE conversion problem.

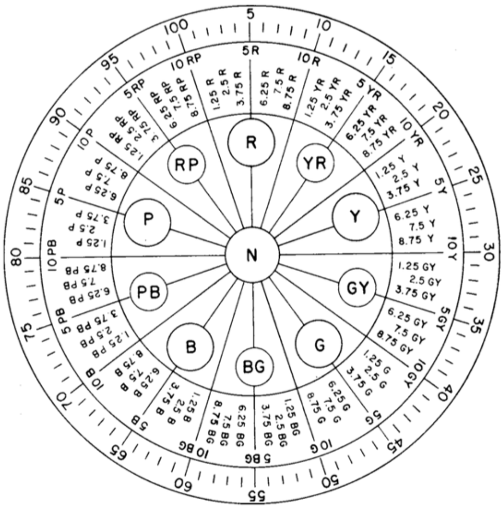

Fig. 1. Designation Systems for Munsell Hue, from (ASTM Standard D 1535-14 2014).

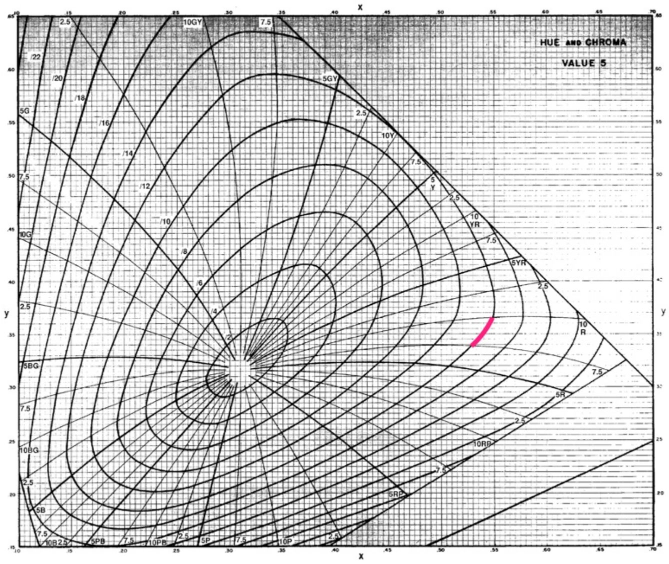

Fig 2 Munsell Renotation Data with Munsell Value 5 from (ASTM Standard D 1535-14 2014). In red the constant chroma arc between 7.5R 5/12 and 10R 5/12.

3. The Munsell to CIE conversion problem

The Munsell to CIE conversion problem can be stated as follows: given a Munsell notation with hue *pH*, value *V* and chroma *C*, find the corresponding CIE Yxy coordinates (Centore 2011):

$$\begin{aligned} Y &= f_1(pH, V, C) \\ x &= f_2(pH, V, C) \\ y &= f_3(pH, V, C). \end{aligned} \tag{1}$$

If the Munsell notation belongs to the MRD, the conversion can be easily carried out by an exhaustive search in the MRD database. Conversely, for a generic Munsell notation outside the MRD, it is necessary to rely on (1) to find the corresponding CIE coordinates.

The luminous reflectance factor *Y* depends only on the value *V*; hence, one can write $Y = f_1(V)$. A conversion table to directly obtain *Y* from *V* is available in (ASTM Standard D 1535-14 2014).

Unfortunately, useful approximations of the functions f_2 and f_3 in (1) are not available (Centore 2011). Thus, there are two possible ways to convert hue and chroma to the CIE xy parameters.

One approach is the graphical procedure outlined in (Newhall et al. 1943) and recommended by (ASTM Standard D 1535-14 2014).

The second one consists in a tri-dimensional search and interpolation process based on the MRD, digitally implemented by a computer program (Centore 2011).

When converting a generic Munsell notation, any available procedure (be it graphical or software-based) necessarily starts by transforming the closest adjacent Munsell notations having multiple-of-two chroma and integer value: for a detailed discussion of this aspect, see (ASTM Standard D 1535-14 2014), (Centore 2011), (Paglierani and Valan 2018, Conference Paper).

Once such adjacent Munsell notations have been converted, their CIE coordinates can be easily interpolated, so as to obtain the coordinates of the generic Munsell notation to convert (ASTM Standard D 1535-14 2014).

As a consequence, in the following, we will only consider (without loss of generality) the conversion of Munsell notations lying on the constant-chroma ovoids (at multiple-of-two chroma) drawn in the integer-value planes shown in the MRD charts (Centore 2011).

3.1 *The graphic conversion procedure*

The first step in the graphic conversion procedure consists in determining the chart (identified by an integer value from 1 to 9) and the ovoid in that chart (determined by a multiple of two chroma) on which the Munsell notation to convert lies. Then, one must find the arc containing such Munsell specification, delimited by the two closest MRD notations (Centore 2011).

As an example, we will consider the problem of determining the CIE coordinates of the Munsell notation 8.75R5/12, which is an intermediate color sample physically available in some editions of the MBC.

The MRD notations closest to 8.75R5/12 and with the same value and chroma are 7.5R5/12 and 10R5/12, as one can see in the zoomed chart (value 5) shown in fig.3.

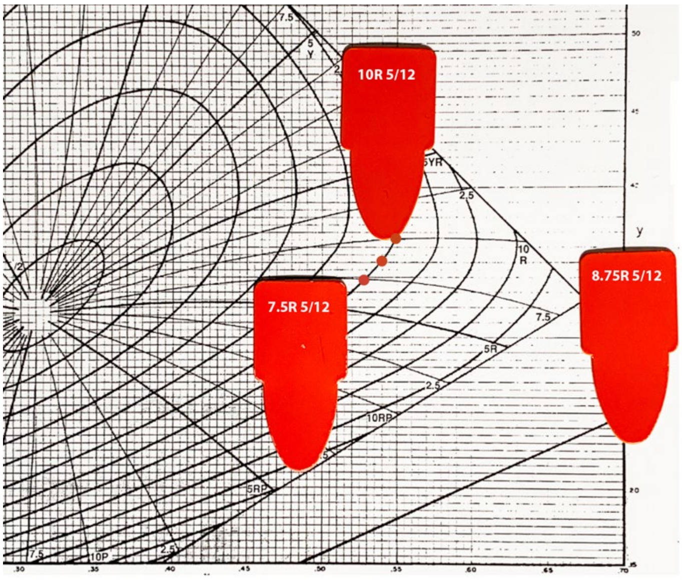

Fig 3. Munsell notation to convert (8.75R 5/12) and the two closest MRD notations with the same value and chroma.

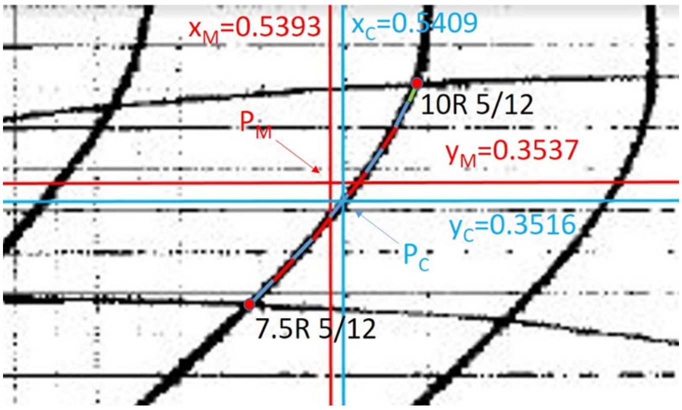

Fig 4. Graphical interpolation of the Munsell specification 8.75R5/12 obtained with PowerPoint.

Once the position of 8.75R5/12 on the arc between 7.5R5/12 and 10R5/12 is known, its xy coordinates can be straightforwardly obtained on a paper print of the chart, or digitally on a Personal Computer, e.g. using a drawing program such as PowerPoint. The searched xy values are the coordinates of the point corresponding to 8.75R5/12, read on the x and y axis of the chart.

However, determining the accurate position of such a point on the arc is less trivial. To overcome this difficulty, we need to assume a linear relationship between arc lengths in the CIE xy plane and numeric hues.

From an operating point of view, this assumption is fundamental to locate the color position in the CIE xy plane, and proceed with the interpolation process. On the other hand, the non uniformity of the CIE xy plane is well-known (Wyszecki and Stiles 2000); hence, assuming a linear relationship between hue and arc length in the CIE xy plane could give rise to observable non-linear effects, if a statistical analysis of the converted data were performed.

Nonetheless, this type of statistical analysis is out of the scope of this paper. Thus, in the following, we will accept the linearity assumption discussed above as a necessary working hypothesis, and postpone this type of investigation (together with its extension to more recent, approximately uniform color spaces such as CIELAB) to the next steps of this research work.

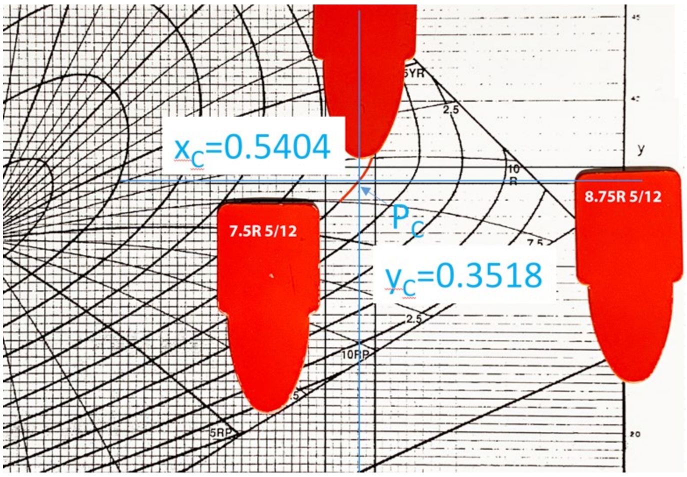

Fig. 5. Graphical interpolation of the Munsell specification 8.75R5/12 obtained with Adobe Illustrator.

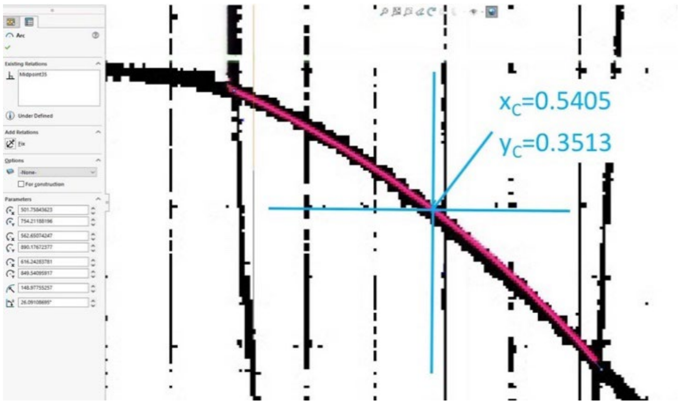

Fig. 6. Graphical interpolation of the Munsell specification 8.75R5/12 obtained with SolidWork

In the considered conversion example, the hue of 8.75R5/12 is exactly intermediate between 7.5R and 10R; hence, the searched point on the chart divides in two equal parts the arc between 7.5R and 10R.

In general, if *Hx* is the hue of the Munsell notation to convert, *H1* and *H2* the hues (*H2* > *H1*) of the adjacent MRD notations, and *L* the length of the arc between *H1* and *H2*, the semi-arc length *Lx* from *H1* to *Hx* can be evaluated as:

$$Lx = \frac{Hx - H1}{H2 - H1} L. \tag{2}$$

To evaluate (2), the length *L* of the entire arc from *H1* to *H2* is needed. Such a value can be obtained by approximating the arc as a sequence of linear segments of fixed length *Lu*. Fig.4 shows an example of arc approximation with a sequence of juxtaposed linear segments (in red and blue) of constant length *Lu*, superimposed to the arc in the MRD chart. In this case, the length *L* was obtained with Power Point. Notice that the final segment (in green) has a shorter length than *Lu*, to adjust the sequence of approximating segments to the arc. The length *Lx* of the semi-arc can be obtained in the same way.

To verify the reproducibility of the proposed graphical approach, a second operator independently (i.e. without any information coming from the previous PowerPoint-based conversion experiment) transformed the Munsell notation 8.75R 5/12 to CIE coordinates using the Adobe graphic design tool Illustrator. The output of the procedure is summarized in fig. 5.

Finally, a third operator independently performed the same conversion, using a Computer Aided Design tool, i.e. SolidWork. In this case, the lengths *L* and *Lx* were obtained by approximating the arc as a circle, by means of the built-in functionalities provided by SolidWork. The circular curve interpolating the arc and the obtained results are shown in fig.6.

As one can easily verify, even if performed with different tools and carried out by different independent operators, the results of the three conversions are very close.

3.2 *Liner/radial digital interpolation*

The MKT algorithm extends the purely linear interpolation technique suggested in (Rheinboldt and Menard 1960) by using radial interpolation where the MRD constant-chroma ovoids exhibit a higher curvature (Centore 2011).

The MKT is open source, available in Python (Colour Science for Python 2019) or Matlab (The Munsell and Kubelka-Munk toolbox 2019). Its details are thoroughly described and discussed in (Centore 2011).

The algorithm mimics the graphical procedure described in the previous paragraph by digital means, using the two MRD samples, *p*1*H*1*V*/*C* and *p*2*H*2*V*/*C*, adjacent to *pHxV*/*C*, and their coordinates (*x1*,*y1*) and (*x2*,*y2*).

To decide if linear or radial interpolation should be applied, a lookup table is used (Centore 2011). If linear interpolation applies, the algorithm transforms the hues into real number in the range (0,10]. The searched coordinates are calculated as:

$$(x, y) = \frac{H2 - Hx}{H2 - H1}(x1, y1) + \frac{Hx - H1}{H2 - H1}(x2, y2)$$

For radial interpolation, the xy coordinates (*x1*,*y1*) and (*x2*,*y2*) of *p1H1V*/*C* and *p2H2V*/*C* must be first transformed into polar coordinates (*θ1*,*ρ1*) and (*θ2*,*ρ2*), with respect to the Illuminant C neutral point. Moreover, MKT transforms hues in Temporary Hue Angles, *TH*. The polar coordinates of *pHxV*/*C* are finally obtained as:

$$(\theta, \rho) = \frac{TH2 - THx}{TH2 - TH1}(\theta 1, \rho 1) + \frac{THx - TH1}{TH2 - TH1}(\theta 2, \rho 2)$$

and then transformed in xy coordinates.

3.3 *Spline interpolation*

The spline interpolation technique is described in (Paglierani and Valan 2018, Conference Paper). While the MKT algorithm uses only the adjacent points to interpolate a notation, the spline technique can use all the points belonging to a constant chroma ovoid (or even to constant-hue radial lines). The basic idea behind this algorithm is to express any MRD chart ovoid, identified by constant integer value *V* and multiple-of-two chroma *C*, by approximate mathematical formulas, which can be expressed in parametric form as:

$$x = g_1(H), \qquad y = g_2(H) \tag{3}$$

To evaluate the xy coordinates, we substitute in (3) the unknown functions g_1, g_2 with their approximations $\hat{g}_1$ and $\hat{g}_2$, obtained by applying spline interpolation to the subset of MRD points lying on the found ovoid (Press et al. 2007). To this end, one must collect the xy coordinates (*xi*,*yi*) and numeric Munsell Hues *Hi* of the MRD points lying on the identified ovoid in the vectors *x*=[*x1*,…*xN*]T, y=[*y1*,…*yN*], and *H*=[*H1*,…*HN*]. The desired xy coordinates are:

$$x = \hat{g}_1(H), y = \hat{g}_2(H)$$

where:

$$\hat{g}_1 = spline(H, x), \hat{g}_2 = spline(H, y),$$

while:

$$q = spline(t, z)$$

represents the cubic spline interpolant polynomial *q* of the data values *z*, at data sites *t* (Press et al. 2007).

Since the ovoids are closed curves, it is necessary to extend and wrap the vectors *x*, *y* and *H*, repeating the initial and final values (Paglierani and Valan 2018 Conference paper). The interpolation procedure is finally performed by (6)-(7), with the extended vectors *x*', *y*' and *H*' taking the place of *x*, *y* and *H*.

4 Numerical and experimental results

To analyze the performance of the considered graphical and digital interpolation techniques, a set of intermediate MBC color chips were used. Such sequence of Munsell specifications is reported in the first column of Table I; for the sake of simplicity, they all have value equal to 5 and chroma equal to 12.

For each Munsell specification, the corresponding CIE xy coordinates were obtained by applying the graphical procedure described in Section 3.1, using PowerPoint. The results are shown in the second column of Table I, while the third and fourth columns report the corresponding digitally interpolated values provided by the MKT and the Spline algorithms, respectively. Finally, the fifth column contains the results of the measurements performed on the physical chips of the MBC, obtained by using the Konica-Minolta spectrophotometer CM-2600d (Konica Minolta 2014).

Preliminary tests showed a high repeatability of the CIE coordinate measurements. For this reason, the measurement process was repeated four times for each physical chip; the fifth column in Table I shows the average measured coordinates. The standard deviations obtained from all the performed measurements resulted very similar for the *x* and *y* coordinates, close to $\sigma x = \sigma y = \sigma = 0.0001$.

As an example, the CIE coordinates obtained by the graphical procedure and the averaged coordinates provided by the Konica Minolta instrument are shown in fig.4, in light blue and red, respectively.

From Tab. 1, it was possible to evaluate the geometric distance between the CIE coordinates provided by the different approaches. The final results are reported in Tab. 2. As one can immediately notice, the maximum geometric distance occurs between graphical interpolation data and instrumental measurements. Conversely, the data obtained by the two digital algorithms achieve the minimum distance. Finally, one can observe that, quite surprisingly, the digital interpolation techniques, though based on the MRD, provide data closer to the spectral measurements than to the graphical interpolation results.

5. Conclusions

This paper has presented a graphical technique, based on digital drawing tools, to perform Munsell to CIE data conversion. The procedure has been described through a practical example, in which a Munsell notation has been transformed to CIE coordinates by three independent operators using different digital tools, so as to verify its reproducibility.

The procedure has also been applied to a set of color chips available in some editions of the Munsell Color Book, whose CIE coordinates are not known. Moreover, instrumental measurements of the chip CIE coordinates have been performed, using a Konica Minolta CM-2600d spectrophotometer.

The obtained data have been analyzed and discussed. The interpolation results provided by the two digital interpolation techniques result quite close. The maximum geometric distance is observed between graphical interpolation data and instrumental measurements. The results of the digital algorithms are closer to the instrumental measurements than to the graphical interpolation results.

The presented procedure can be used to assess the conversion performance achieved by computer programs, and to further analyze the relationship between Munsell notations and CIE coordinates as enforced by the Munsell Renotation Data.

Further steps in this research activity can be observer tests to assess the linearity of interpolated data in terms of appearance, the main characteristics of the Munsell Book of Colors, and investigating the possible use of other color spaces to improve the conversion process of the Munsell notations.

Munsell Spec	**Graphical interpolation** (x,y)	**MKT interpolation** (x,y)	**Spline Interpolation** (x,y)	**Spectral Measurements** (x,y)
8.75R 5/12	0.5409, 0.3516	0.5384, 0.3519	0.5395, 0.3520	0.5393, 0.3537
1.25YR 5/12	0.5442, 0.3613	0.5485, 0.3784	0.5502, 0.3789	0.5467, 0.3786
3.75YR 5/12	0.5464, 0.4027	0.5455, 0.4025	0.5454, 0.4025	0.5487, 0.4021
1.25G 5/12	0.2694, 0.5421	0.2616, 0.5431	0.2650, 0.5431	0.2644, 0.5360
3.75PB 5/12	0.1875, 0.1852	0.1856, 0.1875	0.1847, 0.1874	0.1849, 0.1899
6.25PB 5/12	0.2016, 0.1837	0.2039, 0.1839	0.2029, 0.1840	0.2082, 0.1933
3.75RP 5/12	0.3829, 0.2422	0.3831, 0.2424	0.3829, 0.2424	0.3817, 0.2399
6.25RP 5/12	0.4166, 0.2600	0.4172, 0.2602	0.4163, 0.2599	0.4202, 0.2607

Tab. 1. Simulation and Experimental Data

	Graphical interpolation	MKT interpolation	Spline Interpolation	Spectral Measurements
Graphical Interpolation	x	0.0070	0.0069	0.0084
MKT Interpolation	0.0070	X	0.0015	0.0051
Spline Interpolation	0.0069	0.0015	x	0.0053
Spectral Measurements	0.0084	0.0051	0.0053	x

Tab. 2. Interpolation technique comparison via geometric distance

6. Conflict of interest declaration

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this manuscript. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors

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