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Elaborating a Change Process Model for Elementary Mathematics Teachers' Beliefs and Practices

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This report focuses on the processes of change in beliefs and practices experienced by practicing elementary school teachers during a sixteen-session course using two of the modules from the *Developing Mathematical Ideas (DMI)* materials. We identify a collection of six metaphors for knowing, learning, and teaching mathematics to succinctly describe and categorize teachers' beliefs. We present three case studies representing a continuum of change in beliefs observed among the participants of the DMI course. We relate this continuum both to the beliefs teachers brought to the course and to their degree of engagement in various components of a change process model. Using this model of change, we analyze the ways in which teachers expressed interest in change, problematized their beliefs, experimented with possible solutions, and reflected on experimental results leading to changes in beliefs and practices. The results of our analysis indicate that variations in change among participants can be explained by variations in their levels of engagement in particular elements of the change model by the learning activities of the DMI course.

Paradigmatic and systemic changes recommended in the standards of the National Council of Teachers of Mathematics (NCTM, 2000) and supported by teacher enhancement grants from the National Science Foundation involve complex issues associated with teacher beliefs, teacher knowledge, and changes in teaching practices. Friel and Bright (1997) integrated and summarized a number of issues of particular importance for these changes, including: (1) working to change teacher beliefs is the starting point for most professional development, and (2) changing beliefs and experimenting with teaching practices are intertwined in an iterative process.

In discussing programs that are successful in changing beliefs, Richardson (1996) singled out "programs that approach learning to teach in a constructivist manner" as being particularly successful in "engaging their participants in examining and changing their beliefs and practices" (p. 113). Among the common features of successful constructivist professional development programs which she listed, two are of particular importance for this study: (1) participant's beliefs and understandings are a major concern in the program, and (2) the goal of the program is to facilitate participants' understanding of their beliefs as well as their considering and experimenting with new beliefs and practices. Richardson (1996) also noted, "attitudes and beliefs are important concepts in understanding teachers' thought processes, classroom practices, change, and learning to teach" (p. 102).

In the context of an innovative elementary mathematics professional development program using two modules from *Developing Mathematical Ideas* (DMI) (Schifter, Bastable, & Russell, 1999a and 1999b), this study (1) provides a framework for organizing, describing, and analyzing teachers' beliefs about knowing, learning, and teaching mathematics; (2) explores a sequence model for the processes of teacher change; and (3) reports effects of a specific teacher development program on participating teachers' beliefs and practices.

Conceptual Framework

As we frame the discussion that follows, we draw from research on teachers' beliefs, propose metaphors that summarize teachers' beliefs and mental models for teaching mathematics, and summarize research on the processes of changing beliefs through professional development experiences. This literature frames our decision to use case-study methodology to study the effects of the course on teachers' beliefs and practices as well as our presentation and interpretation of the results.

Teachers' Beliefs and Practices

Although it seems obvious that teachers' personal beliefs and experiences effect how they teach, Thompson (1992) pointed out that there are numerous ways of interpreting the idea of a belief. We focus here on the beliefs of teachers in our study in three areas: their views of what it means to know mathematics, to learn mathematics, and to teach mathematics.

A useful way of talking about such beliefs is the approach taken by Bullough (1992) and his associates (Bullough & Stokes, 1994). Bullough discussed *metaphors* as mental images or models that briefly summarize the elaborate and complex theories, assumptions, and understandings upon which people act. He used the notion of personal teaching metaphors as one image that helps teachers discuss and reflect upon their assumptions about teaching and their roles as teachers. Bullough & Stokes (1994) discussed at some length one particular metaphor of a teacher as "'one who knows,' or master" and students as "disciples, imitators of higher authority." They went on to suggest that "for the master, teaching is telling," knowledge is "fixed and stable," and the teaching and learning process is "a matter of compelling others to dance to the beat of the master's drum" (p. 201).

This master teaching metaphor is coherent with traditional views of knowing and learning mathematics. Kuhs and Ball (1986) summarized this view as follows: "rules are the basic building blocks of all mathematical knowledge and all mathematical behavior is rule-governed; knowledge of mathematics is being able to get answers and do problems using the rules that have been learned; computational procedures should be automatized; it is not necessary to understand the source or reason for student errors; further instruction on the correct way to do things will result in appropriate learning; in school, knowing mathematics means being able to demonstrate mastery of the skills described by instructional objectives" (p. 2, bullets removed).

This description has much in common with Lampert's (1990) description of the cultural assumptions about mathematics in schools: "*doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher" (p. 32). Discussions by Stigler and Hiebert (1999) and Ma (1999) suggest that this traditional view still dominates the culture of mathematics teaching.

In contrast to the master metaphor, research from a social constructivist perspective (Vygotsky, 1978; Bakhtin, 1981; Rogoff & Wertsch, 1984; Wertsch, 1985, 1991; Tharp & Gallimore, 1989; Rogoff, 1990) provides an alternative model for teaching centered on *joint productive activity* and *instructional conversation* (Tharp, 1997; Dalton, 1998). This view of teaching, which could be summarized by a *facilitator* metaphor, is founded on alternative beliefs about what knowledge is, how it is learned, and the role of teachers in guiding and facilitating learning. Learning from this perspective involves social interactions among students and more capable others (teachers and peers) working together in authentic, goal-directed activities (joint productivity) and conversing about what they are doing, thinking, and learning (instructional conversations). The role of the teacher during these instructional conversations is to listen carefully to what students are saying and to use questioning techniques to guide students' thinking as they develop understanding and connect it to individual and community knowledge (Tharp 1997). This view of teaching is clearly evident in the NCTM *Professional Standards for Teaching Mathematics* (1991).

In Table 1 [see Appendix D], we provide a set of six metaphors that summarize the traditional and standards-based views of school mathematics. The traditional view includes (1) a *toolbox* metaphor for knowing mathematics, (2) a *behaviorist* metaphor for learning, and (3) the *master* teaching metaphor. The standards-based view of school mathematics includes (1) a *flexible problem solving with understanding* metaphor for knowing mathematics, (2) a *social constructivist* metaphor for learning mathematics, and (3) a *facilitator* metaphor for teaching mathematics. For us, these two sets of metaphors summarize the "from what" and "to what" of the current mathematics education reforms.

Processes of Changing Beliefs and Practices

In the case of school mathematics, changing from the traditional beliefs and practices summarized by one set of metaphors to fundamentally different standards-based beliefs and practices summarized by another set of metaphors constitutes a revolutionary paradigm shift rather than incremental change. Beginning to share new exemplars for what it means to know mathematics, to learn mathematics, to teach mathematics, and to practice in this community is a part of the basic approach of both the NCTM Standards and the DMI materials. As we examine the processes for encouraging and supporting these changes, we can benefit from research on learning and professional development of teachers.

The same research that justifies the shift to social constructivist views of learning for students in school mathematics supports a shift in designing professional development for teachers. Social constructivist views of learning support the perspective that changes in teachers' beliefs will be experiential, developmental, and gradual. This is consistent with Guskey's (1986) model of the process of teacher change (see Figure 1 [see Appendix D]), which claimed that changes in teachers' beliefs (Step 4) followed improvements in student outcomes (Step 3) produced by changes in classroom practices (Step 2) teachers made in response to professional development experiences (Step 1).

Recognizing the complexity of teacher change, Clarke and Hollingsworth (2002) recently reviewed Guskey (1986) and other models for teacher change (Johnson & Owen, 1986; Lappan et al., 1988; Cobb, Wood, & Yackel, 1990; Clarke & Peter, 1993) and proposed what they called an interconnected model of teacher professional growth. Describing four domains affecting a teacher's world (external, personal, practice, and consequence), they focused on two processes of change: reflection and enactment. Their model particularly emphasizes how these two mechanisms of change produce particular *change sequences* affecting these four domains, allowing for development of what they call *growth networks*. Although they thoroughly elaborated their model with examples from their empirical studies, they did not provide examples for how their model could be used to analyze and explain variations in change across individuals.

In searching for a model that is sufficiently detailed to explain variations across participants, we noticed that the Guskey (1986) model could be used to apply a change sequence to Dewey's (1933) model of reflective thought, which included five phases or non-hierarchical elements, including: problematizing the situation and recognizing the conditions of the problem; recognizing possible solutions to a problem; generating hypotheses for possible solutions to the problem; reasoning about these hypotheses to determine their potential for success; and testing one or more of these hypotheses in the context of the problem.

To problematize is to recognize a situation as a problem and to acknowledge the conditions in the situation that affect the problem. Problematizing current practices is required to translate general interest or curiosity into an interest in changing practices. In applying Dewey's model to preservice teacher education, Mewborn (1999) emphasized the importance of problematizing teaching situations. If problematizing is not sufficient to develop interest in complex fundamental change then interest narrows to simple incremental change within current paradigms and practices. Obviously, unless a situation is problematized, it is difficult to formulate hypotheses and test possible solutions.

Examining potential solutions is relatively simple for incremental changes and relatively complex for fundamental changes. Incremental change poses only minor changes in strategies within current metaphors as possible solutions. Fundamental change requires development of understanding of new principles, metaphors, and curriculum needed to support major changes in practices needed to solve more fundamental and more complex problems. Similarly, interest in incremental improvement leads to simple experiments with minor changes, while interest in fundamental changes leads to complex experiments with major changes.

Dewey's (1904/1965, 1933) arguments for reflective thinking support the notion that teachers need to develop habits of reflection to be able to do the level of problematizing needed to improve their teaching and students' learning. He argued that teachers who avoid reflective thinking develop an intellectual dependency on and interest in explicit directions from others on what and how to teach. This dependence can reinforce the master metaphor and focus teachers' interest on acquiring additional activities and teaching strategies to be accommodated within existing practices. This could support minor, incremental changes in teaching, but would not lead to fundamental changes in practices that produce big benefits in student learning, such as understanding mathematics.

Both Dewey and Guskey also discussed elements of *interest* in change. Dewey (1933) mentioned curiosity as one of three personal resources essential for reflective thinking. Guskey (1986) was more specific, claiming that most teachers engage in professional development because of their interest in students and their desire to become better teachers to the benefit of their students. This desire to improve student outcomes provides both the basis for problematizing current practices and for changing beliefs when evidence of improved student outcomes can be tied to specific changes in teaching practices.

Again considering both Dewey (1933) and Guskey (1986), reflecting on teacher experiences and student outcomes leads to decisions about whether experiments were successful or not for improving students' learning and were practical or not for long-term use. Success then leads to changes in beliefs; failure leads to ending the experiment, reversion to previous practices, and no changes in beliefs. The complexity of the needed reflection depends on the complexity of both the problem posed and the solutions attempted in the teaching experiments.

Summary

The literature summarized above suggests to us that two important characteristics would mediate the effects of the DMI course: (1) beliefs about knowing, learning, and teaching mathematics; and (2) sequenced processes for changing beliefs and practices. These two themes framed our exploration of variations in the impact of the DMI course on teachers' beliefs and practices. For the purposes of this study, we have chosen to formulate and empirically support a time-sequenced (and therefore linear) process model that supports the detailed analysis of variations in teacher change yet fits within Clarke and Hollingsworth's more global (and therefore interconnected and iterative) model of professional growth as a specific *change sequence* between two domains (the personal domain and the domain of professional practice).

Integrating the frameworks for change from Dewey and Guskey provides the components of a model for analysis of the essential elements and processes of changing beliefs, including:

- Teachers' interests in change;
- The extents to which they problematize current practices and pose possible solutions;
- Their activities in exploring and testing these alternative practices; and
- Their reflective analyses of the benefits of these changes for students, leading to lasting changes in beliefs and practices.

Consequently, this study focuses on (1) describing the beginning and ending beliefs of participants in the DMI course (including summarizing those beliefs with metaphors for knowing, learning, and teaching mathematics) and (2) elaborating and testing a sequenced process model that is useful for explaining the variability in the documented changes in these teachers' beliefs.

Methodology

The DMI Course and Participants

Thirteen inservice elementary teachers participated in a one-semester university course using the *Developing Mathematical Ideas* (DMI) materials developed at the Educational Development Center. Participating teachers were employed in two local school districts. Their teaching experience ranged from 1-32 years and averaged about 10 years. Their teaching assignments were distributed across grades 1-5, with one teacher in a K-6 mathematics lab.

The course used two DMI modules: *Building a System of Tens* (Schifter, Bastable, & Russell, 1999a) and *Making Meaning for Operations* (Schifter, Bastable, & Russell, 1999b). The big ideas in *Building a System of Tens* focused on understanding place-value and *Making Meaning for Operations* focused on understanding addition, subtraction, multiplication, and division operations. These two DMI modules supported a series of sixteen three-hour class sessions designed to help teachers think about these particular *big ideas* of mathematics and to examine how children learn to understand those ideas. The essence of the course experiences included "a deep exploration of mathematics content—including the base-10 structure of our number system, the meaning of operations, and methods for calculating with multi-digit numbers and fractions—as well as analyzing children's thinking about that content" (Davenport, 2001, p. 6).

The DMI materials were designed to help teachers learn additional mathematics content and make mathematical connections, appreciate students' thinking and learn how to foster such thinking, and analyze mathematics lessons and activities to uncover the mathematics students will learn. At the heart of the DMI materials are: (1) sets of classroom episodes (cases) illustrating student thinking (as described by the students' teachers), (2) various learning activities providing opportunities for participants to reflect on their own and their students' understandings of mathematics, and (3) assignments providing opportunities for participants to reflect on teaching mathematics. Participants collected their written work and responses from the facilitator in a portfolio, which provided another tool for reflection.

The DMI materials provided a variety of models for course organization. The course in this study met one evening each week for a full semester and was led by the first author as facilitator. One major role of the facilitator was to organize and moderate small-group and whole-class discussions about: (1) focus questions about the written cases of classroom episodes; (2) videotapes of mathematics classrooms and clinical interviews; (3) samples of student work from teachers' classrooms; and (4) research reports related to students' thinking. The teachers also (5) explored the mathematical ideas in the videotapes; (6) planned, conducted, and analyzed clinical interviews of their own students; (7) wrote additional cases about their students; (8) experimented with the ideas discussed in class while teaching in their own classrooms, and (9) wrote personal reflections.

Data Collection and Analysis

Data for this study were taken from (1) audio taped whole-class and small-group discussions during the course; (2) field notes taken by the authors; (3) written materials collected from the teachers as part of the course; (4) a group interview with participants conducted by an independent evaluator following the course; (5) a post-course observation of participants' teaching practices in their regular classrooms; and (6) a post-observation interview on changes in practices resulting from the DMI course.

We audio taped each class session. During small group discussions, the third author usually sat with a target group and recorded their conversation. These audiotapes were transcribed for analysis. We used our field notes to clarify and provide context for the recorded conversations.

Analysis of the data proceeded in a manner consistent with a naturalistic inquiry approach (Lincoln & Guba, 1985). First, two researchers read the transcripts independently and identified emergent themes. They paid particular attention to teachers' beliefs (especially images and metaphors for teaching) and participation in processes of change. As these researchers discussed their reading of the transcripts, common themes began to emerge and they developed codes for these themes. Second, these same researchers re-read the transcripts and coded the conversations according to the emergent themes. The next step in the analysis of the data involved the isolation and validation of the major themes wherever they appeared in the data by triangulation across the various data sources and across time. Lastly, these themes were used to analyze all of the other data from the study.

At the end of the semester following the conclusion of the DMI course, the second author observed several of the DMI course participants' classroom teaching and then interviewed them about changes in teaching practices they attributed to the DMI course. These interviews were audio taped and transcribed. Some of the observed teaching episodes were also audio taped using a wireless microphone worn by the teacher. Field notes were taken for all observations, providing the primary data source for those cases in which teachers declined to wear the wireless microphone. This data provided an additional source for teachers' reflections and for triangulation and confirmation of the conclusions from the primary data collected during the DMI course.

Results

As we analyzed the data we noticed that the impact of the course varied across the participants. To illuminate this variation, we present three cases of individual teachers (indicated by the pseudonyms Christine, Linda, and Paula). In many ways, the cases of Christine and Paula represent the two extremes of the course's impact, and the effects of the course on Linda are similar to those of the majority of the participants. These three cases also articulate the variety of images and metaphors held by the participants and illuminate the coherence in their mental models. These three individual stories are largely self-narratives by the teachers that we have pieced together from the various data sources indicated. We selected these statements for their consistency across time and data sources.

The descriptions of these three cases (located in the Appendices) are organized around the framework for changing beliefs and practices that emerged from the analysis of the data and the review of the literature: interest in change, problematizing and posing solutions, exploring/testing alternatives, and reflective analysis of benefits and changing beliefs and practices. This section includes brief descriptions of these teachers' beginning beliefs as well as summaries of our claims about these teachers' changes in beliefs and practices.

Christine

At the time of the DMI course, Christine was teaching third grade for the second consecutive year. She had previously taught for ten years in the resource room with students she characterized as two or three years behind grade-level in mathematics.

Beginning beliefs. Christine spoke about her mathematical background and beliefs on several occasions, which clearly indicated a toolbox perspective on knowing mathematics, a behaviorist view of learning mathematics, and a strong commitment to the master metaphor for teaching mathematics, primarily because she had been taught that way. She believed mathematics should focus on skills and facts, expressed the goal of getting correct answers using procedures memorized to the point of automaticity, and believed that standard algorithms were most efficient both for her and for her students. Christine was comfortable with direct instruction of algorithms and believed that it was "the best way" to teach mathematics. This approach had been successful for her as a student, and it provided a foundation for her confidence as a successful mathematics teacher. For example, she said: [I have thought about] why I like algorithms so much. [It's because] I was taught that way. I am comfortable using an algorithm, and I am good at it (Portfolio Assignment 1.7).

See the Case of Christine (Appendix A) for evidence of her engagement in the change processes.

Changes in beliefs and practices. On the whole, the evidence in the Case of Christine indicates that her view of mathematics changed very little as a result of the DMI course. She had enlarged her view of what it means to do mathematics somewhat by allowing invented strategies into her toolbox, but she made little progress as a result of the course toward revising her preferences for behaviorist learning and the master metaphor for teaching. She learned how to ask students to explain their solutions and allowed some temporary informal inventing, but only if it led students to the traditional algorithms which she continued to emphasize through her direct instruction. (See Table 2 [see Appendix D] for a comparison of Christine's beginning and ending beliefs and practices.).

Linda

Linda had taught kindergarten for 16 years and was teaching first grade for the first time during the DMI course.

Beginning beliefs. At the beginning of the course, Linda embraced the traditional algorithmic emphasis in elementary mathematics and felt that it was fast and accurate. She noted that she was able to perform mathematical tasks and arrive at correct answers, but had not developed meaning for the mathematics she had learned to do. Because Linda's beliefs about mathematics were firmly based in algorithmic processes, she was surprised at first by the variation in students' thinking exhibited in the DMI materials. Initially, Linda emphasized direct instruction; she believed that the traditional algorithms needed to be taught directly.

See the Case of Linda (Appendix B) for evidence of her engagement in the change processes.

Changes in beliefs and practices. The evidence shows that during the DMI course, a major focus for Linda became a new view of number—what she called number sense. She came to believe that differences among students' strategies for solving number problems in sensible ways indicated real understanding. She came to desire her own students being able to visualize number in the same ways as the students on the DMI videotapes. As part of this, she wanted her students to be able to explain their thinking and be flexible in working with numbers. Although she came to value invented algorithms as useful for mental computation, she still placed some importance on speed and efficiency using conventional algorithms.

Linda pushed herself to solve problems in non-traditional ways and began to enjoy talking to others about her own mathematical ideas. She began to value these new ways of thinking and started to think she had been cheated by the narrow focus of her early mathematical experiences. By the end of the course, she had expanded her view of learning to include children being able to think, explore, take risks, and struggle with a problem in coming to understand mathematics.

During the course, it became apparent to Linda that children who have developed number sense can approach problems in a variety of ways. Thus she began to value letting children explore their own ways of approaching problems as a way to help them "grasp a concept." At the same time, she felt that this exploration needed "some structure and explanation" and that direct instruction of algorithms still had its place. By the end of the course, changes in her goals for learning allowed changes in her teaching practices. Her expectations had changed from recall of number facts to explaining or modeling answers to story problems that allowed her to assess errors in students' understanding. She began to focus more on questioning individual students' problem solutions and to keep track of their strategies. In order to do this, she began to keep detailed notes on her students daily, rather than just writing down a score. Linda recognized the need to become a better questioner, to help her students articulate their strategies and understandings and explain their thinking processes. Finally, she said that she had begun to question and evaluate how she taught. She noted that she was having fun approaching teaching in this way; she found it both exhilarating and frightening. (See Table 2 [see Appendix D] for a comparison of Linda's beginning and ending beliefs and practices.)

Paula

Paula had been teaching fifth grade for six years and had previously taught grades two and four.

Beginning beliefs. Paula enjoyed mathematics and had taken other courses to help her become a better mathematics teacher. She entered the course with a view of mathematics as a mixture of the traditional toolbox of algorithms and sense making. She said she valued critical thinking and problem solving skills and wanted her students to "understand what they are doing," nevertheless she expressed a concern that her fifth grade students were not proficient in the traditional algorithms. She questioned the traditional emphasis on algorithms, even though she was then teaching from a textbook that emphasized direct instruction of standard algorithms. She expressed the concern that if the goals of mathematics instruction included critical thinking and problem solving, more direct instruction in procedural skills would not help meet those goals. This led her to question the efficacy of the traditional master teaching methods she was still using, but there was no evidence that she understood that achieving these additional learning goals required social constructivist learning processes and alternative teaching methods.

See the Case of Paula (Appendix C) for evidence of her engagement in the change processes.

Changes in beliefs and practices. The evidence shows that she developed a deeper and more interconnected view of mathematics and adopted this as a goal of her instruction. She also valued an increased focus on conceptual understanding. This development was supported by her personal mathematical problem solving in the course. She noted that she was becoming more flexible in her problem-solving approaches, and this in turn allowed her to value flexibility in her students. She articulated this as a change in her philosophy about knowing mathematics.

Paula's beliefs about learning grew deeper and more interconnected. She began to focus more on students' thinking processes and variations in thinking among students evidenced through dialogue and written representations. She viewed this as children taking back the responsibility for their learning.

The changes in Paula's beliefs about teaching focused on movements away from the standard practice of following a page-by-page sequence in the textbook and a concern about "covering the material." She felt that she had moved toward a focus on individual children's thinking, cautious planning of problems, and questioning students to get at their understanding. She began to focus more on individual students' mistakes and problematic thinking in order to help them build a "bridge" to understanding. Because she wanted to press for deeper understanding, she began to be concerned with building deeper, more coherent curriculum materials. She found this approach to teaching very enjoyable, but more importantly, she felt it was a better way of educating students. (See Table 2 [see Appendix D] for a comparison of Paula's beginning and ending beliefs and practices.)

Discussion

These three cases represent a continuum of engagement in the process of change. On the one extreme, Christine started with very traditional beliefs about knowing, learning, and teaching mathematics, and made the least progress toward alternative beliefs and practices. More typical of the results of course participants, Linda started with very traditional beliefs and practices and made significant shifts in both, although only beginning to understand the coherent alternative provided by the standards-based perspective. At the other extreme, Paula started the course already wondering about children's understanding of mathematics and made the most real progress by connecting additional complexity to her previous understanding and developing a robust and coherent philosophy of standards-based mathematics education.

The purpose of this section is to examine how varying levels of engagement in the process of change during the DMI course, linked to various beginning beliefs, resulted in these variations in change. In the process of this analysis, the applicability of the analytical framework proposed from the review of literature and used to organize the descriptions of the cases will also be demonstrated. This analysis is organized in terms of the following four variables in the change process: (1) interest in change, (2) problematizing/posing solutions; (3) exploring/testing alternative practices, and (4) reflective analysis/changing beliefs and practices.

Interest in Change

By enrolling in the course, all of the participants expressed an interest in learning something that could possibly improve or add greater variety to their teaching, constituting at least a low level of interest in change. Most of these interests were expressed in the context of the participants' initial expectations for the course. These expectations focused largely on acquiring immediately applicable "teaching-as-telling" strategies, "exciting" activities, or "make-it-take-it" materials consistent with incremental change in traditional beliefs and practices. Although a few participants came with a desire to explore ways of enhancing their own mathematical understanding and that of their students, most came looking for better ways to be a master teacher rather than for fundamentally different ways to think about mathematics knowing, learning, and teaching.

At first, these expectations created a mismatch between what the course actually offered and what students expected. The critical point here is that almost all of the participant's expectations for the course were closely aligned with the traditional beliefs of knowing, learning, and teaching mathematics. The fact that some teachers made progress in changing their beliefs given these initial expectations is encouraging.

From examining the data, we recognized that these teachers' interest in change, as it originated at the beginning of the course or developed during the course, involved three factors: (1) curiosity, (2) recognition of differences in beliefs and practices, and (3) dissatisfaction with current beliefs and practices.

Curiosity. Some teachers enrolled in the DMI course with high initial levels of curiosity, while others increased in curiosity during the early course sessions. In both cases, this curiosity encouraged their becoming more dissatisfied with the status quo, more aware of alternatives, and, consequently, more interested in change. For example, Linda expressed curiosity about the ways in which some of her students thought about numbers in ways that she could not. At first, she could not understand the thinking of those students. This led her to become curious about their thinking and to question, "What is it that they know? What are they trying to do? What is in this problem that is causing them to think this way?" She became anxious to explore children's thinking and her own understanding of mathematics in order to answer some of these questions. This contributed to her increased interest in change.

Recognition of differences in beliefs and practices. This factor in developing greater interest in change became evident in the discourse during class sessions that verbalized, compared, and contrasted the variety of beliefs and opinions about teaching practices held by course participants (e.g., the value of using manipulatives) or evident in the DMI cases (e.g., facilitating invention of strategies from conceptual understandings). Teachers in the course had many opportunities to see and discuss these differences of opinion about various aspects of teaching. Recognition of these differences also contributed to increased interest in change.

For example, regarding the value of using manipulatives, Christine expressed the opinion that "when you do use manipulatives you've got to drill and drill so that they understand exactly what those manipulatives are representing." Paula expressed a different view. "I read a research article saying manipulatives are not the answer—be careful and sure that they understand. The emphasis here is on the child's understanding and the meaning the child makes of the materials they use" rather than on the use of the manipulatives, which can be proceduralized without understanding. This discussion situated one of the differences between Christine's and Paula's beliefs about knowing mathematics: Christine used manipulatives in particular ways to further her goals of learning procedures and Paula recognized that manipulatives had to be used in different ways to meet her goal of developing conceptual understanding.

Regarding the example of facilitating children's invention of strategies from understandings as evidenced in the DMI cases, Christine analyzed the teaching practices evident in those cases from the perspective of standard algorithms as the best pathway to correct answers, with her interest in students' invented strategies only as temporary alternative pathways to the same destination, pathways with which particular students may be more comfortable. However, she paid little attention to the specific thinking and understanding students used to invent those strategies. Linda, in comparison, was fascinated by the student thinking in the cases and responded that she wanted to become better at helping students communicate their understanding by improving her questioning techniques focused on flexible problem solving with understanding. Paula demonstrated in her response to the cases that she recognized the connection between classroom discussions of students' thinking and their understanding, and she could even see the logic in students' incorrect strategies.

Dissatisfaction with current beliefs and practices. As with curiosity, teachers can come with this factor of interest in change or it can be created in the initial course sessions. More important than curiosity or awareness of differences, this factor is the essential element of an interest in change sufficient to lead to problematizing current beliefs and practices. This factor originates as dissatisfaction with the outcomes of teaching, not yet sufficiently detailed as to have identified the particular problem or cause of the dissatisfaction. For example, engaging in mathematical problem solving and examining children's thinking and understanding of mathematics during the DMI course led Linda and Paula to express dissatisfaction with their own early mathematical experiences. Linda recognized that she had not developed an understanding of the mathematical procedures she had been taught. Paula had found mathematics enjoyable, but recognized that not all of her friends "got it" and was thus dissatisfied with some aspects of her experience. Christine, in comparison, expressed satisfaction with what she had been taught and how she had learned, and was content with teaching her students the same mathematics in the same manner.

In summary, Christine expressed a low level of interest in change, focused primarily as adding some "spice" to her teaching. Linda expressed a moderate level of interest in change, primarily curiosity about children's thinking that was different from her own and improving her questioning techniques to facilitate students' communication. Paula expressed a high level of interest in change, centered on sufficient dissatisfaction with traditional practice for her to be interested in exploring the rationale behind particular alternative teaching practices intended to building conceptual understanding. (See Table 3 [see Appendix D] for a summary of our appraisal of these three teachers' levels of interest in change as low, moderate, and high, respectively.)

Problematizing Current Beliefs and Practices and Posing Possible Solutions

One element of problematizing current beliefs and practices is becoming aware of possible alternatives. The task of incremental change is primarily experimenting with minor improvements to practices within existing beliefs and reflecting upon the results of these incremental changes in practices. However, without problematizing current beliefs and practices, the need for revolutionary change is unrealized and the focus remains on incremental improvement. Two examples are provided here of how revolutionary alternatives problematize traditional beliefs and practices: (1) comparing the learning of algorithms with developing conceptual understanding, and (2) comparing "fast and accurate" computational skills with flexible problem solving.

Comparing algorithms to understanding. Paula questioned the traditional emphasis on algorithms. She expressed a concern that her fifth grade students were not proficient in the traditional algorithms. This led her to question the efficacy of traditional teaching methods. She also expressed the concern that if the goals of mathematics instruction included critical thinking and problem solving, more instruction in procedural skills would not help meet those goals. She even felt constricted by the need to continue to spend time teaching algorithms to satisfy parents and prepare students for standardized tests. In comparison, Linda translated her sensitivity about her own supposed lack of understanding into a greater interest in her students coming to thoroughly understand mathematics and be able to clearly communicate that understanding.

Comparing fast and accurate to flexible. Recognition that the traditional elementary school emphasis on "fast and accurate" computation (automaticity) and direct instruction in "one way" to solve particular problems was challenged by a more interconnected and conceptual view of mathematics, including understanding of concepts, invented strategies, and flexibility in solving nonroutine problems. Paula came to understand the connection between depth of conceptual understanding and degree of flexibility in problem solving strategies. Linda also recognized that children who understand concepts of number show greater flexibility in problem solving and develop very efficient invented strategies for mental computations. She reinforced this perspective as her own understanding grew during the course and she began to develop greater flexibility in her own problem solving. She admitted that her invented mental strategies were often faster than her traditional computational methods. Linda also recognized that this shift in learning goals required a corresponding shift in teaching practices.

Recognizing problems as occurring within one's own experience requires that these problems be stated and that the conditions that affect them be examined carefully and in detail. Thorough statements of problems of practice include the contexts where they typically arise and the relationships between the stated problems and other aspects of practice. Mewborn (1999) indicated that preservice elementary teachers rarely specifically stated problems and even less frequently questioned what was problematic about particular situations for others. However, the inservice teachers participating in the DMI course frequently stated problems specifically and often questioned others about particular teaching situations in which those problems occurred. Their attempts to personally solve these problems were assisted by the readily available alternatives in the DMI materials. Absent these workable solutions, attempting to solve such Significant problems of practice would likely have been very difficult.

In summary, Christine expressed a low level of problematizing and solution posing as she remained confident in her current success with direct instruction and worried about how she could explain a variety of invented strategies without confusing her students further. However, Linda focused on how to create an appropriate environment for student exploration and sharing of strategies, reflecting a high level of problematizing knowing, learning, and teaching and posing solutions in each of those areas. Paula began to consider changes in her curriculum to provide problem situations that could develop critical thinking, problem solving, and understanding, also reflecting a high level of problematizing and solution posing in all three areas of beliefs and practices. See Table 3 [see Appendix D] for a summary of our appraisal of these three teachers' levels of problematizing/posing solutions as low, high, and high, respectively.

Exploring/Testing Alternative Beliefs and Practices

To engage in exploring/testing alternatives, the process of problematizing and posing possible solutions in the DMI course needed to create sufficient hope that the revolutionary alternatives would result in better outcomes and more satisfying results to generate the needed experimentation. Yet at this stage of the process, many of the teachers' understandings of the alternative practices were minimal. Actually experimenting with these alternatives helped them become more knowledgeable about the specifics of the alternatives.

One of the strengths of the DMI course was that it provided two sites for the experimenting/testing of solutions to problematic practice. One was the DMI classroom, where peers had conversations about elementary mathematics and children's solutions. The second site was the participants' own classrooms, where they interviewed their own students and tried out different teaching ideas. Participants' experiences suggested that these two laboratories were portable in the sense that they could remember the first one as they tried out ideas in the second, and vice versa. This provided a very rich context for reflection on practice.

Many of the participants indicated that the things they read in the cases and saw on the videotapes were actually happening in their classrooms and interviews with their students. By design, the teachers' classrooms provided laboratories for validating what was taking place in the DMI course. Having both sites available for exploration of ideas—the DMI course and individual classrooms—allowed participants to reflect on their mathematical thinking and their teaching in both venues. The extent to which participants enjoyed talking to each other about their mathematical ideas and the results of their experiments were evidenced through the participants' dialogue during class sessions and their written responses in portfolios.

Change processes in the DMI classroom involved what Clarke and Hollingsworth (2002) referred to as the teacher's personal domain, while change processes occurring in the school classrooms involved the teacher's domain of practice. Being able to problematize, experiment, and reflect in both the DMI class and one's own classroom provided an iterative element with a short turn-around time and increased the impact of the DMI course on teachers' beliefs and practices.

As expected, attempts to experiment with alternative practices were constrained by a variety of factors even after current practices had been problematized. In particular, participants' conceptions (formed during several years of school mathematics instruction) and traditional expectations of administrators or the teacher next door both served to discourage some attempts to experiment with alternative beliefs and practices.

In summary, Christine tried asking some of her students to share their thinking, but her focus in teaching mathematics continued to be on the correct answer and providing students with hints if they didn't get to the answer she wanted. Linda departed from her emphasis on direct instruction and tried letting children explore their own ways of approaching problems as a way to help them "grasp a concept." In doing so, she began to focus more on questioning individual students' problem solutions and keeping daily detailed notes on her students' strategies rather than just recording their scores for number of correct answers. Paula began to focus specifically on developing understanding; cautiously planning problems to provide a deeper, more coherent curriculum; and questioning students to get at their understanding. (See Table 3 [see Appendix D] for a summary of our appraisal of these three teachers' levels of experimenting/testing alternatives as moderate, high, and high, respectively.)

Reflective Analysis of Benefits and Changing Beliefs and Practices

We have learned that reflecting on practices and changing beliefs is a complex activity. It involves comparative analysis of the evidence of results from current and alternative beliefs and practices and making connections among these experiences. Reflecting also involves making judgments about the relative efficacy and relative satisfaction from these alternatives and making generalizations about their potential efficacy in somewhat different circumstances or contexts. In addition, reflecting involves examining the level of implementation of traditional or alternative practices for opportunities for incremental change within existing or alternative beliefs. Interest in change and problematizing are highly interactive, with new awareness of possible alternatives strengthening dissatisfactions and leading to additional problematizing of ineffective practices.

All of these elements of the change process occurred in both laboratories: the DMI course and teachers' classrooms. For some, well-developed theories emerged from their analysis of personal experiments with alternative practices. For others, the effects of the DMI course were less dramatic.

In summary, although Christine engaged in some exploring of alternative practices, her low interest in change and low problematizing of current beliefs and practices left her without a significant problem to be solved by alternative practices.

Consequently, she saw only limited usefulness for students' invented strategies and made only insignificant changes in her beliefs, curriculum, and teaching practices. She thought students' strategies were interesting and worth sharing in class occasionally, but failed to recognize them as the foundation for building students' understanding.

In comparison, Linda and Paula both recognized the results of their experiments as beneficial for students, and made significant progress in changing beliefs and practices involving knowing, learning, and teaching mathematics. Linda began to expand her students' opportunities to explore mathematics in a supportive environment and to build their number sense, particularly for use in mental computations. She increased her ability to ask good questions, rephrase what students said, and assess errors in their thinking. However, she also held on to some traditional tools and emphases in the curriculum. This represents significant progress given her moderate interest in change and her current understanding of the concepts of the standards-based perspective.

Paula's reflections on the benefits of her experiments resulted in high levels of change in her beliefs about knowing, learning, and teaching mathematics. Her teaching metaphor became that of a facilitator who poses interesting problems, uses questioning to find out what students think, and leads them to build deep, well-connected understandings. She enlarged her curriculum to include more emphasis on understanding big ideas, problem solving, and critical thinking. She also recognized that understanding occurs through problem solving, dialogue about solution processes, and creating written representations for those solutions. All of these changes were responsive to her initial interest in the whys of the standards-based perspective and her ability to develop a new personal theory of mathematics education. (See Table 3 [see Appendix D] for a summary of our appraisal of these three teachers' levels of reflecting/ changing beliefs and practices as low, moderate, and high, respectively.

Conclusions

We learned from this study that metaphors can provide concise yet powerfully descriptive summaries of widely held beliefs about knowing, learning, and teaching mathematics. Although the titles of these metaphors immediately conjure up particular images in our minds, the ways we used these metaphors still allowed us to describe additional details about specific teachers' beliefs so that our overall characterization of any particular teacher's beliefs remained true to the individual subject. Yet, for purposes of comparing beginning and ending beliefs, analyzing change, and summarizing the effects of this particular professional development experience, the level of simplification afforded by these metaphors proved to be very helpful.

Overall, it has been encouraging to see that some teachers made real progress in changing their beliefs during the DMI course, even though they were initially looking for additional strategies to use while teaching within their existing metaphors. Examining the processes through which these teachers changed their beliefs has also been a useful tool for understanding the nature of the changes and the variations across individuals.

Figure 2 [see Appendix D] shows our completed process model for describing and analyzing the variations in changes in beliefs and practices observed in the participants of the DMI course and evidenced by the three cases presented. It was clear to us that participants' beliefs about knowing, learning, and teaching mathematics were complex, varied, and interrelated. Creating and using this model has allowed us to elaborate on key elements and processes of teacher change which integrate the literature on professional development with our empirical experiences researching the effects of the DMI course.

Acceptance of redefined learning goals emphasizing flexible problem solving with understanding, such as those found in NCTM's *Principles and Standards* (2000), requires corresponding shifts toward paradigmatic theories of learning and teaching practices that are coherent with and capable of achieving those new learning goals. The results of this study suggest that one's teaching practices are unlikely to be problematized unless beliefs about knowing and learning mathematics are problematized first. Without problematizing beliefs about knowing and learning mathematics, interest tended to follow the typical pattern of remaining focused on incremental change within current teaching metaphors rather than participation in the paradigmatic changes offered by the DMI course. Important elements of interest in change, which is essential to this problematizing process, included curiosity, recognition of differences, and dissatisfaction with current beliefs and practices.

In analyzing variations in the amount of change achieved by participants in the DMI course, we observed that participants with low interest in change and/or low levels of engagement in these change processes, particularly in problematizing and experimenting, made only limited changes in their beliefs and practices. Conversely, those participants who had higher levels of initial interest and more fully engaged in each of the change processes, experienced greater changes in beliefs and practices.

Implications

Although situated in the context of a specific professional development course in elementary mathematics education, the use of metaphors as a tool for describing and categorizing complex, interconnected beliefs is generally applicable. With some modification to the metaphors for knowing, this collection of metaphors may be useful in examining teachers' beliefs about knowing, learning, and teaching in other content areas that also focus on developing conceptual understanding and thinking processes. Use of metaphors like these can facilitate analyses of the types of changes that can be achieved by various professional development activities.

This change process model merges practical elements of professional development targeted at changing beliefs and practices with a potentially generalizable framework from the literature on teacher education. To the extent that this change model has elaborated a change sequence that has been useful for us in explaining variations in the outcomes of this DMI course, this process model may be helpful to others interested in instigating or studying paradigmatic changes in teachers' beliefs.

Additional work clearly remains. This study represents our effort at developing theory grounded in the specifics of three individual cases that are representative of the participants in one university classroom engaged together in one series of teaching/learning experiences. Additional utility can be added to this theory by further development and testing of suitable collections of metaphors for knowing, learning, and teaching. Utility can also be improved by development and testing suitable methods for gathering comparable data relevant to teachers' participation in the various components of this change process, which would allow comparison of the results of this study to data from larger numbers of participants in similar systematic professional development activities.

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Appendix A

The Case of Christine

Interest in Change. Christine expressed some curiosity about whether there were other ways to teach mathematics that could provide ideas she could incorporate into her current practices in an incremental rather than revolutionary way. She wrote:

> Most of my students did well in math. I could bring their math skills up to grade level. I thought I was a good math teacher. I took this class because I thought there was always room for improvement—a way of learning to teach math in a better and more exciting way. (Portfolio Assignment 1.5)

Christine's expectations for the course included learning more about how students learn to think mathematically, along with some skills to make her a better math teacher. She hoped to transfer those skills into her classroom so that her students would be better at mathematics. She was only mildly curious about fundamentally different approaches to teaching mathematics that place greater emphasis on understanding mathematics concepts than on memorizing facts and algorithms.

Problematizing and Posing Solutions. Various tasks and activities during the course provided opportunities for Christine to question what she believed and knew about children's thinking. As she began studying the DMI cases of teachers' experiences with children's thinking, she expressed surprise and amusement at the different ways the children solved problems, some of which seemed "weird and difficult" to her (Portfolio Assignment 1.2). She wondered if the tediousness and complexity of the student's work at solving what she thought were simple math problems was not the reason why some students hated mathematics. She questioned the underlying causes of some students' difficulties. Maybe "it is because they do not have a good understanding of basic place value systems…. With this new way of teaching math, will more students change their minds about math? Will they do better in math? Is there a better way, a most efficient way, to solve math problems? I wonder!" (Portfolio Assignment 1.3)

Exploring/Testing Alternatives. As Christine engaged in discussions about various strategies for solving problems, she observed how interesting it was "to see how different people had their own math style in solving a problem." But her primary interest was in how this should affect her teaching. "In third grade, if you present a problem like this, do you explain all this that we have been doing? Or do you just say you can't do that? Or do we just go on and confuse them more by showing all these ways? I'm not sure they're ready for this" (DMI Session Discourse).

> More questions surfaced for Christine as she shared her experiences from interviewing some of her students.
> From the interview I found out I can't assume that students can do those simple, simple things. I just was shocked at what they didn't know. I wish I could do this with every one of my kids, so I could see just where they mess up. (DMI Session Discourse)
> Christine reflected in her portfolio on one student's work that really concerned her. She titled it "Did I do the right thing?"
> Jessica was getting very discouraged. I could tell that she did not want to continue any longer. My question: Did I do the right thing to have her come up in front of the class to solve the problem? It was obvious that she had no understanding of the question. I was surprised at what she came up with. I tried to find out more by probing, redirecting, and asking questions to clear up her confusions. How do I get her to ask her own questions? How do I get her to stick with a problem even when it's difficult without getting discouraged? Am I embarrassing her in front of the class? Should I give her more hints to come up with the answer, or should I allow her time to figure it out herself? (Portfolio Assignment 2.2)

Christine struggled to generate thoughtful potential solution paths to her problematic activity of attending to differences in the classroom. Although she perceived the need to expose her students to different solution strategies, her metaphors limited the kinds of solutions she could seek. Rather than question her metaphors, she tried solutions that left her teaching metaphor intact, essentially deciding just to tell these other strategies to students. Her choices of which strategies to show in class were also linked to her metaphor for mathematics; she chose to show only those strategies that had a clear and direct path to the traditional algorithm. This resulted in a relatively superficial change in her practice.

Reflective Analysis of Benefits and Changing Beliefs and Practices. After one classroom episode, Christine decided to interview Jessica. Christine had just finished a multiplication unit where her students first memorized the multiplication tables, then they all passed the "daily times test." She then taught a unit about the meaning of the multiplication process. Christine wrote that she thought Jessica appeared to understand the basic concept of multiplication. However, during the interview it became clear to Christine that Jessica hadn't remembered anything they had discussed in class and was unable to explain what "times" meant. Christine wrote:

> Now I am getting a little discouraged. Maybe I did not teach her well. Maybe she did not pay attention in class. Maybe she needs more one-on-one help in math. I was surprised at her [lack of] understanding. (Portfolio Assignment 2.5)

Christine began reflecting on her own mathematical thoughts. She recorded in her portfolio that she had never done this before. It was enlightening to her to see how she solved problems differently than her colleagues and how that made her more aware of her thinking. However, throughout the entire course Christine still chose a standard algorithm as her preferred approach to solving problems. She said she felt her ways of problem solving had not changed and she was still "most comfortable" with standard algorithms. At times she would look at a problem from what she called "a different way," but she said she felt it took much longer.

As the course progressed, Christine broadened her learning goals for students a little bit, but continued to frame her teaching with the master metaphor. For example, during the first of the two DMI modules she recorded in her portfolio "the ultimate goal of doing mathematics is to get correct answers. However, in order to get correct answers understanding plays an important role" (Portfolio Assignment 1.5). She had noticed that children and adults could use different solution strategies in solving problems and still come up with the same answer. She provided some room in her goals for invented strategies as alternatives to standard algorithms. She even wanted her students to be exposed to what she called "different ways" of solving problems. However consistent with her master metaphor of teaching, she beganshowing those strategies to her students and encouraging them to try them. The value of these invented strategies for her was as a "stop gap" measure until students learned the more efficient standard algorithms. She said:

> This class has helped me see a variety of ways children solve problems. Maybe children can figure out problems other ways until they have had enough practice with the math facts and have drilled standard algorithms long enough that they will become more efficient with them, because many of these creative ways are so time consuming. But it is a way they can solve the problems without getting them marked wrong. Who cares anyway how you get it. As long as the answer is correct it doesn't really matter how you got it—just get it! (DMI Session Discourse)

As Christine reflected back over the first half of the course, she wrote that several things had changed for her. She felt she had learned more about the variety of children's invented strategies. She noticed she had begun to ask students to tell how they had come up with an answer as opposed to just asking them "What is the answer?" In doing so, she believed she had "added a lot to her instruction."

> I tend to show students different ways of solving math problems, to expose them to various ways. And I have gained a lot of respect for the many different ways problems can be solved, though some methods I do not really agree with. It has added a little more spice to my math curriculum. (Portfolio Assignment 1.7)

As Christine saw other participants in the course bring up new ways to do mathematics, she continually questioned whether they were better than her traditional methods. At the end of the course, she revisited her beliefs about standard algorithms and wrote, "Now, I don't think it is the best way of doing math. It may be one of the many ways of doing math, but it surprised me to admit that it is only one of many ways of doing math" (Portfolio Assignment 2.8).

Five months after the completion of the DMI course, Christine taught a mathematics lesson during which she posed eight division story problems and asked students to solve them using manipulatives, pictures, graph paper, or traditional algorithms. She asked students to explain their solution strategies, and after the initial solution, asked for students who did it another way. For each of the eight problems, Christine asked for additional strategies until she had someone share the traditional long-division algorithm. If no one shared the standard algorithm she would do so, emphasizing how the traditional algorithm was faster, particularly with larger numbers (Observation Field Notes).

> During the post-observation interview that focused on Christine's perceptions of what had changed in her teaching as a result of the DMI course, she responded:
> Most of the time I just pick out story problems from what we call the curriculum…. And once in a while I add a little bit to it…but most of the time I just follow the curriculum... I just ask more often, "Explain to me how you got this answer. Why do you say this?" So I ask more for the understanding... I try to use a lot of the visual aids. I try to… meet the different learning styles of the kids by using different ways of teaching. Some

may be more comfortable with the traditional way, so I am still doing that. I feel like they still need to be exposed to that. I think they still need to know that because of standardize testing. They go to the computer room, and they still give you basic division. You still have to do that. At the same time, I think the understanding part is very important, too, and so I like to see them doing it different ways. I want them to say, “Math is not just one way of doing it.” (End-of-Year Interview Transcript)

Appendix B

The Case of Linda

Interest in Change. Linda expressed an interest in coming to understand the mathematics she already knew how to do, but her new assignment in first grade focused her interest in learning new things that were immediately applicable to her teaching. "I had hoped for activities and ideas that I could incorporate into my classroom.... More like a big workshop, where you take the ideas and run with them; like a make-it-take-it" (Portfolio Assignment 1.2). Linda said she felt her way of looking at skills would be challenged in the course, but she still considered "the old tried and true ways" faster and more accurate than the children's invented strategies introduced during the first class session. She felt skills still needed to be taught directly.

Linda expressed curiosity about some children's mathematical thinking when she said that her students thought about numbers in ways that she could not. At first, she did not understand how they were thinking, and she was curious about how they were able to manipulate numbers in their heads. This led her to question, "What is it that they know? What are they trying to do? What is it in this problem that is causing them to think this way?"

Problematizing and Posing Solutions. As the course progressed, Linda gained an ease and confidence that allowed her to share her previous experiences with mathematics as a student and her perceptions of how those experiences had affected her current feelings toward mathematics.

> When I first came to this class, I was terrified. I wanted to keep my mouth shut because I was afraid someone would discover how little I know about mathematics…. When I was in school a very long time ago, the system expected me to know how to get the answers, and if I got the answer and had it right, that was the important thing…. I really feel bad looking back. I wanted to understand the algorithms, but I never could…. I don't think I ever really understood why you multiplied this number and then you moved over a space and multiplied the next one. But you obeyed and did it. (DMI Session Discourse)

Linda began to raise questions and to reflect on her assumptions about learning mathematics as she read cases about other teachers' classroom experiences, viewed video recordings of children explaining their mathematical solutions, and discussed children's thinking with peers.

> This bothered me last week, because it just seems so much faster and easier to do it my way—using the standard algorithm. And why shouldn't I teach the kids that way? But then I watch these kids. They're thinking, and they really picture the numbers much better than I do…. What I am finding I dearly love about this class is I can sit here and I look at these kids and I'm forced to answer these focus questions. I'm forced to step inside their shoes and say, "Okay, what do they know? What are they trying to do? What is in this problem that is causing them to think this way?" (DMI Session Discourse)

Linda continued to question her beliefs about how children learn mathematics, and she commented on these beliefs in her portfolio. She noted that children who understand numbers show great flexibility in their approaches to problems and become very efficient in mental math. She remained uncertain about the need for direct instruction in algorithms even as her own computational strategies became more flexible.

> I have found myself trying to stretch and think [like] the children in the case studies…. In regards to addition where regrouping is required, I must admit that I have even found it faster than the traditional method when doing mental math. If it is written, I am still more comfortable with the old way. I still feel a need to check my answers using the traditional borrow and carry method. (Portfolio Assignment 1.5)

Linda, who problematized not only mathematics and the learning of mathematics but also what it meant to teach, took the greatest risk in generating possible solution paths. She abandoned many of her earlier conceptions for new ways of teaching. Her shift from telling things to students to attending more to children's thinking allowed her to consider a wide variety of practices as part of her solution: (1) keeping more detailed notes on children; (2) learning how to question children in order to help them articulate their thinking; (3) de-emphasizing the traditional algorithms; and (4) allowing children to struggle with problems without stepping in to help. All of these paths led Linda out of her comfort zone, but she could see that these paths also led to a significant solution to the problems in her practice.

Exploring/Testing Alternatives. As Linda made progress in her own mathematical thinking she began to show some willingness to consider alternative approaches to teaching, and she began to confront the dilemma of how to reconcile her preference for direct instruction and what she was learning about teaching for understanding.

> I still find myself questioning how much instruction is necessary from the teacher. "How do I create an

> environment where my students are willing to explore and to take risks? Is this method effective for children of all ability levels, or am I losing some of my class that doesn't understand what their peers are doing?" (Portfolio Assignment 1.5)

Partway through the semester the course participants were asked to identify the one case that had the greatest impact on them and describe why they had selected that case. It was at this time that we began to see more changes in Linda's personal metaphor. Linda commented during class about the case she had selected.

> The first case had the greatest impact on me because I'm trying to teach my first graders number sense—very basic. But it moved me out of my comfort zone enough that I decided that I wanted my children to be able to visualize number like the students in those cases…. This class is helping me realize, as a primary teacher, we need to help our children develop number sense and do a better job helping children see relationships between numbers at a much younger age…. I'll probably never teach them the algorithms. (DMI Session Discourse)

Reflective Analysis of Benefits and Changing Beliefs and Practices. By halfway through the course Linda had made considerable progress from reflecting on the mathematics she wanted her students to know, what experiences would provide her students the opportunity to learn those things, and what would be her role as a teacher. In her portfolio she wrote:

> I think that my objectives have changed somewhat from things like "the student will demonstrate their knowledge of addition by adding numbers to 12" to "the student will be able to explain or model answers to story problems using addition." I think that being able to explain what they are doing is not only helpful for them but for me as a teacher, so that I can see where they might have errors in their understanding of a process…. I struggle with how long to let them explore with some of these concepts before I demonstrate a method. I still feel some things in first grade need to be taught. You need to give them a beginning point. For instance, I need to teach that addition means to combine and subtraction means to take away. (Portfolio Assignment 1.6)

However, Linda expressed some uncertainty about how to proceed as she began to move away from her expert metaphor. She recognized that rebuilding her teaching practice could be a lonely and uncertain process, even though she had seen alternative practices modeled in the videotapes and cases. "I feel like I'm kind of hanging out there, like Columbus, hoping that I don't fall off the edge and take my class with me. It's exhilarating and also very frightening" (Portfolio Assignment 1.6).

Linda started to notice that changes in her teaching were beginning to have an important impact on the environment in her classroom and the thinking that her students were beginning to do. This required her to begin supporting students thinking in ways that were still new to her.

> It's amazing that some of the things that are happening in these cases have actually been happening in my classroom. My children are finally starting to be able to tell me how they solved their problems…. It takes real restraint to sit back and let a child struggle with a problem and not point out what they should do. (DMI Session Discourse)

Near the end of the first unit, Linda reflected on her personal knowledge growth with respect to various cases and how her growth had enlightened her thinking about her students' understanding of mathematics.

> Each reading assignment has helped me view mathematical processes in a new light. I think the case studies in Section 1 have had the greatest impact on my thinking and teaching…. I am trying to become better at questioning my students to help them articulate what they are doing. Perhaps the reason I felt the greatest impact from the first case study is because it moved me out of my comfort zone and made me question and evaluate how I teach math in my classroom. (DMI Session Discourse)

During the second DMI module, Linda noted several ideas that had become important to her. Gaining confidence as a problem solver and addressing her past experiences with mathematics had empowered alternative images of herself as a learner and teacher of mathematics.

> I am finding it exhilarating to talk with other teachers about the teaching of mathematical ideas. I have become concerned about individual understanding, as I have done interviews and case studies of my students. And I have learned to recognize the value of each voice in my class (Portfolio Assignment 2.3).
>
> I have found the focus questions stimulating. I love to come home and share them with my family. I would not have felt this way at the beginning of this class. At that time I felt insecure, and I was resisting the fact that my way of doing things was being challenged. (Portfolio Assignment 2.6)
>
> I hope that I will be able to continue to grow in my ability to ask questions, rephrase what the children tell me,

and help them to explain their thinking and process in arriving at answers to problems. (Portfolio Assignment 2.8) I've now come to the point where I feel that my comments are somewhat of value, and I feel much more comfortable. I'm not afraid to make a few mistakes and say, "What do you mean?" This class turned out to address a bigger picture than I had envisioned, and it turned out to be very exciting. It was such a revelation for my own self awareness that it became very exciting to see where I was coming from, from my past experiences as a child growing up with mathematics, and my feelings toward mathematics now. I have almost come to feel like I had been cheated. I will never look at mathematics the same way as I did before I had this class. (DMI Session Discourse)

Linda's shifts in mathematics learning goals for students provided a foundation for this shift in her image of a teacher as one who ask questions, rephrases what children say, and helps them explain their thinking and problem-solving processes.

Five months after the completion of the DMI course, Linda taught a first-grade mathematics lesson in which she asked students to use the "rodeo math" manipulatives they had created the previous day (consisting of a cowboy and multiple pants, shirts, and hats with numerals written on each piece of clothing). Working in groups of 2-3, students exchanged pieces of clothing on their cowboys to produce addition number sentences. One student recorded the number sentences and all students in the group computed the sums. Linda encouraged students to look for patterns among the combinations in order to get all of the possible combinations. She asked students whether the same numbers in different orders were the same sum (e.g., Could $5 + 2 + 3 = 10$ be $5 + 3 + 2 = 10$ instead?). She also asked, "Which way could you dress the cowboy to get the highest number" (Observation Field Notes).

During the post-observation interview, Linda commented about valuing students' voices as they shared their solution strategies, but also showed that she was focusing on what children were thinking and asking follow-up questions. She said, "I can see that they have a much deeper knowledge of what they are doing if they can explain it and not just go through a ritual.... I am trying very hard to... ask questions and not tell answers.... That is a very difficult skill. I still struggle with it sometimes—not just telling them 'This is where you've made your mistake.'"
She had added two more realistic problem-solving tasks to the curriculum. She said, "We have a standard math book, and I have tried to stretch them beyond that and let them do things that I wouldn't have done before. I took problems that were in the DMI book and brought them back, and I let them try to do them... This morning... we talked about the Jazz game, and I said, 'If at the end of the first quarter, the Jazz had 20 points and Carl Malone had made 5 of those points, how many points had the other people made?' It was interesting to watch them figure that out, and they all got the same answer. They used several different methods, and each of them shared it" (Post-Observation Interview Transcript).

She also commented on the dilemma she still faced in reconciling basic skills versus invented strategies: "I've felt like they passed the [state] core test very well. I don't know that they're any better, really, than other students... But I hope they are freer to explore. One thing that I still struggle with is that in first grade I think they have to have some basic skills. I am still trying to balance between drill and exploration. I still feel that... some of this has to be mastered for them to be able to explore" (Post-Observation Interview Transcript).
She expressed plans for continued experimenting: "I've been thinking that perhaps I will change a little bit more. Maybe I've tried to take too big of a leap too fast... I'm going to start right at the first day of school, start doing some addition, some subtraction, all with these few numbers and just let it grow as we go along... Now that I'm seeing... the full scope of the curriculum a little better, it's a little clearer to me. I think that I'm going to try to do all of it in connection with the few things that we've learned and then move on. That's my thought for next year" (Post-Observation Interview Transcript).

Appendix C

The Case of Paula

Interest in Change. From the beginning it was clear Paula questioned the metaphor of teacher as master and allowed room in her view of learning for sense making. Paula recorded five of her expectations for this course in her portfolio.

- Explore the *whys* of basic algorithms.
- Explore the *whys* of student processing.
- Collect new games, techniques, and thought provoking problems to incorporate into my instruction.
- Explore the *hows* in the management of so many toys, so many ideas, so many children, and so little time.
- Enhance my own mathematical processing skills. (Portfolio Assignment 1.2)

Later, Paula said she hoped "Teaching to the Big Ideas" would help her "create, duplicate, borrow, beg, or steal some intervention strategies" that would answer her concerns (Portfolio Assignment 1.5). Paula also expressed her concern with helping older learners "rebuild" a system of tens (Portfolio Assignment 2.3).

Problematizing and Posing Solutions. Paula's experience as a student differed from Linda's. As a child, Paula was able to perform algorithms efficiently and enjoyed doing so. But she remembered noticing not all of her classmates could. This raised questions in her mind about whether too much emphasis was being placed on computational algorithms.

> When most of us went to school, we learned one way to add, one way to subtract, one way to multiply, and one way to divide. Several of us noticed, however, that not all of our friends we went to school with "got it." The question I have is why is the entire nation seemingly committed to the *algorithm* anyway? Is it a universal truth or just a way to fit into society? Is an algorithm a tool? Is an algorithm really a good thing? Is there only one algorithm? I think we teach algorithms because we want to hold our jobs, so that people won't say we were lousy teachers. Really, it's so children's parents can help them with their homework, right? What it all comes down to is how we are held accountable. It feels like we are being held back, because we have to teach this way so the children can pass tests. It's too bad. (DMI Session Discourse)

Exploring/Testing Alternatives. At the beginning of the course Paula indicated she was open to alternatives for what to teach and how to teach it and was willing to pursue what teaching for understanding might mean.

> Most of my fifth grade students are not proficient in traditional algorithms. Can I reinforce the students' algorithms best through story context, or symbol context, or pictorial context, or a variety of all three? Is there a time when bare mathematical symbol context is better? What I'm most concerned about is that they are able to develop critical thinking in their problem solving, and they won't get there if we keep saying that the standard algorithm is the only way. In the long run they need to understand what they are doing. (DMI Session Discourse)

As the course progressed, Paula noted in her portfolio that her understanding of mathematics was deepening and her rigidity had given way to greater flexibility. She commented that breaking down various problems had given her a much better understanding of how algorithms work, the value of base-ten knowledge, and the connected and integrated nature of mathematics. She expressed similar ideas during class:

> I'm noticing that mathematics is not just single ideas taught separately in isolation, but rather more of a connected and integrated whole. It's making me re-think what mathematics is—its wholeness versus its parts. It seems like the underlying factor for everything is base ten. We revisit it over and over again, just in different ways. (DMI Session Discourse)

Along with shifting her beliefs about mathematics, Paula also changed what she did in the classroom. As she reflected on what she had begun to do differently, she wrote about increased frequency of "flights away from the book, workbook, and workshop files" with more time spent on exploring and dissecting students' mistakes on assignments and tests (Portfolio Assignment 1.6). Later she noted:

> I have found that I can lead a whole class discussion with only two or three problems and use the children's ideas to develop a much better session than a page of math problems can accomplish. What many of them are doing is rebuilding an understanding of mathematics. However, it takes longer than I anticipated. (Portfolio Assignment 2.3)
> They are taking back the responsibility for learning, and I am becoming convinced that this is not only a "funner" way to teach math but a better way as well. (Portfolio Assignment 2.7)

Paula attributed some changes in her teaching practices to a greater emphasis on children's thinking and understanding.

> I am thinking more about students' thinking individually rather than covering the mathematics. Not just to say, "Okay, that's checked off" and continue going on hoping everybody in the class has gotten what I planned them to get. But more, "What did that student individually get?" You have to stop and reflect about what you are seeing, and it takes time. I noticed in the last few days I have been more cautious in my planning as to what kind of problems can get at thinking, that will make their mathematical understanding deeper. What I would like is to help bridge their thinking and to diagnose how to help them straighten out some of the problems they are having in that thinking. (DMI Session Discourse)

Reflective Analysis of Benefits and Changing Beliefs and Practices. At the end of the course, Paula reflected in her portfolio on what had changed for her during the semester. She wrote that she knew her students better than before. She noticed her students could now track their thinking processes and pinpoint the variations in their ideas. She also noticed that their understanding of the mathematics was deeper and their dialogues and written representations were more focused. Paula had gained confidence in her ability to question and pose interesting problems. She commented in class that she had started to teach mathematics more as connected big ideas rather than separate disconnected pieces, which was consistent with changes in her philosophy of mathematics education.

> I thought this class was going to be more about ideas on how to teach. I think I had an idea that there was going to be a little more theory behind it, and not just ideas, but I think the thing that was most helpful was it helped me develop my philosophy of mathematics education. And, I think until you understand what your philosophy of something is, you are not as powerful at it. I think I had a philosophy, but it changed by the process that we went through and discussions we had. I knew questioning was important, but I now see it spilling

> over into other subjects. The type of questions that you ask children to get them to understand is permeating my whole life, and I will never be the same. (DMI Session Discourse)

By the end of the course, Paula had built upon her views of how children learn to redesign her teaching metaphor as one who uses questions to find out what students are thinking and then poses additional questions to lead students to build new understandings of well-connected mathematical ideas.

Five months after the end of the DMI course, Paula taught a mathematics lesson that posed a problem that focused on vocabulary and language development associated with the mathematical concepts of two- and three-dimensional shapes connected to the science of soap bubbles and strongest shapes. She fostered development of a problem-solving community as she engaged her students in a social constructivist experience and used questioning to prompt and facilitate students' thinking about deep understandings of the mathematics in the lesson. (Observation Field Notes)

During the post-observation interview focused on changes in Paula's teaching practices resulting from the DMI course, she commented on how she now attended to children's understanding and helped them explore their mistakes.

> I spend a lot more time discussing their ideas. I don't do as many problems per se, but we spend more time on specific problems. We just go broader because we take the time to listen and talk about all the different ways they would approach it. . . . I find myself asking more questions and trying to generate more questions from them. . . . I am trying to build on what each child says and to connect it to either someone else in the room, something they said earlier, or a particular path I am hoping they will go down. . . . I try to point out to them how valuable it is when they find and correct themselves and figure out why they did it wrong. I use a phrase this year I have never used before. I will say, 'Do you realize the most important thing that has happened today just happened for you in our class, because you corrected yourself, and you just figured out what you have done wrong. And that is the most valuable thing.' I found that the more that I use that phrase the more children start saying that. They would try to find the mistakes they had made and then bring it to my attention. 'Look, this is what I did wrong. That is the most important thing I could do today. I just found what I did wrong.' And that cognition. . . [and] celebration—of 'Oh, I got that wrong, and now if I figure out why I got it wrong she is going to be even happier than if I had gotten it right'— is very different, and that is attributed directly to the [DMI] class. (Post-Observation Interview Transcript)

Paula recognized that changes to her fifth-grade curriculum were essential to achieving her goals for learning.

> I haven't changed all my problems. I'm locked into district core in math, and the majority of the core deals with computation out of context. I do have to continue to work with them on math out of context, but I have given more real life math problems at other times in the day, such as the warm-up activity in the morning. I used to spend quite a bit of time with grammar or daily oral language, and now probably fifty percent of it would be a math contextual question or game, because I have found that if you put math in the context of real life that builds their grammar skills. If the math is in the context of a story, then you are killing two birds with one stone. I want more of the kinds of problems [we saw in the DMI course]. I want more of them because I am too exhausted to think them up myself. I am

> stretched too thin to keep generating them, and I have generated quite a few. I will take situations in the literature book we'll be reading and use that as a springboard for a math problem, or a field trip we've been on. I did a lot more of that this year. Almost every field trip or outside experience we did I tied back into a math problem after we had done it, and I watch for those opportunities to do that. . . . Still, I haven't moved everything completely to a big idea. (Post-Observation Interview Transcript)

Her comments during the interview also indicated she understood that shifts in teaching required to facilitate learning flexible problem solving with understanding were revolutionary in nature and as yet unsupported by continuing standardized testing. In addition, she expressed some concern over vocal groups expressing opposing viewpoints that indicated that her changes in beliefs and practices were very fragile due to the limited evidence of their effectiveness. She said, "I think math education is [students'] ticket to the future, and my consciousness is open and sensitive to the need of better math education. I know what we have done traditionally all these years isn't working as well as it should, and it is exciting but also frustrating to be caught in the middle of the revolution and not know if what you are doing is really [helping]" (Post-Observation Interview Transcript).

Appendix D

List of Table and Figure Captions

Table 1. Metaphors

Topic	Traditional School Mathematics	Standards-Based School Mathematics
Knowing **Mathematics**	**Toolbox:** Knowing mathematics is having a toolbox filled with a collection of facts, definitions, rules, and efficient computational procedures (e.g., standard algorithms) to be applied in computing correct answers by matching the intended, well-practiced tool to a familiar type of problem.	**Flexible Problem-Solving with Understanding:** Knowing mathematics is having and being able to flexibly use a complex, interconnected web of understanding of concepts, procedures, and problem-solving experiences to convert new, nonroutine, culturally valued, real-world problems into mathematical abstractions that can be solved using concept-driven, sensible strategies.
Learning **Mathematics**	**Behaviorist:** Learning mathematics involves memorization and practice, which strengthens mental associations between the generalized knowledge and specific procedures that have been demonstrated by the teacher and the typical problem types to which those preferred procedures are routinely applied.	**Social Constructivist:** Learning mathematics involves constructing a complex web of knowledge through social negotiation of meaning for mathematical language and symbols; construction of shared understandings of mathematical concepts that can generate possible problem-solving strategies for nonroutine problems; and developing flexibility in thinking and communicating mathematically through participation in a cultural community.
Teaching **Mathematics**	**Master:** Teaching mathematics consists of direct instruction (ala master-apprentice or master-disciple) in which the teacher shows students preferred procedures; tells facts, definitions, and rules; assigns practice of these generalized responses; assigns applications to particular contexts (e.g., word problems); and tests for computational speed and proficiency by counting correct answers to familiar problems.	**Facilitator:** Teaching mathematics consists of posing worthwhile mathematical tasks, facilitating students' problem-solving efforts, questioning students' understanding and thinking, and orchestrating discourse to facilitate and guide students' construction of more complex understandings; and facilitating students' reflection on their experiences so that they build connections among their context-specific conceptions and produce generalizations that are sufficiently well-connected to particular contexts and experiences that they can be assessed as being generative of new strategies for solving new problems in unfamiliar contexts.

Table 2. Changes in Metaphors/Beliefs

Name (pseudonym)	Beliefs About *Knowing* Math		Beliefs About *Learning* Math		Beliefs About *Teaching* Math	
	Beginning	Ending	Beginning	Ending	Beginning	Ending
Christine	**Toolbox:** Successful in math; liked algorithms, correct answers, and automaticity	**Toolbox:** Enriched by allowing some student-invented strategies if they led to traditional algorithms	**Behaviorist:** Emphasized repetitive practice; felt successful bringing resource students up to grade level	**Behaviorist:** Focused on additional procedures rather than concepts or student thinking	**Master:** With room to improve teaching skills and become more exciting	**Master:** With some showing of invented strategies (allowed students to become part of the show-and-tell)
Linda	**Toolbox:** Not very successful in math; felt safe by following traditional emphases on speed and efficiency	**Flexible PS/U and Toolbox:** Liked number sense and invented strategies for mental math and seeing relationships; held on to some traditional tools and emphases	**Behaviorist:** Focused on correct answers without under-standing	**Social Constructivist:** Began to value each voice; concerned about student explorations, thinking, and understanding	**Master:** Looking for additional strategies and activities	**Facilitator:** Increased ability to ask good questions, rephrase what students said, support explanations, and assess errors in thinking
Paula	**Toolbox and Flexible PS/U:** Very successful with algorithms but noticed others were not; had some interest in problem solving and understanding	**Flexible PS/U:** Focused on understanding big ideas, problem solving, and critical thinking	**Behaviorist:** Some interest in sense making	**Social Constructivist:** Recognizing more individual responsibility for learning that occurs through dialogue and written representations	**Master:** Focus on textbook sequence; looking for more successful strategies and a more individual focus	**Facilitator:** Pose interesting problems; use questioning to find out what students think; and lead them to build deep, well-connected understandings

Table 3. Interest in Change, Problematizing, Exploring, and Analyzing

Name (pseudonym)	Interest in Change	Problematizing Current Beliefs and Practices and Posing Possible Solutions	Exploring/Testing Alternative Practices	Reflective Analysis of Benefits and Changing Beliefs and Practices
Christine	Low: looking for spice	Low: confident in current success with low-performing students	Moderate: tried asking children to explain their strategies	Low: questioned benefit of children explaining thinking; saw limited usefulness for alternative strategies, only as temporary transitions to traditional algorithms
Linda	Moderate: looking to develop greater confidence in her own understanding of mathematics; curious about children's thinking different from her own	High: questioned her own understanding, the process by which she had failed to learn to understand math, and her traditional approach to teaching; developed a strong interest in change to develop number sense and relational thinking	High: questioned students' thinking; kept track of children's responses; supported children with problem solving and communicating thinking; assessed to understand students' processes and conceptual understanding	Moderate: gained confidence in her ability to problem solve; became concerned about individual students' understanding; changed her goals for learning; broadened her view of the curriculum
Paula	High: dissatisfied with traditional practice; interested in exploring whys of new practices	High: concerned with inequity of traditional practices; interested in teaching for understanding	High: modified curriculum and teaching to focus on big ideas, discourse, and written representations; attended to individual children's thinking and understanding	High: concluded questioning develops deep understanding in mathematics and other content areas; students who understand are able to analyze their own errors; formed a new philosophy of mathematics education

Figure 1. Guskey's (1986) Model of the Process of Teacher Change

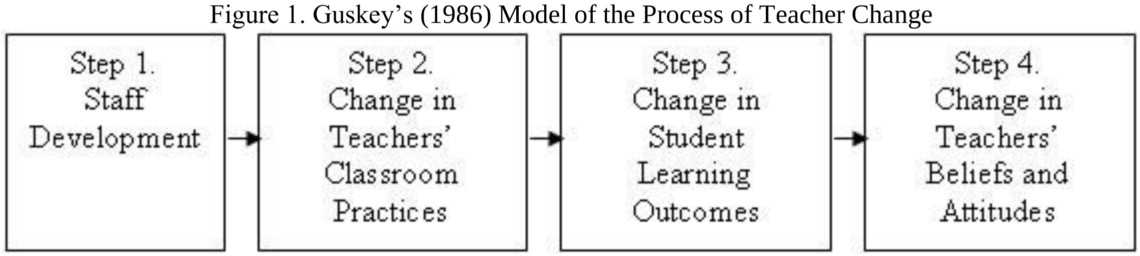
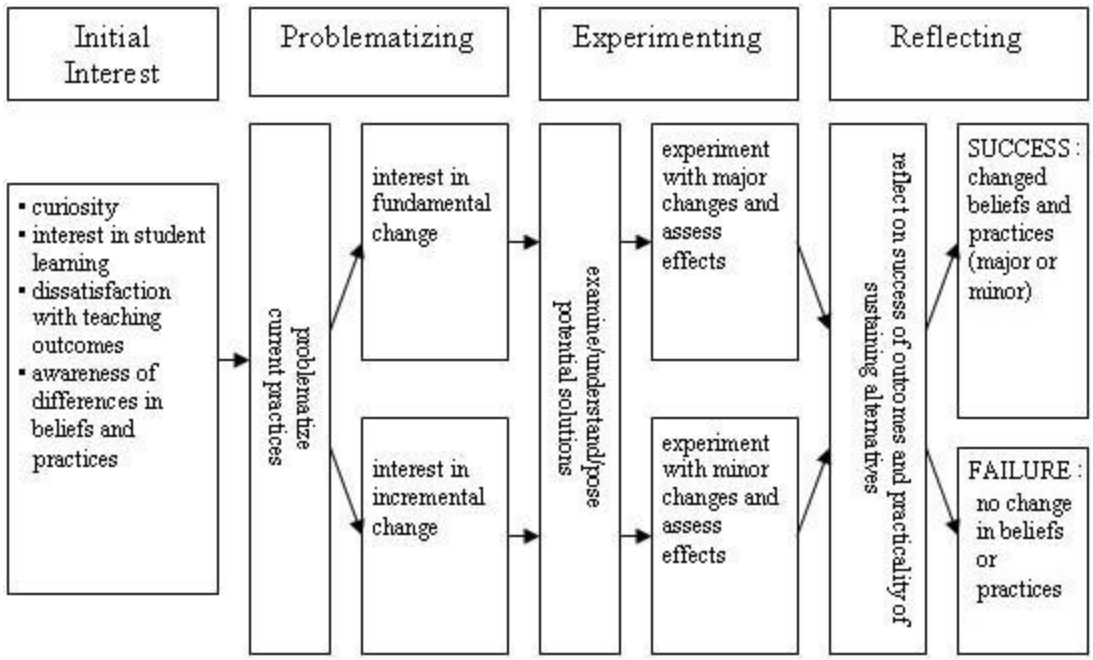


Figure 2. Change Process Model for Teachers' Beliefs and Practices



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