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A Q-RUNG ORTHOPAIR BASIC PROBABILITY ASSIGNMENT AND ITS APPLICATION IN MEDICAL DIAGNOSIS

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Abstract: *Dempster-Shafer theory is widely used in decision-making and considered as one of the potential mathematical tools in order to fuse the evidence. However, existing studies in this theory show disadvantage due to conflicting nature of standard evidence set and the combination rule of evidence. In this paper, we have constructed the framework of q-rung evidence set to address the issue of conflicts based on the q-rung fuzzy number due to its more comprehensive range of advantage compared to the other fuzzy or discrete numbers. The proposed q-rung evidence set has the flexibility in assessing a parameter through the q-rung orthopair basic probability assignment consisting of membership and non-membership belief degree. Moreover, as the proposed q-rung orthopair basic probability assignment consists of pair of belief degrees, the possibility of conflicts cannot be ignored entirely. In this regard, a new association coefficient measure is introduced where each component of the belief degrees is modified through the weighted average mass technique. This paper uses various concept such as fuzzy soft sets, Deng entropy, association coefficient measure and score function for decision-making problem. Firstly, to obtain the initial q-rung belief function, we have implemented the Intuitionistic fuzzy soft set to assess the parameter of the alternatives and Deng entropy to find the uncertainty of the parameters. Secondly, the association coefficient measure is used to avoid the conflict through the modified form of evidence. Finally, we combined the evidence and found the score value of the Intuitionistic fuzzy numbers for the ranking of the alternatives based on the score values of alternatives. This study is validated with the case study in the medical diagnosis problem from the existing paper and compared the ranking of alternatives based on the score function of belief measures of the alternatives.*

Key words: *Fuzzy soft set, q-rung belief function, Association coefficient measure, Medical diagnosis.*

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1. Introduction

Decision-making in real life problems depends on the knowledge and information of the parameters, which affects the decision alternatives. In order to model the information with less uncertainty, various theories are discussed in literature to deal with the uncertainty in which Probability theory, Fuzzy set theory, and evidence theory are a few of them that are very popular in decision-making. Fuzzy set theory was first introduced by L. Zadeh in 1965, and it was further generalized into Interval-valued fuzzy set (Gorzałczany, 1987)., Intuitionistic fuzzy set (Atanassov, 1986), Pythagorean fuzzy set (Yager, 2013), q-rung fuzzy set (Yager, 2016), and Picture fuzzy set (Cuong, 2015), etc. The extended theories of fuzzy sets are applied in various directions by integrating with the fuzzy soft set theory, Dempster-Shafer theory, and multi-criteria decision-making. Molodtsov, (1999) proposed soft set theory for the adequate parametrization of the parameters, and fuzzy soft set was later developed by Maji et al. (2001). The proposed theories are widely used in decision-making problem. Das et al. (2013) studied intuitionistic multi-fuzzy sets in group decision-making. Çelik & Yamak, (2013) applied fuzzy soft set theory in medical diagnosis using fuzzy arithmetic operations. Peng et al. (2015) used Intuitionistic fuzzy soft set in the decision-making after the introduction of Intuitionistic fuzzy soft (Maji, 2001). We have also developed the concept of Bell-shaped fuzzy soft sets (Dutta & Limboo, 2017) and applied it in medical diagnosis. Das et al. (2017) proposed robust decision-making using intuitionistic fuzzy numbers. Krishankumar et al. (2019) proposed the q-rung orthopair fuzzy set with partially known weight in the evaluation of renewable energy sources. Hussain et al. (2020) proposed the q-Rung orthopair fuzzy soft average aggregation operator and their application in multicriteria decision-making. Mishra et al. (2021) extended the fuzzy decision-making framework using hesitant fuzzy sets for the drug selection of coronavirus disease (COVID-19).

Dempster-Shafer theory has been applied with its discrete basic probability assignment, a generalization of classical probability theory, and was first introduced in the year 1967 (Dempster, 1967; Shafer, 1976). In literature, this theory offers various applications in decision-making problems of many fields of sensor data fusion (Jiang et al. 2016; Xiao & Qin, 2018; Xiao, 2019), medical diagnosis (Li et al. 2015; Xiao, 2018; Chen et al. 2019), target recognition (Pan & Deng, 2019), multi-criteria decision-making (Li & Deng, 2019) etc., though it has produced some conflicts in the evidence combination rule (Zadeh, 1978). The conflicts and counterinitiatives nature of evidence have been addressed for several decades. Researchers developed another form of combination rule (Dubois & Prade, 1986; Yager, 1987; Inagaki, 1991; Zhang, 1994) after realising that the conflicts can be resolved by either the method of pre-processing or modifying the basic probability assignment (BPA) before applying Dempster's combination rule (Murphy, 2000; Deng et al. 2004; Jiang, 2016; Xiao & Qin, 2018). In the application of medical diagnosis, Li et al. (2015) used fuzzy soft set (FSS) and Dempster-Shafer theory (DST) in medical diagnosis problem and compared the result of the case study that put forward in Basu et al. (2012) with the earlier technique. Wang et al. (2016) proposed a method of using ambiguity measure, fuzzy soft set and Dempster-Shafer theory in the medical diagnosis. Xiao, (2018) proposed the fuzzy preference relation and fuzzy soft set technique in medical diagnosis. Chen et al. (2019) proposed the weighted average mass technique in medical diagnosis and compared with the earlier method. Zhou et al. (2020) proposed a novel divergence measure of Pythagorean fuzzy set (PFS) connecting the belief function and PFS with the application in the medical diagnosis.

In evidence theory, Interval-valued and fuzzy form of BPA is more efficient than the precise BPA to reflect the uncertainty of a parameters. In this regard, Yager, (2001) put forward interval-valued belief structure (IBS) and Yager, (2014) proposed the intuitionistic view of the Dempster-Shafer belief structure. Song et al. (2015) proposed a new distance measure between Intuitionistic beliefs functions based on the new similarity measure of IFS with its properties. Apart from this, Li & Deng, (2019) proposed the concept of Intuitionistic evidence set in the form of support and non-support belief degree of BPAs and applied the concept in multi-criteria decision-making (MCDM). Gao & Deng, (2020) introduced the quantum model of the mass function and established the relationship between quantum mass function and classical mass function with examples. Xiao, (2020) generalizes the DST by introducing the complex basic probability assignment and applied it in medical diagnosis problem to show the efficiency of the proposed algorithm. Kar et al. (2021) used picture fuzzy set based fusion operator and Dempster-Shafer theory in the medicine selection of covid 19 disease. In this paper, the main motivation and contributions of the paper are as follows:

- To address the issue of conflict in evidence theory is still an open issue and the assessment of the alternatives from human cognitive subjective knowledge in more flexible way is always helpful in precise decision-making process.
- We have introduced a new form of evidence theory called q-rung evidence set of classical evidence theory in the form of pair of support and non-support degree of basic probability assignment. The q-RoBPA is effective due to its flexibility offered to decision maker for the assessment of the alternatives based on the support as well as non-support degree of belief in the assessment of a problem.
- Since, the proposed q-rung orthopair BPA is based on the pair of two belief degree, therefore their conflict nature cannot be fully ignored. For this, we have further proposed an association coefficient measure to handle the uncertainty involved in the q-Rung basic probability assignment (q-RoBPA).
- The present method is based on the use of novel q-RoBPA and association coefficient measure in the medical diagnosis case study that put forward in Basu et al. (2012) and compared its belief measure with the existing method to show its efficiency and superiority.

The present paper is organised as follows: In the section 2, the basic overview of the preliminary concepts of Dempster-Shafer theory, and some theories related to uncertainty measure viz. Deng entropy, distance measure and association coefficient measure with related property are put forward in section 3. Section 4 carried out the fundamental ideas of fuzzy set theory and fuzzy soft set with related properties. In section 5, we have proposed the new form of evidence theory and introduced an association coefficient measure to handle the uncertainty in the process of conflict management. In section 6, we have put forward an algorithm to implement the proposed association coefficient measure on the q-rung orthopair basic probability assignment and also, an application in medical diagnosis is carried out in section 7. Section 8 concludes the paper.

2. Preliminaries

In this section, we have put forward some fundamental concepts of DST and uncertainty measure of evidence.

2.1. Dempster-Shafer Theory: Basic Concept

Definition 1 (Dempster, 1967; Shafer, 1976**)**: Let $X = \{x_1, x_2, ..., x_N\}$ be a collection of mutually exclusive and exhaustive elements and the collection of all the hypotheses F_i is defined by the power set of X such that $2^X = \{\phi, \{x_1\}, ..., \{x_n\}, \{x_1, x_2\}, ..., X\}$. Then, X is called as the frame of discernment (FOD) such that $2^X = \{F_1, F_2, ..., F_i, ..., F_{2^N}\}$.

Definition 2 (Dempster, 1967; Shafer, 1976): The basic probability assignment (BPA) is a function $m : 2^X \to [0,1]$ that satisfies the condition

$$m(\phi) = 0 \text{ and } \sum_{F_i \subseteq X} m(F_i) = 1 \tag{1}$$

where the collection $2^X = \{F_1, F_2, ..., F_i, ..., F_{2^N}\}$ is 2^N possible propositions.

Definition 3 (Dempster, 1967; Shafer, 1976): The belief measure of F on X is a function $Bel : 2^X \to [0,1]$ which satisfies the condition

$$Bel(\phi) = 0, \; Bel(F) = \sum_{F_i \subseteq F} m(F_i) \tag{2}$$

Plausibility measure of F is also a function $Pl : 2^X \to [0,1]$ which satisfies the condition

$$Pl(\phi) = 0, Pl(F) = \sum_{F \cap F_i \neq \phi} m(F_i) \tag{3}$$

Definition 4 (Dempster, 1967)**:** The Dempster's combination rule for combining two BPAs m_1 and m_2 is a joint function $m_1 \oplus m_2 : 2^X \to [0,1]$ defined as

$$(m_1 \oplus m_2)(F) = \frac{1}{(1-K)} \sum_{F_i \cap F_j = F} m_1(F_i) m_2(F_j) \tag{4}$$

In addition, $(m_1 \oplus m_2)(\phi) = 0$ and $K = \sum_{F_i \cap F_j = \phi} m_1(F_i) m_2(F_j)$ represents the conflict coefficient between m_1 and m_2. Two pieces of evidence are said to be in conflict whenever $K = 1$. The counter-intuitiveness and conflicts of BPAs are reduced and managed with the help of various methods of uncertainty measure.

3. Some Uncertainty Measure in Dempster Shafer Theory

Deng entropy (Deng, 2015) is used to measure the uncertainty contained in the BPAs, whereas distance measure (Jousselme, 2001; Cheng, 2019), similarity measure (Xiao, 2018) and divergence measure (Fei et al. 2018; Xiao, 2019; Zhou et al. 2020) are the measure to distinguish two belief functions, and is also used to modify the conflicting evidence.

Definition 5 (Deng, 2015): Let m be the BPA on the discernment frame X. The Deng's entropy E_d of m is defined as

$$E_d(m) = \sum_{\forall F_i \in 2^X} m(F_i) \log_2 \left(\frac{m(F_i)}{2^{|F_i|} - 1} \right) \tag{5}$$

In particular, if $\left|F_i\right| = 1$, then Deng's belief entropy reduces to

$$E_d(m) = \sum_{\forall F_i \in 2^X} m(F_i)\log_2\left(m(F_i)\right) \tag{6}$$

Definition 6 (Jousselme, 2001)**:** Consider m_1 and m_2 be two BPAs defined on the FOD X. Then, the Jousselme's evidence distance between two BPAs is defined as

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^t \underline{\underline{D}}(\vec{m}_1 - \vec{m}_2)} \tag{7}$$

where $\vec{m}_1$ and $\vec{m}_2$ be the BPAs in the vector form. The matrix $\underline{\underline{D}}$ is the Jaccard's matrix of order $2^N \times 2^N$ whose elements $\underline{\underline{D}}(F_i, F_j)$ are the Jaccard's similarity coefficient defined by

$$\underline{\underline{D}}(F_i, F_j) = \frac{\left|F_i \cap F_j\right|}{\left|F_i \cup F_j\right|}, \text{ for all } F_i, F_j \in 2^X \tag{8}$$

Definition 7 (Cheng, 2019)**:** Cheng's distance measure between two BPAs m_1 and m_2 is defined as

$$d(m_1, m_2) = \sqrt{\frac{1}{2}(\vec{m}_1 - \vec{m}_2)^t D_\alpha(\vec{m}_1 - \vec{m}_2)} \tag{9}$$

where D_α is the matrix of order $2^N \times 2^N$ whose entries $D_\alpha(F_i, F_j)$ are defined as

$$D_\alpha(F_i, F_j) = \frac{\left|F_i \cap F_j\right|}{\left|F_i\right|} \times \frac{\left|F_i \cap F_j\right|}{\left|F_j\right|}, \text{ for all } F_i, F_j \in 2^X \tag{10}$$

Definition 8 (Jiang, 2016)**:** The Jiang's correlation coefficient measure between m_1 and m_2 is defined as

$$r(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1)}\sqrt{c(m_2, m_2)}}, \tag{11}$$

where $c(m_1, m_2) = \sum_{i=1}^{2^N-1}\sum_{j=1}^{2^N-1} m_1(F_i) m_2(F_j) \times \frac{|F_i \cap F_j|}{|F_i \cup F_j|}$ (12)

Definition 9 (Pan & Deng, 2019)**:** Pan & Deng's association coefficient measure between m_1 and m_2 is defined as

$$a(m_1, m_2) = \frac{r(m_1, m_2)}{\frac{1}{2}\{r(m_1, m_1) + r(m_2, m_2)\}} \tag{13}$$

where $r(m_1, m_2) = \sum_{i=1}^{2^N-1}\sum_{j=1}^{2^N-1} m_1(F_i) m_2(F_j) \times \left(\frac{2^{|F_i \cap F_j|} - 1}{2^{|F_i|} - 1}\right) \times \left(\frac{2^{|F_i \cap F_j|} - 1}{2^{|F_j|} - 1}\right)$ (14)

4. Fundamental concepts of Fuzzy sets and q-Rung fuzzy set

This section puts forward some fundamental concepts of fuzzy set theory and fuzzy soft set with related properties and operations.

Definition 10 (Zadeh, 1965): A fuzzy set $\tilde{A}$ in the universal set X is defined as $\tilde{A}=\left\{\left\langle x,\mu_{\tilde{A}}(x)\right\rangle : x\in X\right\}$, where $\mu_A : X\to[0,1]$ is the membership function of the fuzzy set $\tilde{A}$.

Definition 11 (Yager, 2016): A q-rung orthopair fuzzy set M in the universal set X defined as $M=\left\{\left\langle x,\mu_M(x),\nu_M(x)\right\rangle : x\in X\right\}$, where $\mu_M : X\to[0,1]$ is the membership function and $\nu_M : X\to[0,1]$ is the non-membership function that satisfying the condition $0\le \left\{\mu_M(x)\right\}^q+\left\{\nu_M(x)\right\}^q \le 1$. The hesitancy degree of the q-rung fuzzy set (q-RoFS) M is defined as

$$\pi_M(x)=\sqrt{1-\left(\left\{\mu_M(x)\right\}^q+\left\{\nu_M(x)\right\}^q\right)},\ q\ge 1 \tag{15}$$

For fixed $q=1$, M reduces to Intuitionistic fuzzy Set (IFS) (Atanassov, 1986) and for $q=2$, M reduces to Pythagorean fuzzy set (PFS) (Yager, 2013).

Definition 12 (Yager, 2016; Hussain, 2020)**:** Let $A, B\in q\text{-}RoFS(X)$ such that $A=\left\{\left(x,\mu_A(x),\nu_A(x)\right) : x\in X\right\}$ and $B=\left\{\left(x,\mu_B(x),\nu_B(x)\right) : x\in X\right\}$, then we have

a) $A\subseteq B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$, $\forall x\in X$.

b) $A=B$ iff $\mu_A(x)=\mu_B(x)$ and $\nu_A(x)=\nu_B(x)$, $\forall x\in X$.

c) $A^C=\left\{\left(x,\nu_A(x),\mu_A(x)\right) : x\in X\right\}$, where A^C is the complement of A.

d) $A\cap B=\left\{\left(x,\mu_A\wedge\mu_B(x),\ \nu_A\vee\nu_B(x)\right) : x\in X\right\}$, where $\mu_A\wedge\mu_B(x)=\min\{\mu_A(x),\mu_B(x)\}$ and $\nu_A\vee\nu_B(x)=\max\{\nu_A(x),\ \nu_B(x)\}$

e) $A\cup B=\left\{\left(x,\mu_A\vee\mu_B(x),\ \nu_A\wedge\nu_B(x)\right) : x\in X\right\}$, where $\mu_A\vee\mu_B(x)=\max\{\mu_A(x),\ \mu_B(x)\}$ and $\nu_A\wedge\nu_B(x)=\min\{\mu_A(x),\nu_B(x)\}$.

f) The score function (Peng et al. 2018) of the q-RoFSS A is given by

$$S(A)=\left\{\mu_A(x)\right\}^q-\left\{\nu_A(x)\right\}^q+\left(\frac{e^{\{\mu_A(x)\}^q-\{\nu_A(x)\}^q}}{e^{\{\mu_A(x)\}^q-\{\nu_A(x)\}^q}+1}-\frac{1}{2}\right)\{\pi(x)\}^q,\ q\ge 1$$

Definition 13 (Maji et al. 2001): Let $\mathcal{F}(U)$ be the collection of fuzzy sets over the universal set U and E be the set of parameters with $A\subseteq E$. A fuzzy soft set over U is a pair $\left(F,A\right)$, where $\tilde{F}$ is a function given by $F:\ A\to\mathcal{F}(U)$.

In general, if $\mathcal{F}(U)=IFS(U)$, $\mathcal{F}(U)=PFS(U)$ and $\mathcal{F}(U)=q\text{-}RoFS(U)$, then $\left(F,A\right)$ is accordingly called as the Intuitionistic fuzzy soft set (IFSS) (Maji et al. 2001) or Pythagorean fuzzy soft set (PFSS) (Peng et al. 2015) or q-rung orthopair fuzzy soft set (q-RoFSS) (Hussain et al. 2020) over the universal set U respectively.

Definition 14 (Maji et al. 2001): Let $\left(F,X_1\right)$ and $\left(F,X_2\right)$ represents the two distinct FSSs over the universe U and for all $x_1\in X_1$, $x_2\in X_2$, we have

(i) $\left(H, X_1 \times X_2\right) = (F, X_1) \wedge \left(G, X_2\right)$, where $H(x_1, x_2) = F(x_1) \cap G(x_2)$

$\left(H, X_1 \times X_2\right) = (F, X_1) \vee \left(G, X_2\right)$, where $H(x_1, x_2) = F(x_1) \cup G(x_2)$

5. q-Rung Evidence Set

In this section, we have defined a new form of evidence set named q-rung evidence set inspired from the work of Li & Deng, (2019) and put forward all the related definitions of Dempster-Shafer theory in this form.

Definition 15: Let X be the frame of discernment. A q-rung basic probability assignment or q-rung orthopair basic probability assignment (q-RoBPA) on X is defined as the pair $m = \left\langle m^+, m^- \right\rangle$ in which the function m has the following conditions

The support belief degree $m^+ : 2^X \to [0,1]$ satisfies the conditions

$$\sum_{A \subseteq X} m^+(A) = 1 \tag{16}$$

The non-support belief degree $m^- : 2^X \to [0,1]$ satisfies the conditions

$$\sum_{A \subseteq X} m^-(A) = 1 \tag{17}$$

For all $A \neq X$, $\{m^+(A)\}^q + \{m^-(A)\}^q \leq 1$, where q tends to ∞.

Definition 16: Let $m = \left\langle m^+, m^- \right\rangle$ is a q-RoBPA on the FOD X. Then, the belief measure of the focal element A is pair $\left\langle Bel^+, Bel^- \right\rangle$ that satisfies the following conditions:

$$Bel^+(A) = \sum_{B \subseteq A} m^+(B) \text{ and } Bel^-(A) = \sum_{B \subseteq A} m^-(B), \tag{18}$$

Plausibility measure of A is a pair of two function $\left\langle Pl^+, Pl^- \right\rangle$ which satisfies:

$$Pl^+(A) = \sum_{A \cap B \neq \phi} m^+(B) \text{ and } Pl^-(A) = \sum_{A \cap B \neq \phi} m^-(B) \tag{19}$$

where Pl^+, $Bel^+ : 2^X - \phi \to [0,1]$ are the support degree whereas Pl^-, $Bel^- : 2^X - \phi \to [0,1]$ are the non-support degree of belief and plausibility functions respectively.

Definition 17: Let $m_1 = \left\langle m_1^+, m_1^- \right\rangle$ and $m_2 = \left\langle m_2^+, m_2^- \right\rangle$ be two distinct q-RoBPA on the FOD X. The combination of m_1 and m_2 is the new mass function defined as:

$$m_1 \oplus m_2(A) = \left\langle m^+(A), m^-(A) \right\rangle, \ \forall A \in 2^X - \phi \tag{20}$$

where $$m^+(A) = \frac{\sum_{B \cap C = A} m_1^+(B) m_2^+(C)}{1 - \sum_{B \cap C = \phi} m_1^+(B) m_2^+(C)} \text{ and } m^-(A) = \frac{\sum_{B \cap C = A} m_1^-(B) m_2^-(C)}{1 - \sum_{B \cap C = \phi} m_1^-(B) m_2^-(C)} \tag{21}$$

Example 1: Consider $X = \{x_1, x_2, ..., x_n\}$ be a FOD having two q-RoBPAs m_1 and m_2 for the focal elements F_1, F_2, F_3 are given below

$m_1(F_1) = \left\langle 0.6, 0.1 \right\rangle$, $m_1(F_2) = \left\langle 0.05, 0.8 \right\rangle$, $m_1(F_3) = \left\langle 0.35, 0.1 \right\rangle$

$m_2(F_1) = \left\langle 0.7, 0.2 \right\rangle$, $m_2(F_2) = \left\langle 0.1, 0.6 \right\rangle$, $m_2(F_3) = \left\langle 0.3, 0.2 \right\rangle$

Using the equations (20) and (21), the combined q-RoBPA for focal elements are given by

$$m(F_1)=\langle 0.8485, 0.0307\rangle,\ m(F_2)=\langle 0.0101, 0.9831\rangle \text{ and } m(F_3)=\langle 0.1414, 0.0015\rangle$$

Counter-Intuitiveness: If $\sum_{B\cap C=\phi} m_1^+(B)m_2^+(C)=1$ and $\sum_{B\cap C=\phi} m_1^-(B)m_2^-(C)\neq 1$, then the combination rule is conflicting from its membership belief degree of q-RoBPA. On the other hand, if $\sum_{B\cap C=\phi} m_1^+(B)m_2^+(C)\neq 1$ and $\sum_{B\cap C=\phi} m_1^-(B)m_2^-(C)=1$ then the fusion rule is conflicting from its non-membership end. Again, if both $\sum_{B\cap C=\phi} m_1^+(B)m_2^+(C)=1$ and $\sum_{B\cap C=\phi} m_1^-(B)m_2^-(C)=1$, then the evidence is in conflicting from both the end.

The conflict of the above type can be handled by pre-processing the q-RoBPAs via the weighted average technique similar to followed in BPA in the classical evidence theory. The weight of the evidence is obtained by using the various uncertainty measure. We have proposed a new association coefficient measure of belief functions below and shows its efficiency and applicability in conflict resolution in the new q-RoBPA with example 2.

5.1. Association coefficient measure

Let m_1 and m_2 denotes the BPAs on the FOD X. The association coefficient measure is defined as

$$a(m_1,m_2)=\frac{r(m_1,m_2)}{\frac{1}{2}\{r(m_1,m_1)+r(m_2,m_2)\}} \tag{22}$$

where $r_{Pro}(m_1,m_2)=\sum_{i=1}^{2^N-1}\sum_{j=1}^{2^N-1} m_1(F_i)m_2(F_j)\times\frac{(2^{|F_i\cap F_j|}-1)^3}{(2^{|F_i|}-1)(2^{|F_j|}-1)(2^{|F_i\cup F_j|}-1)}$, (23)

$r(m_1,m_2)$ can also be represented as $r(m_1,m_2)=m_1(F_i)Dm_2(F_j)$ with

$$D=\frac{(2^{|F_i\cap F_j|}-1)^3}{(2^{|F_i|}-1)(2^{|F_j|}-1)(2^{|F_i\cup F_j|}-1)}, \tag{24}$$

where D is a positive-definite matrix of order $(2^N-1)\times(2^N-1)$ and it can be expressed as the product of the invertible matrix and its transpose i.e., $D=Q^tQ$.

The proposed association coefficient measure $a(m_1,m_2)$ will trivially holds the following properties:

(i) Symmetricity $a(m_1,m_2)=a(m_2,m_1)$.
(ii) Boundedness $0\le a(m_1,m_2)\le 1$.
(iii) $a(m_1,m_2)=1 \Leftrightarrow m_1=m_2$
(iv) $a(m_1,m_2)=0 \Leftrightarrow F_i\cap F_j=\phi$ for all $F_i,F_j\in 2^X$.

Example 2: Consider the $X=\{1,2,3,\ldots,10\}$ be a FOD and two q-RoBPAs m_1 and m_2 for the events is given below

$m_1(\{2,3,4\})=\langle 0.05, 0.2375\rangle$, $m_1(\{7\})=\langle 0.05, 0.2375\rangle$, $m_1(X)=\langle 0.1, 0.225\rangle$, $m_1(A)=\langle 0.8, 0.05\rangle$, $m_1(\{1,2,3,4,5\})=\langle 0, 0.25\rangle$

$m_2(\{2,3,4\}) = \langle 0, 0.25 \rangle$, $m_2(\{7\}) = \langle 0, 0.25 \rangle$, $m_2(X) = \langle 0, 0.25 \rangle$, $m_2(A) = \langle 0, 0.25 \rangle$, $m_2(\{1,2,3,4,5\}) = \langle 1, 0 \rangle$

The effectiveness of the new proposed association coefficient measure is shown through the ten cases of different focal elements of A. We have noticed that the conflict degree $1 - a(m_1, m_2)$ of q-RoBPA is minimum for the variable event A at {1,2,3,4,5} from both the pair of support as well as non-support belief degree as given in Table 1.

Table 1. Conflict coefficient measure of q-RoBPA

Sl. No.	Variable Set	Conflict for support belief degree	Conflict for non-support belief degree
1	{1}	0.9959	0.2125
2	{1,2}	0.9877	0.2104
3	{1,2,3}	0.9476	0.1997
4	{1,2,3,4}	0.7729	0.1547
5	{1,2,3,4,5}	0.0325	0.0064
6	{1,2,3....,6}	0.5216	0.1602
7	{1,2, 3, ...,7}	0.9393	0.1989
8	{1,2, 3,....,8}	0.9825	0.2070
9	{1,2, 3, ...,9}	0.9934	0.2019
10	{1,2, 3, ..., 10}	0.9963	0.1789

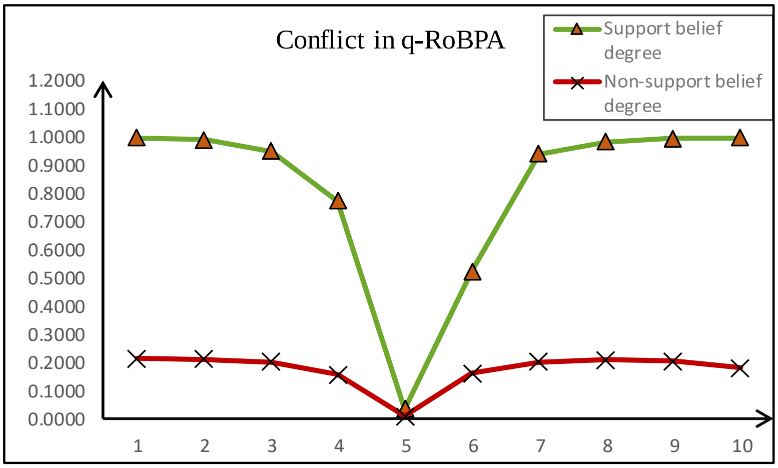

Figure 1. Comparison of conflicts for support and non-support belief degree

From the graphical representation, it is noticed that the trends of conflict coefficient of q-RoBPA first decrease with the addition of elements, attains minimum valued at A equals to {1,2,3,4,5} and increases with the addition of extra elements. The association coefficient measure is effective in handling conflict and modifying the q-RoBPA.

6. Application of q-RoBPA and Association coefficient measure in Decision-Making.

In this section, we have proposed a methodology focussing on applying the association coefficient measure on the new q-RoBPA. The methodology and algorithm are as follows.

6.1. Methodology and Algorithm

Let E be the finite set of parameters consisting of disease's symptoms $A=\{e_1,e_2,...,e_m\}$ and decision-making parameters $B=\{f_1,f_2,...,f_n\}$ respectively. Suppose (F,A) be the fuzzy soft set for representing the state of symptoms of disease and (G,B) be the fuzzy soft set representing the decision-making tools. The steps for the generation of initial q-RoBPA for the diseases $D=\{x_1,x_2,...,x_l\}$ are as follows:

Step I: Input two q-RoFSS (F,A) and (G,B) for the assessment of symptoms and decision-making tool relative to the disease as

$$F(e_i)=\left\{\frac{x_k}{\left\langle \mu_{e_i},\nu_{e_i}\right\rangle}: i=1,2,...,m\right\} \text{ and } G(f_j)=\left\{\frac{x_k}{\left\langle \mu_{f_j},\nu_{f_j}\right\rangle}: j=1,2,...,n\right\},$$

for all x_k , the fuzzy soft set can also be written in the matrix form as follows

$$(F,A)=\begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & \cdots & e_m \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_l \end{array} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{l1} & a_{l2} & \dots & a_{lm} \end{bmatrix} \end{array}; \quad (G,B)=\begin{array}{c c} & \begin{array}{cccc} f_1 & f_2 & \cdots & f_n \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_l \end{array} & \begin{bmatrix} b_{11} & b_{11} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ b_{l1} & b_{l2} & \dots & b_{ln} \end{bmatrix} \end{array}$$

where $a_{ij}=\left\{\left(x_i,\left\langle \mu_{e_j},\nu_{e_j}\right\rangle\right)\right\}$ and $b_{ij}=\left\{\left(x_i,\left\langle \mu_{f_j},\nu_{f_j}\right\rangle\right)\right\}$ are the relative q-rung fuzzy number of x_i with respect to the different parameters.

Step II: Construct $t=mn$ number of new parameters $p_1=e_1\wedge f_1$, $p_2=e_1\wedge f_2$,..., $p_t=e_i\wedge f_j$,..., $p_t=e_m\wedge f_n$ based on the "AND" operation of the FSSs. In matrix form, we have

$$M=\begin{array}{c c} & \begin{array}{ccccccc} p_1 & p_2 & \cdots & p_i & p_{i+1} & \cdots & p_t \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_l \end{array} & \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} & c_{1i+1} & \dots & c_{1t} \\ c_{21} & c_{22} & \dots & c_{2i} & c_{2i+1} & \dots & c_{2t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ c_{l1} & c_{l2} & \dots & c_{li} & c_{li+1} & \dots & c_{lt} \end{bmatrix} \end{array},$$

where $c_{ij}=\left\{\left\langle x_i,\min\left(MF(a_{ij}),MF(b_{ij})\right),\max\left(NMF(a_{ij}),NMF(b_{ij})\right)\right\rangle : a_{ij},b_{ij}\in[0,1]\right\}$ represents the membership value of x_i with respect to the new parameters.

Step III: Construct the information structure image matrix M of alternatives by the membership degree as follows

$$M = \begin{array}{c|cccccc|} & p_1 & p_2 & \cdots & p_i & p_{i+1} & \cdots & p_t \\ x_1 & \tilde{c}_{11} & \tilde{c}_{12} & \cdots & \tilde{c}_{1n} & \tilde{c}_{1i+1} & \cdots & \tilde{c}_{1t} \\ x_2 & \tilde{c}_{21} & \tilde{c}_{22} & \cdots & \tilde{c}_{2i} & \tilde{c}_{2i+1} & \cdots & \tilde{c}_{2t} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ x_l & \tilde{c}_{l1} & \tilde{c}_{l2} & \cdots & \tilde{c}_{li} & \tilde{c}_{li+1} & \cdots & \tilde{c}_{lt} \end{array}, \quad \tilde{c}_{ij} = \left\langle \frac{\tilde{\mu}_{p_j}}{\sum_{j=1}^{t} \tilde{\mu}_{p_j}}, \frac{\tilde{\nu}_{p_j}}{\sum_{j=1}^{t} \tilde{\nu}_{p_j}} \right\rangle$$

where $\{\tilde{c}_{ij}\} = \{\tilde{c}_{1j}, \tilde{c}_{2j}, ..., \tilde{c}_{lj}\}$ is the information structure image sequence (Li et al. 2015).

Step IV: Build t pieces of q-rung belief function m_i generated from the new parameters as follows

$$m_1(x_1) = \left\langle \bar{\tilde{c}}_{11}^{+}, \bar{\tilde{c}}_{11}^{-} \right\rangle, m_1(x_2) = \left\langle \bar{\tilde{c}}_{21}^{+}, \bar{\tilde{c}}_{21}^{-} \right\rangle, ..., m_1(x_i) = \left\langle \bar{\tilde{c}}_{i1}^{+}, \bar{\tilde{c}}_{i1}^{-} \right\rangle, ..., m_1(x_l) = \left\langle \bar{\tilde{c}}_{l1}^{+}, \bar{\tilde{c}}_{l1}^{-} \right\rangle$$

$$m_2(x_1) = \left\langle \bar{\tilde{c}}_{12}^{+}, \bar{\tilde{c}}_{12}^{-} \right\rangle, m_2(x_2) = \left\langle \bar{\tilde{c}}_{22}^{+}, \bar{\tilde{c}}_{22}^{-} \right\rangle, ..., m_2(x_i) = \left\langle \bar{\tilde{c}}_{i2}^{+}, \bar{\tilde{c}}_{i2}^{-} \right\rangle, ..., m_2(x_l) = \left\langle \bar{\tilde{c}}_{l2}^{+}, \bar{\tilde{c}}_{l2}^{-} \right\rangle$$

....

....

....

$$m_t(x_1) = \left\langle \bar{\tilde{c}}_{1t}^{+}, \bar{\tilde{c}}_{1t}^{-} \right\rangle, m_t(x_2) = \left\langle \bar{\tilde{c}}_{2t}^{+}, \bar{\tilde{c}}_{2t}^{-} \right\rangle, ..., m_t(x_i) = \left\langle \bar{\tilde{c}}_{it}^{+}, \bar{\tilde{c}}_{it}^{-} \right\rangle, ..., m_t(x_l) = \left\langle \bar{\tilde{c}}_{lt}^{+}, \bar{\tilde{c}}_{lt}^{-} \right\rangle$$

Here $m_j(D) = \left\langle 1 - \sum_{i=1}^{l} \bar{\tilde{c}}_{ij}^{+}, 1 - \sum_{i=1}^{l} \bar{\tilde{c}}_{ij}^{-} \right\rangle$, $\bar{\tilde{c}}_{ij}^{+} = \tilde{c}_{ij}^{+}(1 - \bar{E}_d^{+}(p_i))$, $\bar{\tilde{c}}_{ij}^{-} = \tilde{c}_{ij}^{-}(1 - \bar{E}_d^{-}(p_i))$ and $\bar{E}_d(m_i) = -\sum_i m(x_i) \log_2 \left(\frac{m(x_i)}{2^{|x_i|} - 1} \right)$ is the Deng's belief entropy (Deng, 2015) of the parameters.

Step V: The support degree of the q-RoBPA m_i, $i = 1, 2, ..., t$ are defined as

$$Sup(m_i) = \left(Sup(m_i^{+}), Sup(m_i^{-}) \right),$$

where $Sup(m_i^{+})$ and $Sup(m_i^{-})$ represents the support of membership belief function and non-membership belief function such that

$$Sup(m_i^{+}) = \sum_{j=1, j \neq i}^{t} a_{Pro}(m_i^{+}, m_j^{+}) \text{ and } Sup(m_i^{+}) = \sum_{j=1, j \neq i}^{t} a_{Pro}(m_i^{-}, m_j^{-})$$

Step VI: The degree of credibility of q-RoBPAs is given by

$$Crd(m_i) = \left(\frac{Sup(m_i^{+})}{\sum_{i=1}^{n} Sup(m_i^{+})}, \frac{Sup(m_i^{-})}{\sum_{i=1}^{n} Sup(m_i^{-})} \right),$$

where the credibility $Crd(m_i)$ is the weight of m_i such that $\sum_{i=1}^{n} Crd(m_i) = 1$.

Step VII: The weighted average mass $WAM(m)$ of the evidences m is given by

$$WAM(m) = \left(WAM(m^+), WAM(m^-)\right)$$

Step VIII: The weighted average mass of alternatives is combined separately for the membership and non-membership belief degree up to $n-1$ by using the Dempster's combination rule.

Step IX: Rank the alternatives based on the score values of its belief measure by using the equation

$$S(\{x_i\}) = \mu(x_i) - \nu(x_i),$$

Thus, the patient is suffering from x_i if $\max\{S(x_1), S(x_2), \ldots, S(x_l)\}$ with support belief measure $\{Bel^+(x_i) : i = 1, 2, \ldots, l\}$ and non-support belief measure $\{Bel^-(x_i) : i = 1, 2, .., l\}$.

7. Numerical Example in Medical Diagnosis

In this section, we have implemented the various concepts related to uncertainty to execute the case study in medical diagnosis taken from the example 6.2 in Basu et al. (2012).

Consider a patient is under observation have noticed some symptoms among the several symptoms namely fever s_1, running nose s_2, weakness s_3, oro-facial pain s_4, nausea or vomiting s_5, swelling s_6 and trismus s_7 respectively. The set $D = \{d_1, d_2, d_3, d_4\}$ of four possible diseases associated with the set of symptoms, where d_1, d_2, d_3 and d_4 stands for the disease Acute dental abscess, Migraine, Acute sinusitis and Peritonsillar abscess respectively. Let the symptoms and other decision-making tools history (h), physical examination (p) and lab investigation (l) together forms the set of parameters E such that $E = \{s_1, s_2, \ldots, s_7, h, p, l\}$ associated with possible disease.

An expert assessed a patient's disease possibility based on the responses made by the patient against his symptoms, history, physical examination and laboratory investigation etc. Let the parameters $A = \{s_1, s_2, \ldots, s_7\}$ and $B = \{h, p, l\}$ forms two q-rung orthopair fuzzy soft sets such that

$(F, A) = \{F(s_1), F(s_2), \ldots, F(s_7)\}$ and $(G, B) = \{G(h), G(p), G(l)\}$,

where the membership degree is defined by generalised form of fuzzy sets i.e., Intuitionistic, Pythagorean or q-rung orthopair fuzzy number as

$$F(s_1) = \left\{\frac{d_1}{\langle 0.6, 0.4\rangle}, \frac{d_2}{\langle 0.2, 0.8\rangle}, \frac{d_3}{\langle 0.3, 0.7\rangle}, \frac{d_4}{\langle 0.4, 0.6\rangle}\right\}, \quad F(s_2) = \left\{\frac{d_1}{\langle 0, 0.7\rangle}, \frac{d_2}{\langle 0, 0.7\rangle}, \frac{d_3}{\langle 0.7, 0.4\rangle}, \frac{d_4}{\langle 0, 0.7\rangle}\right\},$$

$$F(s_3) = \left\{\frac{d_1}{\langle 0.6, 0.4\rangle}, \frac{d_2}{\langle 0.1, 0.8\rangle}, \frac{d_3}{\langle 0.3, 0.4\rangle}, \frac{d_4}{\langle 0.2, 0.7\rangle}\right\}, \quad F(s_4) = \left\{\frac{d_1}{\langle 0.9, 0.1\rangle}, \frac{d_2}{\langle 0.9, 0.1\rangle}, \frac{d_3}{\langle 0.8, 0.2\rangle}, \frac{d_4}{\langle 0.7, 0.3\rangle}\right\},$$

$$F(s_5) = \left\{\frac{d_1}{\langle 0, 0.9\rangle}, \frac{d_2}{\langle 0.8, 0.2\rangle}, \frac{d_3}{\langle 0.3, 0.6\rangle}, \frac{d_4}{\langle 0.1, 0.8\rangle}\right\}, \quad F(s_6) = \left\{\frac{d_1}{\langle 0.7, 0.3\rangle}, \frac{d_2}{\langle 0, 0.9\rangle}, \frac{d_3}{\langle 0.4, 0.6\rangle}, \frac{d_4}{\langle 0.6, 0.4\rangle}\right\},$$

$$F(s_7)=\left\{\frac{d_1}{\langle 0.8,0.2\rangle},\frac{d_2}{\langle 0,0.8\rangle},\frac{d_3}{\langle 0,0.8\rangle},\frac{d_4}{\langle 0.5,0.4\rangle}\right\};\ G(h)=\left\{\frac{d_1}{\langle 0.6,0.2\rangle},\frac{d_2}{\langle 0.8,0.1\rangle},\frac{d_3}{\langle 0.8,0.2\rangle},\frac{d_4}{\langle 0.6,0.2\rangle}\right\},$$

$$G(p)=\left\{\frac{d_1}{\langle 0.8,0.2\rangle},\frac{d_2}{\langle 0.3,0.5\rangle},\frac{d_3}{\langle 0.4,0.6\rangle},\frac{d_4}{\langle 0.8,0.2\rangle}\right\},G(l)=\left\{\frac{d_1}{\langle 0.4,0.6\rangle},\frac{d_2}{\langle 0.6,0.4\rangle},\frac{d_3}{\langle 0.7,0.4\rangle},\frac{d_4}{\langle 0.3,0.7\rangle}\right\}$$

Since, the patient expressing three symptoms fever s_1, running nose s_2, and facial pain s_4, the nine possible pairs of parameters p_i is represented by the pairs (s_1,h), (s_1,p), (s_1,l), (s_2,h), (s_2,p), (s_2,l), (s_4,h), (s_4,p), (s_4,l) respectively. We consider the set $D=\{d_1,d_2,d_3,d_4\}$ as the frame of discernment and each pair is represented as the evidence. The matrix of membership degree of d_i relative to the joint parameters obtained from the q-RoFSSs (F,A) and (G,B) is given by

$$M=\begin{array}{c} \\ d_1\\ d_2\\ d_3\\ d_4\end{array}\begin{array}{c}\begin{array}{ccccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9\end{array}\\ \begin{bmatrix}\langle 0.6,0.4\rangle & \langle 0.6,0.4\rangle & \langle 0.4,0.6\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1\rangle & \langle 0.0,1.0\rangle & \langle 0.6,0.4\rangle & \langle 0.8,0.2\rangle & \langle 0.4,0.6\rangle\\ \langle 0.2,0.8\rangle & \langle 0.2,0.8\rangle & \langle 0.2,0.8\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1\rangle & \langle 0.0,1.0\rangle & \langle 0.8,0.2\rangle & \langle 0.3,0.7\rangle & \langle 0.6,0.4\rangle\\ \langle 0.3,0.7\rangle & \langle 0.3,0.7\rangle & \langle 0.3,0.7\rangle & \langle 0.7,0.3\rangle & \langle 0.4,0.6\rangle & \langle 0.7,0.3\rangle & \langle 0.8,0.2\rangle & \langle 0.4,0.6\rangle & \langle 0.7,0.3\rangle\\ \langle 0.4,0.6\rangle & \langle 0.6,0.4\rangle & \langle 0.3,0.7\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.0,1.0\rangle & \langle 0.6,0.4\rangle & \langle 0.7,0.3\rangle & \langle 0.3,0.7\rangle\end{bmatrix}\end{array}$$

The fuzzy information structure image matrix M is constructed based on the step I-III given by

$$\tilde{M}=\begin{bmatrix}\langle 0.40,0.16\rangle & \langle 0.40,0.16\rangle & \langle 0.3333,0.2143\rangle & \langle 0.0,0.3030\rangle & \langle 0.0,0.2778\rangle & \langle 0.0,0.303\rangle & \langle 0.2143,0.3333\rangle & \langle 0.3636,0.1111\rangle & \langle 0.20,0.30\rangle\\ \langle 0.1333,0.32\rangle & \langle 0.1333,0.32\rangle & \langle 0.1667,0.2857\rangle & \langle 0.0,0.3030\rangle & \langle 0.0,0.2778\rangle & \langle 0.0,0.303\rangle & \langle 0.2857,0.1667\rangle & \langle 0.1364,0.3889\rangle & \langle 0.30,0.20\rangle\\ \langle 0.20,0.28\rangle & \langle 0.20,0.28\rangle & \langle 0.25,0.25\rangle & \langle 1.0,0.0909\rangle & \langle 1.0,0.1667\rangle & \langle 1.0,0.0909\rangle & \langle 0.40,0.16\rangle & \langle 0.1818,0.3333\rangle & \langle 0.35,0.15\rangle\\ \langle 0.2667,0.24\rangle & \langle 0.2667,0.24\rangle & \langle 0.25,0.25\rangle & \langle 0.0,0.3030\rangle & \langle 0.0,0.2778\rangle & \langle 0.0,0.0303\rangle & \langle 0.40,0.16\rangle & \langle 0.3182,0.1667\rangle & \langle 0.15,0.35\rangle\end{bmatrix}$$

Now, the generated initial q-RoBPA m_i can be obtained in Table 2 and evaluated based on the Step I-IV and the initial q-RoBPAs are modified by using the association coefficient measure defined in equation (22)-(24).

Table 2. Q-rung orthopair basic probability assignments of alternatives.

	d_1	d_2	d_3	d_4	D
m_1	$\langle 0.3346,0.1419\rangle$	$\langle 0.1115,0.2838\rangle$	$\langle 0.1673,0.2484\rangle$	$\langle 0.2231,0.213\rangle$	$\langle 0.1635,0.1129\rangle$
m_2	$\langle 0.3346,0.1419\rangle$	$\langle 0.1115,0.2838\rangle$	$\langle 0.1673,0.2484\rangle$	$\langle 0.2231,0.213\rangle$	$\langle 0.1635,0.1129\rangle$
m_3	$\langle 0.2767,0.1897\rangle$	$\langle 0.1384,0.2528\rangle$	$\langle 0.2076,0.2213\rangle$	$\langle 0.2076,0.2213\rangle$	$\langle 0.1697,0.1149\rangle$
m_4	$\langle 0,0.2701\rangle$	$\langle 0,0.2701\rangle$	$\langle 1,0.0801\rangle$	$\langle 0,0.2701\rangle$	$\langle 0,0.1087\rangle$
m_5	$\langle 0,0.2462\rangle$	$\langle 0,0.2462\rangle$	$\langle 1,0.1477\rangle$	$\langle 0,0.2462\rangle$	$\langle 0,0.1137\rangle$
m_6	$\langle 0,0.2701\rangle$	$\langle 0,0.2701\rangle$	$\langle 1,0.0801\rangle$	$\langle 0,0.2701\rangle$	$\langle 0,0.1087\rangle$
m_7	$\langle 0.1774,0.2964\rangle$	$\langle 0.2366,0.1482\rangle$	$\langle 0.2366,0.1482\rangle$	$\langle 0.1774,0.2964\rangle$	$\langle 0.1720,0.1108\rangle$

	d_1	d_2	d_3	d_4	D
m_8	$\langle 0.3039, 0.0993\rangle$	$\langle 0.1140, 0.3476\rangle$	$\langle 0.1519, 0.2979\rangle$	$\langle 0.266, 0.149\rangle$	$\langle 0.1642, 0.1062\rangle$
m_9	$\langle 0.1666, 0.2424\rangle$	$\langle 0.2499, 0.2424\rangle$	$\langle 0.2916, 0.2154\rangle$	$\langle 0.125, 0.1885\rangle$	$\langle 0.1668, 0.1113\rangle$

Therefore, the association coefficient measure matrix A^+ and A^- of the belief function for the support belief degree as well as non-support belief degree is given by

$$A^+ = \begin{array}{c} \\ m_1^+ \\ m_2^+ \\ m_3^+ \\ m_4^+ \\ m_5^+ \\ m_6^+ \\ m_7^+ \\ m_8^+ \\ m_9^+ \end{array} \begin{array}{c} \begin{array}{ccccccccc} m_1^+ & m_2^+ & m_3^+ & m_4^+ & m_5^+ & m_6^+ & m_7^+ & m_8^+ & m_9^- \end{array} \\ \begin{bmatrix} 1 & 1 & 0.986 & 0.2731 & 0.2731 & 0.2731 & 0.8917 & 0.9934 & 0.8387 \\ 1 & 1 & 0.986 & 0.2731 & 0.2731 & 0.2731 & 0.8917 & 0.9934 & 0.8387 \\ 0.986 & 0.986 & 1 & 0.3437 & 0.3437 & 0.3437 & 0.9488 & 0.9818 & 0.9106 \\ 0.2731 & 0.2731 & 0.3437 & 1 & 1 & 1 & 0.3938 & 0.2488 & 0.4792 \\ 0.2731 & 0.2731 & 0.3437 & 1 & 1 & 1 & 0.3938 & 0.2488 & 0.4792 \\ 0.2731 & 0.2731 & 0.3437 & 1 & 1 & 1 & 0.3938 & 0.2488 & 0.4792 \\ 0.8917 & 0.8917 & 0.9488 & 0.3938 & 0.3938 & 0.3938 & 1 & 0.8938 & 0.9854 \\ 0.9934 & 0.9934 & 0.9818 & 0.2488 & 0.2488 & 0.2488 & 0.8938 & 1 & 0.8283 \\ 0.8387 & 0.8387 & 0.9106 & 0.4792 & 0.4792 & 0.4792 & 0.9854 & 0.8283 & 1 \end{bmatrix} \end{array},$$

and

$$A^- = \begin{array}{c} \\ m_1^- \\ m_2^- \\ m_3^- \\ m_4^- \\ m_5^- \\ m_6^- \\ m_7^- \\ m_8^- \\ m_9^- \end{array} \begin{array}{c} \begin{array}{ccccccccc} m_1^- & m_2^- & m_3^- & m_4^- & m_5^- & m_6^- & m_7^- & m_8^- & m_9^- \end{array} \\ \begin{bmatrix} 1 & 1 & 0.9878 & 0.8956 & 0.9462 & 0.8956 & 0.8692 & 0.9735 & 0.9686 \\ 1 & 1 & 0.9878 & 0.8956 & 0.9462 & 0.8956 & 0.8692 & 0.9735 & 0.9686 \\ 0.9878 & 0.9878 & 1 & 0.9218 & 0.9670 & 0.9218 & 0.9089 & 0.9396 & 0.9783 \\ 0.8956 & 0.8956 & 0.9218 & 1 & 0.9864 & 1 & 0.9558 & 0.8032 & 0.9419 \\ 0.9462 & 0.9462 & 0.9670 & 0.9864 & 1 & 0.9864 & 0.9671 & 0.8753 & 0.9814 \\ 0.8956 & 0.8956 & 0.9218 & 1 & 0.9864 & 1 & 0.9558 & 0.8032 & 0.9419 \\ 0.8692 & 0.8692 & 0.9089 & 0.9558 & 0.9671 & 0.9558 & 1 & 0.7476 & 0.9372 \\ 0.9735 & 0.9735 & 0.9396 & 0.8032 & 0.8753 & 0.8032 & 0.7476 & 1 & 0.9147 \\ 0.9686 & 0.9686 & 0.9783 & 0.9419 & 0.9814 & 0.9419 & 0.9372 & 0.9147 & 1 \end{bmatrix} \end{array}$$

The support degree of the q-rung ortho pair basic probability assignment is now obtained as follows

$$Sup(m_1) = \langle 5.5291, 7.5365\rangle,\ Sup(m_2) = \langle 5.5291, 7.5365\rangle,\ Sup(m_3) = \langle 5.8443, 7.6130\rangle,$$
$$Sup(m_4) = \langle 4.0117, 7.4003\rangle,\ Sup(m_5) = \langle 4.0117, 7.4003\rangle,\ Sup(m_6) = \langle 4.0117, 7.4003\rangle,$$
$$Sup(m_7) = \langle 5.7928, 7.2108\rangle,\ Sup(m_8) = \langle 5.4371, 7.0306\rangle,\ Sup(m_9) = \langle 5.8393, 7.6326\rangle$$

and the corresponding credibility degree of the belief assignments are obtained as follows

$$Crd(m_1) = \langle 0.1210, 0.1125\rangle,\ Crd(m_2) = \langle 0.1210, 0.1125\rangle,\ Crd(m_3) = \langle 0.1270, 0.1136\rangle,$$
$$Crd(m_4) = \langle 0.0862, 0.1104\rangle,\ Crd(m_5) = \langle 0.0862, 0.1104\rangle,\ Crd(m_6) = \langle 0.0862, 0.1104\rangle,$$
$$Crd(m_7) = \langle 0.1260, 0.1076\rangle,\ Crd(m_8) = \langle 0.1190, 0.1049\rangle,\ Crd(m_9) = \langle 0.1276, 0.1139\rangle$$

Now, the weighted average mass WAM(m) of the alternative are obtained as follows

$m(d_1) = \langle 0.1950, 0.2111 \rangle$, $m(d_2) = \langle 0.1194, 0.2604 \rangle$, $m(d_3) = \langle 0.4129, 0.1875 \rangle$, $m(d_4) = \langle 0.1496, 0.2298 \rangle$, $m(D) = \langle 0.1231, 0.1112 \rangle$

The above WAM(m) of alternative is combined for eight times to itself by using the Dempster's combination rule (21) and the final belief measure for d_i is shown below

$m(d_1) = \langle 0.00902, 0.14785 \rangle$, $m(d_2) = \langle 0.00078, 0.53130 \rangle$, $m(d_3) = \langle 0.98794, 0.07461 \rangle$, $m(d_4) = \langle 0.00225, 0.24575 \rangle$, and $m(D) = \langle 0.000002, 0.00001 \rangle$

Since, the final belief measures of alternatives are in the form of Intuitionistic fuzzy number and this can be ranked based on the score of the intuitionistic BPAs, then we have

$S(\{d_1\}) = -\ 0.1393$, $S(\{d_2\}) = -\ 0.5305$, $S(d_3) = \ 0.9133$, $S(d_4) = -\ 0.2435$

Finally, from values of the score functions we can conclude that the patient has the possibility of suffering from the disease of d_3 with the support belief degree 0.98794 and non-support belief degree 0.07455 respectively.

Table 3. Comparison of final belief measure

Different methods	Type of BPA	Ranking order	$Bel(\{d_3\})$	Score value of d_3
Li et al.'s, method (2015)	Discrete	$d_3 > d_1 > d_4 > d_2$	0.8349	NA
Wang et al.'s method (2016)	Discrete	$d_3 > d_1 > d_4 > d_2$	0.9906	NA
Xiao's, method (2018)	Discrete	$d_3 > d_1 > d_4 > d_2$	0.99996	NA
Chen et al.'s method (2019)	Discrete	$d_3 > d_1 > d_4 > d_2$	0.9989	NA
The proposed method	q-rung fuzzy number	$d_3 > d_1 > d_4 > d_2$	$\langle 0.9879, 0.0746 \rangle$	0.9133

From the comparison as shown in Table 3, the proposed method suggests the same decision and follows the same ranking order $d_3 > d_1 > d_4 > d_2$ as earlier method suggests (Li et al. 2015; Wang et al. 2016; Xiao, 2018; Chen et al. 2019). The advantages of the proposed methodology over the others are that rest of the methods is based on the discreate number while our proposed method has more flexible one in the sense that the assessment of alternatives with respect to the parameters can be made based on membership and non-membership belief degree. In addition, the conflicts in the belief if exist will handled by the proposed association coefficient measure. However, the methodology has certain limitation as assessment of alternatives is based on the human subjective expertise, the assessment may give false decision if the decision-maker deliberately provides some false assessment membership belief degree.

8. Conclusion

In this paper, we have proposed a new q-rung orthopair basic probability assignment consisting of membership and non-membership belief degree to provide decision-maker's flexibility in assigning his belief degree to a proposition. We have investigated various essential concepts of the classical Dempster-Shafer theory and related uncertainty measures in the literature of evidence theory. To cope with uncertainty in the q-RoBPA, we have further implemented our novel association coefficient measure to obtain the pre-process evidence. Finally, a methodology is developed to apply the proposed q-RoBPA and association coefficient measure with a hypothetical case study in medical diagnosis, compared with the existing results. This study reveals that the decision of an alternative from the proposed algorithm follows the same ranking order as earlier did with its support degree of belief and non-support degree of belief. From the study, a medical expert can make an action plan for the patient's treatment, which has high possibility of belief degree based on the appropriate and flexible assessment from his previous experience in the field.

The proposed methodology offers comprehensive advantages to the decision-maker for the assessment of an alternative through the membership as well as non-membership belief degree. The q-Rung orthopair basic probability assignment can easily represent the decision-maker's views on the alternatives from his experience where he has scope to assign membership degree for favourable case and non-membership belief degree for non-favoured cases from same type of situation. From the perspective of the limitations of the methodology, as the belief degree assessment is based expert's knowledge and information, so the intentional false exercise of information sharing may give abrupt results. In this regard, the experienced and loyal decision expert is required for the assessment of these alternatives. In the future, more general basic probability assignment based on the picture fuzzy set and spherical fuzzy sets, etc may be used to get more accurate, precise result and implement the method for a large real-time statistical data set of medical decision-making.

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