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FP-INTUITIONISTIC MULTI FUZZY N-SOFT SET AND ITS INDUCED FP-HESITANT N-SOFT SET IN GROUP DECISION-MAKING

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Abstract: *Intuitionistic fuzzy sets (IFSs) can effectively represent and simulate the uncertainty and diversity of judgment information offered by decision-makers (DMs). In comparison to fuzzy sets (FSs), IFSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this research work, we offer an approach for solving group decision-making problems (GDMPs) with fuzzy parameterized intuitionistic multi fuzzy N-soft set (briefly, FPIMFNSS) of dimension q by introducing its induced fuzzy parameterized hesitant N-soft set (FPHNSS) as an extension of the multi-fuzzy N-soft set (MFNSS) based group decision-making method (GDMM). In this study, we use the proposed GDMM to solve a real-life GDMP involving candidate eligibility for a single vacant position advertised by an IT firm and compare the ranking performances of the proposed GDMM with the Fatimah-Alcantud method.*

Key words: *Decision Making, Fuzzy set, Soft set, N-soft set, Intuitionistic fuzzy set.*

1. Introduction

Soft set theory (SST) was first presented by Molodtsov (1999) as a fundamental and useful mathematical method for dealing with complexity, unclear definitions, and unknown objects (elements). Since there are no limitations to the description of elements in SST, researchers may choose the type of parameters that they need, significantly simplifying decision making problems (DMPs) and making it easier to make decisions in the absence of partial knowledge, it is more effective. While several mathematical tools for modeling uncertainties are available, such as operations analysis, probability theory, game theory, FS theory, rough set theory, and interval-valued fuzzy set (IVFS), each of these theories has its own set of problems. Furthermore, all of these theories lack parameterization of the tools, which means

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they can't be used to solve problems, especially in the economic, environmental, and social realms. In the sense that it is clear of the aforementioned difficulties, SST stands out.

The SST is extremely useful in a variety of situations. Molodtsov (1999) developed the basic results of SST and successfully applied it to a variety of fields, including the smoothness of functions, operations analysis, game theory, Riemann integration, probability, and so on. Later, Maji et al. (2003) presented several new SST concepts, such as subset, complements, union, and intersection, as well as their implementations in DMPs. Ali et al. (2009) identified some more operations on SST and demonstrated that De Morgan's laws apply to these new operations in SST. To solve the DMPs, Maji et al. (2002) used SST for the first time. Recently, several authors later looked into the more broad properties and applications of SST. Fatimah et al. (2018, 2019) studied the concepts of probabilistic SST and dual probabilistic SST in DMPs with positive and negative parameters, and Alcantud (2020) introduced soft open bases and presented a new construction of soft topology from bases for topology.

Many academics are interested in hybrid models, as seen by the aforementioned references. Many hybridization options for the recently created N-soft sets (NSSs) (Fatimah et al. 2018) model. This model's primary role is to broaden the scope of SST applications, which deal with qualities that resemble the universe of discourse. Because many real-world examples have insisted on their applicability, this paradigm constitutes a practical expansion of SST (Fatimah et al., 2018; Alcantud et al., 2020; Kamachi & Petchimuthu, 2020; Kushwaha et al., 2020). In addition, it has demonstrated its theoretical flexibility: the model is adaptable to hybridization with alternative theories of ambiguity and uncertainty. Akram et al. (2018, 2019, 2019a, 2019b, 2021), Chen et al. (2020), Liu et al. (2020), and Riaz et al. (2020) have built hybrid structures that incorporate other notable properties of approximation knowledge structures. An N-soft structure (Riaz et al., 2019) exists as a natural extension of soft topology and is a natural extension of topological studies (Alcantud et al., 2020; Terepeta, 2019; Youssef and Webster, 2022).

The idea of the FS was started by Zadeh (1965), thereafter, many new approaches and ideas have been offered to deal with imprecision and ambiguity, such as the hesitant fuzzy sets (HFSs) (Torra, 2010), multi-fuzzy sets (MFSs) (Sebastian & Ramakrishnan, 2011), IFSs (Atanassov, 1986), intuitionistic multi-fuzzy sets (IMFSs) (Shinoj & John, 2012) and so on (Abdulkareem et al., 2020, 2021; Azam & Bouguila, 2019, 2020; Mohammed & Abdulkareem, 2020). FS has a wide range of applications, including databases, neural systems, pattern recognition, medicine, fuzzy modelling, economics, and multicriteria DMPs (Alcantud and Torra, 2018; Al-Qudah & Hassan, 2017, 2018). Maji et al. (2001) described fuzzy soft set (FSS), which is a hybrid of FS and SST. FSS based decision-making method (DMM) was first proposed by Roy and Maji (2007). Thereafter, the applications of FSSs have been gradually concentrated by using these concepts. Feng et al. (2010) introducing an adjustable DMM to solve FSS-based DMPs. Thereafter, Zhu and Zhan (2015) described and presented the t-norm operations on fuzzy parameterized FSSs, as well as shown their applications in DMPs. Çağman et al. (2010, 2011) introduced the concept of FP-FSS and its applications in DMPs and later on proposed a new idea of FP-SST and shown some applications in DMPs. Das and Kar (2015) presented the concept of HFSS and studied its application in DMPs. Alcantud and Mathew (2017) recently defined separable FSS with its applications in DMPs for both positive and negative qualities. Alcantud et al. (2017) developed a methodology for asset assessment using an FSS (flexible FSS) based DMM. Al-Qudah and Hassan (2018) presented the theory of complex multi-FSS as well as studied its entropy and similarity measure. Based on revised aggregation operators, Peng and Li (2019) suggested a DMM using HFSS. Lathamaheswari et al. (2020) presented the theory of triangular interval type-2 FSS and also, shown its applications. Petchimuthu et al. (2020) defined the mean operators and generalized products of fuzzy soft matrices and discussed their applications in MCGDM. Paik and Mondal, (2020) introduced a distance-similarity method to solve fuzzy sets and FSSs based DMPs. Paik and Mondal (2021) had shown the representation and application of FSSs in a type-2 environment. Močkoř and Hurtik (2021) used the concept FSSs in image processing applications. Gao and Wu (2021) defined the notion of filter with its applications in topological spaces formed by FSSs. Dalkılıç and Demirtaş (2021) introduced the idea of bipolar fuzzy soft D-metric spaces. Dalkiliç (2021) defined topology on virtual FP-FSSs. Bhardwaj and Sharma (2021) described an advanced uncertainty measure using FSSs and shown its application in DMPs. Atanassov (1986) suggested the notion of IFS as a generalization of FS. Maji et al. (2001, 2004) defined intuitionistic-FSS (IFSS) as an important mathematical method for solving DMPs in an uncertain situation by combining SST with IFS. Das and Kar (2014) proposed a GDMM in medical system using IFSS and Das et al. (2014) suggested a MAGDM based on interval-valued intuitionistic fuzzy soft matrix. Later on, Das et al. (2018) proposed a DMM based on intuitionistic trapezoidal FSS and Krishankumar et al. (2019) presented a framework for MAGDM using double hierarchy hesitant fuzzy linguistic term set. Also, Das et al. (2019) presented the concept of correlation measure of HFSSs as well as their applications in DMPs.

The topic of intertemporal FSS selection was first raised by Alcantud and Muoz Torrecillas (2017). The algorithms for IVFSSs in stochastic MCDM and neutrosophic soft DMM were introduced by Peng and others (2017, 2017a). Furthermore, based on CODAS and WDBA with novel information measures, Peng and Garg (2018) suggested unique algorithms for IVFSSs in emergency DMPs. In the case of SST, Zhan and Alcantud (2019) provide an updated assessment of the parameter reduction literature. One or more of the following constraints limited the majority of SST researchers (for example (Ma et al., 2017) or other updated hybrid model summaries): The evaluations can either be binary integers between 0 and 1, or real values between 0 and 1, such as FSSs or separable FSSs (Maji et al., 2001).
Both scenarios are discussed by Alcantud and Santos-García (2017), which includes an examination of partial data. In scenarios such as social assessment systems or the presentation of ranking positions in ordinary life, however, we encounter information with a different framework that is not binary. NSSs (Fatimah et al., 2018) are, nonetheless, the accurate formal expression of the concept of a parameterized description of the universe of objects based on a finite number of ordered grades and the other extended structures of SST that have been linked to social choice were mentioned by Fatimah et al. (2018, 2019). The idea of parameter reduction in NSSs was recently presented by Akram et al. (2020). When the membership degrees of the alternatives are not uniquely defined, such as due to group knowledge or hesitancy (2011, 2015), HFSs (2019) are useful. Hesitancy is a model that can be combined with other key structures, for contemporary examples see, Fatimah and Alcantud (2018), Liu & Zhang (2017, 2017a), Peng et al. (2017, 2018). Recently, Krishankumar et al. (2021, 2021a) presented a decision framework under probabilistic hesitant fuzzy environment with probability estimation for multi-criteria decision making and introduced the idea of Interval-valued probabilistic hesitant fuzzy set-based framework for group decision-making with unknown weight information. Fatimah

and Alcantud (2021) introduced the concept of MFNSS and developed a GDMM as a generalization of the successful idea of NSS and MFS, for solving GDMPs.

IFSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by DMs. In comparison to FSs, IFSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this paper, we offer an approach for solving GDMPs with FPIMFNSS, by extending the MFNSS based GDMM. The new structure combines the advantages of IMFS with those of FP-soft sets and NSSs, three structures that have received a lot of attention in current years. The constructed method in this paper is very advantageous for solving GDMPs. In this study, we use the proposed GDMM to solve a real-life GDMP involving candidate eligibility for a single vacant position advertised by an IT firm and compare the ranking performances of the proposed GDMM with the Fatimah-Alcantud method. The following is the structure of this paper: The essential concepts and conclusions of FS, IFS, soft set, IMFS, NSS, MFNSS, and IFNSS are presented in Sec. 3, which will be important in later discussions. In Sec. 4, we define FPIMFNSS and its induced FPHNSS as an extension of the MFNSS, along with some fundamental features. In Sec. 5, we present an advanced and adjustable GDMM for solving GDMPs based on FPIMFNSSs. In Sec. 6, we show the validity of our proposed GDMM with the help of one real-life example, and in Sec. 7, we address comparison analysis with the Fatimah-Alcantud method. Finally, in Sec. 8, we bring the paper to a conclusion and our future work.

2. Literature review

Torra (2010) was the first to propose the idea of HFSs. Torra and Narukawa (2009) described certain new operations on HFSs and used them in DMPs. Xia and Xu (2011) proposed hesitant fuzzy information aggregation in DMPs. Zhu et al. (2014) suggested a technique for deriving a ranking from hesitant fuzzy preference relations under group DMPs. Das and Kar (2014) proposed a GDMM in medical system using IFSS and Das et al. (2014) suggested a MAGDM based on interval-valued intuitionistic fuzzy soft matrix. Liu and Zhang (2017, 2017a) proposed an MCDM technique with neutrosophic hesitant fuzzy heronian mean aggregation operators and also, developed an extended MCDM technique with the help of neutrosophic hesitant fuzzy information. Peng and Dai (2017) proposed hesitant fuzzy soft DMMs based on COPRAS, MABAC, and WASPAS with combined weights. Fatimah and Alcantud (2018) initiated the idea of expanded dual HFSs and Peng and Li (2019) proposed a hesitant fuzzy soft DMM with the help of revised aggregation operators, CODAS and WDBA. Sebastian and Ramakrishnan (2011) defined MFS as an extension of FS. Thereafter, Shinoj and John (2012) developed the concept of IF-multisets and applied it in medical diagnosis. Yang et al. (2013) developed the theory of MFSS and its application in DMPs. Dey and Pal (2015) developed the concept of generalized MFSS and its application in DMPs.

Fatimah et al. (2018) first introduce the idea of NSSs and based on NSSs they proposed DMMs. Later on, Akram et al. (2018) initiated the theory of a novel model of fuzzy NSSs with its applications in DMPs. Riaz et al. (2019) introduced the theories of N-soft topologies and shown their applications in MCGDM. Akram et al. (2019) developed some group DMMs based on HNSSs. Later on, Akram et al. (2019) presented a novel structure of hesitant fuzzy NSSs with its applications in DMPs. Akram and Adeel (2019) proposed the TOPSIS approach to MCGDM with the help of an interval-valued hesitant fuzzy N-soft environment and Akram et al. (2019) developed a new hybrid DMM with IF-N-soft rough sets. Das et al. (2018) proposed a

DMM based on intuitionistic trapezoidal fuzzy soft set and Krishankumar et al. (2019) presented a framework for MAGDM using double hierarchy hesitant fuzzy linguistic term set. Also, Das et al. (2019) presented the concept of correlation measure of hesitant fuzzy soft sets as well as their applications in DMPs. Recently, Riaz et al. (2020) defined neutrosophic NSS and applied it with the TOPSIS method for MCDM. Kamacı and Petchimuthu (2020) introduced bipolar NSS theory with its applications and Liu et al. (2020) proposed an MCDM method based on neutrosophic vague NSSs. Chen et al. (2020) presented a group DMM based on generalized vague NSSs. Akram et al. (2020) presented the parameter reductions in NSSs and shown their applications in DMPs and Alcantud et al. (2020) presented an NSS approach using rough set. Recently, Krishankumar et al. (2021, 2021a) presented a decision framework under probabilistic hesitant fuzzy environment with probability estimation for multi-criteria decision making and introduced the idea of Interval-valued probabilistic hesitant fuzzy set-based framework for group decision-making with unknown weight information. Fatimah and Alcantud (2021) defined the theory of MFNSS and its applications to DMPs.

It is clear from the continuing literature analysis that previous researchers built tools for various real-world scenarios using NSS, FP-SST, or MFSS. Previous research studies have not combined NSS, FP-SST, and MFSS in an IF setting. IFSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by DMs. In comparison to fuzzy sets, IFSs are highly beneficial for expressing vagueness and uncertainty more accurately. Thus, in this paper, we offer an approach for solving GDMPs with FPIMFNSS by introducing its induced FPHNSS as an extension of the MFNSS based GDMM. MFNSS is a fantastic and useful tool to deal with GDMPs, but it has some limitations. As a result of combining the three models, this study effort focuses on GDMPs in the real world. Consequently, the current gap in the real-time practical implementation of combined NSS, FP-SST, and MFSS in an IF setting as a FPIMFNSS would be filled by this integrated NSS, FP-SST, and MFSS model. This proposed model would provide the most up-to-date viewpoint and motion for dealing with real-world GDMPs.

3. Preliminary

Let us consider Ω represents the starting universe and Q represents a nonempty set of parameters. Let the power set of Ω is denoted by P(Ω) and P⊆Q. Let, $q, N \in \{2,3,4,5,....\}$ and $R = \{0,1,2,3,4,5,....,N-1\}$.

Definition 3.1 (Zadeh, 1965). An FS Z on Ω is a set with a structure $Z = \{(\mathrm{o}, \mu_Z(\mathrm{o})) : o \in \Omega\}$, where the real-valued function $\mu_z : \Omega \to [0, 1]$ is said to be the membership function and $\mu_z(\mathrm{o})$ is called the degree of membership for each object $o \in \Omega$.

Assume that, in this research paper FS(Ω) means the collection of all FSs on Ω.

Definition 3.2 (Atanassov, 1986). An IFS Z on Ω is a set with a structure Z={ $\langle \mathrm{o}, \mu_Z(\mathrm{o}), \nu_Z(\mathrm{o}) \rangle : \mathrm{o} \in \Omega$}, where the real-valued functions $\mu_Z : \Omega \to [0,1]$ and $\nu_Z : \Omega \to [0, 1]$ means the membership function and the non-membership function respectively, and $\mu_z(\mathrm{o})$, $\nu_z(\mathrm{o})$ are called the degree of membership and the degree of non-membership for each object $o \in \Omega$, satisfying the condition $0 \le \mu_Z(o)+\nu_Z(o) \le 1$ for each object $o \in \Omega$.

Assume that, in this research paper IFS(Ω) means the collection of all IFSs on Ω.

Definition 3.3 (Molodtsov, 1999). A soft set over the nonempty universe Ω is a pair (ψ, P), where ψ is a mapping defined by ψ: P→P (Ω).

Definition 3.4 (Shinoj and John, 2012). An IMFS Z on Ω is a set with a structure $Z=\left\{\left\langle o,(\mu_1(o),\nu_1(o)),(\mu_2(o),\nu_2(o)),....,(\mu_q(o),\nu_q(o))\right\rangle : o \in \Omega\right\}$, where the real-valued functions $\mu_k : \Omega \to [0,1]$, $\nu_k : \Omega \to [0,1]$ satisfying the condition $0 \le \mu_k(o)+\nu_k(o) \le 1$ for k =1, 2, 3,...,q and for each $o \in \Omega$.

In this research paper, $IMFs(\Omega)^q$ means the collection of all IMFSs on Ω.

Definition 3.5 (Fatimah et al., 2018). A triple (Ψ, P, N) is said to be an NSS on Ω, where $\Psi : P \to 2^{\Omega\times R}$ is a function, satisfying the condition, for each p ∈ P and o ∈ Ω there exists a unique couple $(o,r_p) \in \Omega \times R$ such that $(o,r_p) \in \Psi(p)$, $r_p \in R$. The object o belongs to the collection of p-approximations of the universal set Ω with the grade r_p, according to the interpretation of the couple $(o,r_p) \in \Psi(p)$.

Definition 3.6 (Akram et al., 2019). A triple *(ψ, P, N)* is said to be a hesitant N-soft set (simply, HNSS) over *Ω,* where $\psi : P \to 2^{\Omega\times R}$ is a function such that for every p ∈ P and o∈Ω there exists at least one couple $(o,r_p) \in \Omega \times R$ such that $(o,r_p) \in \psi(p)$, $r_p \in R$.

Definition 3.7 (Akram et al. 2019). The collection h satisfying the condition $\phi \ne h \subseteq R = \{0,1,2,3,4,5,....,N-1\}$ is said to be hesitant N-tuple (simply, HNT). Any HNSS has a tabular representation consisting of a matrix whose cells are HNTs.

Definition 3.8 (Fatimah and Alcantud, 2021). Let $MFs(\Omega)^{(N,q)}$ be the set of all q-tuples of triples of objects from $R\times[0,1]$ indexed by *Ω*, i.e., the collection of all objects having the structure $\left\{\left\langle o,(r_1(o),\mu_1(o)),(r_2(o),\mu_2(o)),....,(r_q(o),\mu_q(o))\right\rangle : o \in \Omega\right\}$, where $r_k : \Omega \to R$, and $\mu_k : \Omega \to [0,1]$ are mappings. An *MFNSS* on *Ω* is a pair (Ψ, P), such that Ψ is a mapping $\Psi : P \to MFs(\Omega)^{(N,q)}$ defined by

$$\Psi(p)=\left\{\left\langle o,(r_1(o),\mu_1(o)),(r_2(o),\mu_2(o)),....,(r_q(o),\mu_q(o))\right\rangle : o \in \Omega\right\} \quad (1)$$

Definition 3.9 (Fatimah and Alcantud, 2021). Let (Ψ,P) be an *MFNSS* on *Ω,* where $P=\{p_1,p_2,...,p_m\}\subseteq Q$ and $\alpha=(\alpha_1,\alpha_2,...,\alpha_m)$, a vector of thresholds $\alpha_i \in [0,1]$ associated with each $p_i \in P$ for i=1,2,..m. Then the *HNSS α*-induced by (Ψ,P) is the triple (H_Ψ^α,P,N) where $H_\Psi^\alpha : P \to P(R)$ is a mapping, such that

$$H_\Psi^\alpha(p_j)=\left\{\left\langle o_i,\left\{r_t^{\,j}(o_i):\mu_t^{\,j}(o_i)\ge\alpha_j, t=1,2,...,q\right\}\right\rangle : o_i \in \Omega\right\} \quad (2)$$

Definition 3.10 (Maji et al., 2001). A pair (ψ, P) is called an *IFSS* over Ω, where ψ is a function given by ψ: P→ IFS(Ω).

Definition 3.11 (Akram et al., 2019) A triple (φ,K,N) is known as IFNSS, if K= (Ψ, P, N) is an NSS over Ω and $\varphi : P \to IFS(\Omega \times$ is a mapping, where $IFS(\Omega\times R)$ means the collection of all IFSs on $\Omega\times R$.

Example 3.12. Let us consider that four candidates, denoted by $\Omega=\{o_1,o_2,o_3,o_4\}$, applied for a single vacant position advertised by an IT firm. Then, the selection board of the firm has firstly determined a parameter set $P=\{p_1,p_2,p_3\}$, such that p_1 = experience, p_2 = knowledge of foreign language, and p_3 = knowledge of software, which are used to assign grades to candidates. We can calculate a 5-soft set from Table 1, where

4 stars mean Excellent
3 stars mean very good
2 stars mean good
1 star means regular
Circle means bad.

The set of grades G = {0, 1, 2, 3, 4} can be easily associated with checkmarks as follows:

0 stands for ‘o’
1 stands for *
2 stands for **
3 stands for ***
4 stands for ****.

Then from Definition 3.5, the tabular form of 5-soft set K=(Ψ, P, 5) can be shown in Table 2 and from Definition 3.11 the IF5SS $(\varphi, K, 5)$ is given by Table 3.

Table 1. Information extracted from the related data

Ω	p_1	p_2	p_3
o_1	***	****	*
o_2	*	**	****
o_3	****	***	**
o_4	**	*	****

Table 2. The 5-soft set K=(Ψ, P, 5)

Ω	p_1	p_2	p_3
o_1	3	4	1
o_2	1	2	4
o_3	4	3	2
o_4	2	1	3

Table 3. The IF5SS $(\varphi, K, 5)$

Ω	p_1	p_2	p_3
o_1	(3,0.3,0.5)	(4,0.4,0.5)	(1,0.5,0.4)
o_2	(1,0.3,0.4)	(2,0.3,0.4)	(4,0.5,0.3)
o_3	(4,0.4,0.5)	(3,0.5,0.3)	(2,0.5,0.4)
o_4	(2,0.6,0.3)	(1,0.4,0.5)	(3,0.3,0.5)

4. Theoretical analysis of FPIMFNSS

In this research paper, we consider Ω represents the starting universe and Q represents a nonempty set of parameters. Let $P=\{p_1, p_2,..., p_m\}\subseteq Q$ and $X=\{p^{\mu_X(p)} : p\in P\}$, be an FS over P. Let $q, N\in\{2,3,4,5,....\}$ be two fixed numbers, where q is the dimension of our new structure and N distinguishes how many degrees of satisfaction with the parameters are permitted, allowing us to utilize $R=\{0,1,2,3,4,5,....,N-1\}$ as a collection of ordered grades and P(R) means the power set of R.

Definition 4.1 Let us define $IMFs(\Omega)^{(N,q)}$ as the set of all q-tuples of triples of objects from $R\times[0,1]\times[0,1]$ indexed by *Ω*, i.e., the collection of all objects having the structure $\{\langle o,(r_1(o),\mu_1(o),\nu_1(o)),(r_2(o),\mu_2(o),\nu_2(o)),....,(r_q(o),\mu_q(o),\nu_q(o))\rangle : o\in\Omega\}$, where $r_k:\Omega\to R,\ \mu_k:\Omega\to[0,1],\ \nu_k:\Omega\to[0,1]$ and satisfying $0\le\mu_k(o)+\nu_k(o)\le 1$ for k = 1, 2, 3,....., q. A *FPIMFNSS* of dimension q on *Ω* is a pair (Ψ, X) such that Ψ is a mapping $\Psi: X\to IMFs(\Omega)^{(N,q)}$ defined by $\forall p^{\mu_X(p)}\in X$,

$$\Psi\left(p^{\mu_X(p)}\right)=\{\langle o,(r_1(o),\mu_1(o),\nu_1(o)),....,(r_q(o),\mu_q(o),\nu_q(o))\rangle : o\in\Omega\} \quad (3)$$

Note: Simply, we denote the set of all *FPIMFNSS*s of dimension q over Ω by $d(\Omega,P)^{(N,q)}$ where the parameter set P is fixed.

Remark 4.2 If N=1, the members in $d(\Omega,P)^{(N,q)}$ can be matched to those in $IMFs(\Omega)^q$ and the element of $d(\Omega,P)^{(1,q)}$ is of the form

$$\{\langle o,(0,\mu_1(o),\nu_1(o)),(0,\mu_2(o),\nu_2(o)),....,(0,\mu_q(o),\nu_q(o))\rangle : o\in\Omega\}.$$

We identify it with

$\{\langle o,(\mu_1(o),\nu_1(o)),(\mu_2(o),\nu_2(o)),....,(\mu_q(o),\nu_q(o))\rangle : o\in\Omega\}\in IMFs(\Omega)^q$ in a trivial manner.

Example 4.3 Let $\Omega=\{o_1,o_2,o_3\}$ be the universe of candidates and $P=\{p_1,p_2\}$ is the set of attributes and $X=\{p_1^{0.5}, p_2^{0.7}\}$ be an FS over *P*. A FPIMF5SS of dimension 2 (Ψ, X) on Ω is defined by the assignments

$$\Psi(p_1^{0.5})=\{\langle o_1,(3,0.3,0.5),(4,0.5,0.3)\rangle,\langle o_2,(2,0.3,0.4),(4,0.5,0.3)\rangle,\langle o_3,(2,0.4,0.5),(3,0.4,0.4)\rangle\},$$

$$\Psi(p_2^{0.7})=\{\langle o_1,(1,0.4,0.5),(3,0.5,0.4)\rangle,\langle o_2,(2,0.3,0.4),(4,0.5,0.3)\rangle,\langle o_3,(3,0.5,0.3),(2,0.5,0.4)\rangle\}.$$

The tabular representation of the above FPIMF5SS of dimension 2 (Ψ, X) can be shown in Table 4.

Table 4. The FPIMF5SS of dimension $2(\Psi, X)$

Ω	p_1 0.5	p_2 0.7
o_1	(3,0.3,0.5)(4,0.5,0.3)	(1,0.4,0.5)(3,0.5,0.4)
o_2	(2,0.3,0.4)(4,0.5,0.3)	(2,0.3,0.4)(4,0.5,0.3)
o_3	(2,0.4,0.5)(3,0.4,0.4)	(3,0.5,0.3)(2,0.5,0.4)

Definition 4.4 Let us consider two FPIMFNSSs $(\psi, X), (\varphi, Y) \in d(\Omega, P)^{(N,q)}$, such that

$$\forall p^{\mu_X(p)} \in X,\ \psi\left(p^{\mu_X(p)}\right) = \left\{\left\langle o, (r_1(o), \mu_1(o), \nu_1(o)), \ldots, (r_q(o), \mu_q(o), \nu_q(o))\right\rangle : o \in \Omega\right\},$$

$$\forall p^{\mu_Y(p)} \in Y,\ \varphi\left(p^{\mu_Y(p)}\right) = \left\{\left\langle o, (r_1'(o), \mu_1'(o), \nu_1'(o)), \ldots, (r_q'(o), \mu_q'(o), \nu_q'(o))\right\rangle : o \in \Omega\right\}.$$

Then we say that

[1] Subset: $(\psi, X) \tilde{\subseteq} (\varphi, Y)$ if

(i). X is a fuzzy subset of Y, *i.e.* $\forall p \in P,\ \mu_X(p) \le \mu_Y(p)$

(ii). $\forall p \in P,\ \psi\left(p^{\mu_X(p)}\right) \subseteq \varphi\left(p^{\mu_Y(p)}\right) \Leftrightarrow r_i(o) \le r_i'(o), \mu_i(o) \le \mu_i'(o)$ and $\nu_i(o) \ge \nu_i'(o)$ $\forall o \in \Omega$, and $i = 1, 2, \ldots, q$

[2] Equal set: $(\psi, X) = (\varphi, Y)$ if

(i). $\forall p \in P,\ \mu_X(p) = \mu_Y(p)$

(ii). $\forall p \in P,\ \psi\left(p^{\mu_X(p)}\right) = \varphi\left(p^{\mu_Y(p)}\right) \Leftrightarrow r_i(o) = r_i'(o), \mu_i(o) = \mu_i'(o)$ and $\nu_i(o) = \nu_i'(o)$ $\forall o \in \Omega$, and $i = 1, 2, \ldots, q$

[3] Union: $(\psi, X) \tilde{\cup} (\varphi, Y) = (\rho, Z)$, where $Z = X \cup Y$, and $\cup$ denotes the fuzzy union and $\forall p^{\mu_Z(p)} \in Z$,

$\rho\left(p^{\mu_Z(p)}\right) = \left\{\left\langle o, (r_1''(o), \mu_1''(o), \nu_1''(o)), \ldots, (r_q''(o), \mu_q''(o), \nu_q''(o))\right\rangle : o \in \Omega\right\}$, where $\forall o \in \Omega$,

$r_i''(o) = \max\{r_i(o), r_i'(o)\},\ \mu_i''(o) = \max\{\mu_i(o), \mu_i'(o)\}$ and$\nu_1''(o) = \min\{\nu_i(o), \nu_i'(o)\},\ i = 1, 2, \ldots, q.$

[4] Intersection: $(\psi, X) \tilde{\cap} (\varphi, Y) = (\rho, Z)$, where $Z = X \cap Y$, and $\cap$ denotes the fuzzy intersection and $\forall p^{\mu_Z(p)} \in Z$,

$\rho\left(p^{\mu_Z(p)}\right) = \left\{\left\langle o, (r_1''(o), \mu_1''(o), \nu_1''(o)), \ldots, (r_q''(o), \mu_q''(o), \nu_q''(o))\right\rangle : o \in \Omega\right\}$, where $\forall o \in \Omega$,

$r_i''(o) = \min\{r_i(o), r_i'(o)\},\ \mu_i''(o) = \min\{\mu_i(o), \mu_i'(o)\}$ and $\nu_1''(o) = \max\{\nu_i(o), \nu_i'(o)\},\ i = 1, 2, \ldots, q.$

Definition 4.5 We consider a *FPIMFNSS* $(\Psi, X) \in d(\Omega, P)^{(N,q)}$. Its induced FPHNSS of dimension q is the pair (H_Ψ, X), where $H_\Psi : X \to P(R)$ is a mapping, such that

$$H_\Psi(p_j^{\mu_X(p)}) = \left\{\left\langle o_i, \left\{r_1^j(o_i), r_2^j(o_i), \ldots, r_q^j(o_i)\right\}\right\rangle : o_i \in \Omega\right\} \tag{4}$$

Example 4.6 Let $\Omega = \{o_1, o_2, o_3, o_4\}$ be the universe of candidates and $P = \{p_1, p_2, p_3\}$ is the collection of parameters. We consider the FPIMF5SS of dimension 3 (Ψ, X) on Ω as shown in the Table 5. Then we have the induced FPH5SS of dimension 3 (H_Ψ, X) as in Table 6.

Table 5. The FPIMF5SS of dimension 3 (Ψ, X)

Ω	p1 0.5	p2 0.6	p3 0.7
o1	(3,0.3,0.5) (4,0.5,0.3) (2,0.6,0.3)	(1,0.4,0.5) (3,0.5,0.4) (2,0.5,0.3)	(2,0.5,0.5) (3,0.3,0.4) (2,0.6,0.3)
o2	(2,0.3,0.4) (4,0.5,0.3) (3,0.5,0.3)	(2,0.3,0.4) (4,0.5,0.3) (3,0.5,0.3)	(3,0.5,0.4) (4,0.4,0.3) (2,0.6,0.3)
o3	(2,0.4,0.5) (3,0.4,0.4) (1,0.5,0.4)	(3,0.5,0.3) (2,0.5,0.4) (4,0.5,0.4)	(1,0.5,0.3) (2,0.3,0.5) (3,0.6,0.3)
o4	(1,0.5,0.4) (4,0.6,0.4) (3,0.4,0.5)	(1,0.4,0.4) (2,0.6,0.3) (3,0.4,0.3)	(4,0.3,0.4) (2,0.6,0.3) (3,0.4,0.3)

Table 6. The FPH5SS of dimension 3 (H_Ψ, X)

Ω	p1 0.5	p2 0.6	p3 0.7
o1	{3, 4, 2}	{1, 3, 2}	{2, 3, 2}
o2	{2, 4, 3}	{2, 4, 3}	{3, 4, 2}
o3	{2, 3, 1}	{3, 2, 4}	{1, 2, 3}
o4	{1, 4, 3}	{1, 2, 3}	{4, 2, 3}

Definition 4.7 Let us fix $(\Psi, X) \in d(\Omega, P)^{(N,q)}$ where $P = \{p_1, p_2, ..., p_m\} \subseteq Q$ and $(\alpha, \beta) = ((\alpha_1, \beta_1), (\alpha_2, \beta_2), ..., (\alpha_m, \beta_m))$, a vector of thresholds $\alpha_i, \beta_i \in [0,1]$ associated with each $p_i \in P$ for i=1,2,..m. Then the *(α,β)*-FPHNSS induced by (Ψ, X) is the triple $(H_\Psi, X, (\alpha, \beta))$ where $H_\Psi : X \to P(R)$ is a mapping, such that

$$H_\Psi(p_j^{\mu_X(p)}) = \left\{ \left\langle o_i, \left\{ r_t^{\ j}(o_i) : \mu_t^{\ j}(o_i) \geq \alpha_j \text{ and } \nu_t^{\ j}(o_i) \leq \beta_j, t = 1, 2, ..., q \right\} \right\rangle : o_i \in \Omega \right\}.$$

Example 4.8 Let $\Omega = \{o_1, o_2, o_3, o_4\}$ be the universe of candidates and $P = \{p_1, p_2, p_3\}$ is the set of attributes. We consider the FPIMF5SS of dimension 3 (Ψ, X) on Ω whose tabular information is displayed in Table 5 and let (α,β) = {(0.5, 0.4), (0.5, 0.3), (0.6, 0.4)} be a fixed threshold. Then, we obtain *(α,β)*-FPH5SS whose tabular representation is in Table 7.

Table 7. The *(α,β)*-FPH5SS $\left(H_{\Psi}, X, (\alpha, \beta)\right)$

Ω	p_1 0.5	p_2 0.6	p_3 0.7
o_1	{4, 2}	{2}	{2}
o_2	{4, 3}	{4, 3}	{2}
o_3	{1}	{3}	{3}
o_4	{1, 4}	{2}	{2}

5. GDMM based on FPIMFNSS

Now, we present our machine learning algorithm for solving GDMPs based on FPIMFNSS. The steps of our proposed GDMM listed below:

Algorithm 1

Step1: Enter a nonempty universe $\Omega = \{o_1, o_2, ..., o_n\}$, a set of parameters $P = \{p_1, p_2, ..., p_m\}$, an FS $X = \left\{p^{\mu_X(p)} : p \in P\right\}$ over P, and a group of DMs $\{M_1, M_2, ..., M_q\}$.

Step2: Enter the DMs observations (IFNSSs) $(\psi_1, P), (\psi_2, P), ..., (\psi_q, P),$ as provided by each DM.

Step3: Compute the resultant *FPIMFNSS* (Ψ, X) of dimension q from the IFNSSs $(\psi_1, P), (\psi_2, P), ...,$ and (ψ_q, P)

Step4: Enter a threshold (α,β) = $\left\{\left(\alpha_j, \beta_j\right) \in [0,1] \times [0,1], j = 1, 2, ..., m\right\}$, where $\left(\alpha_j, \beta_j\right)$ associated with each attribute $p_j \in P$.

Step5: Obtain the *(α,β)*-FPHNSS $\left(H_{\Psi}, X, (\alpha, \beta)\right)$ in its tabular form.

Step6: Obtain the scores $\Gamma\left(h_j(o_i)\right)$ of all the HNTs $h_j(o_i)$ in $\left(H_{\Psi}^{\alpha}, P, N\right)$ by taking any operation (say, arithmetic mean, geometric mean, etc.), $\forall o_i \in \Omega,$ and j=1,2,3,….,m

Step7: Compute $u_i = \sum_{j=1}^{m} \mu_X(p_j) \times \Gamma\left(h_j(o_i)\right), \ \forall o_i \in \Omega$

Step8: The best optimal choice is to select o_s if u_s is maximized.

Step9: If o_s has many values, any of o_s may be selected.

Remark 5.1 In the 8th-step of our constructed GDMM, one can return to the 4th or 6th steps and change the threshold (α,β) or operation respectively that he previously used to adjust the final optimal choice, particularly when there are lots of optimal choices to choose from.

6. Result and Discussions

In this part, we use the proposed GDMM to solve a real-life GDMP involving candidate eligibility for a single vacant position in a job posting. Let us consider that four candidates, denoted by $\Omega=\{o_1,o_2,o_3,o_4\}$, applied for a single vacant position advertised by an IT firm. The selection of a candidate in this firm is based on star ratings and gradings given by a selection board comprised of three experts: a Director, a Subject Specialist, and a Chairman. Then, the selection board of the firm has firstly determined a parameter set $P=\{p_1,p_2,p_3\}$, such that p_1 = experience, p_2 = knowledge of foreign language, and p_3 = knowledge of software, which are used to assign grades to candidates.

Suppose, the three experts observations (IF5SSs) are in Tables 8, 9, and 10 respectively and let $X=\{p_1^{0.5},p_2^{0.6},p_3^{0.7}\}$ be an FS over *P*. Then the results of combining the three experts observations, we have the resultant FPIMF5SS (Ψ, X) of dimension 3 as shown in Table 11. Let us consider a threshold (α, β) = {(0.5, 0.4), (0.5, 0.3), (0.6, 0.3)} associated with the parameter set P. Then, we obtain *(α,β)*-FPH5SS as shown in Table 12. We use the arithmetic score on HNTs. Table 13 shows the results of the computations at steps 5 and 6. Step 7 suggests that the candidate o_4 is the best candidate for the vacant post in the IT firm.

Table 8. Director's observation

Ω	p_1	p_2	p_3
o_1	(3,0.4,0.5)	(3,0.4,0.5)	(4,0.5,0.5)
o_2	(3,0.4,0.4)	(3,0.5,0.4)	(3,0.6,0.4)
o_3	(2,0.4,0.5)	(3,0.5,0.3)	(4,0.5,0.3)
o_4	(2,0.5,0.4)	(4,0.6,0.2)	(4,0.5,0.4)

Table 9. Subject specialist's observation

Ω	p_1	p_2	p_3
o_1	(4,0.6,0.3)	(4,0.5,0.4)	(3,0.5,0.4)
o_2	(4,0.5,0.3)	(4,0.6,0.3)	(4,0.5,0.3)
o_3	(4,0.4,0.4)	(4,0.5,0.3)	(2,0.4,0.5)
o_4	(4,0.6,0.4)	(2,0.6,0.3)	(2,0.6,0.2)

Table 10. Chairman's observation

Ω	p_1	p_2	p_3
o_1	(2,0.6,0.3)	(2,0.6,0.3)	(2,0.6,0.3)
o_2	(3,0.6,0.3)	(3,0.5,0.3)	(2,0.6,0.3)
o_3	(1,0.5,0.4)	(4,0.6,0.4)	(3,0.6,0.3)
o_4	(3,0.5,0.4)	(3,0.6,0.3)	(3,0.4,0.3)

Table 11. The FPIMF5SS of dimension 3 (Ψ, X)

Ω	p1 0.5	p2 0.6	p3 0.7
o1	(3,0.4,0.5) (4,0.6,0.3) (2,0.6,0.3)	(3,0.4,0.5) (4,0.5,0.4) (2,0.6,0.3)	(4,0.5,0.5) (3,0.5,0.4) (2,0.6,0.3)
o2	(3,0.4,0.4) (4,0.5,0.3) (3,0.6,0.3)	(3,0.5,0.4) (4,0.6,0.3) (3,0.5,0.3)	(3,0.6,0.4) (4,0.5,0.3) (2,0.6,0.3)
o3	(2,0.4,0.5) (4,0.4,0.4) (1,0.5,0.4)	(3,0.5,0.3) (4,0.5,0.3) (4,0.6,0.4)	(4,0.5,0.3) (2,0.4,0.5) (3,0.6,0.3)
o4	(2,0.5,0.4) (4,0.6,0.4) (3,0.5,0.4)	(4,0.6,0.2) (2,0.6,0.3) (3,0.6,0.3)	(4,0.5,0.4) (2,0.6,0.2) (3,0.4,0.3)

Table 12. The *(α,β)*-FPH5SS $\left(H_{\Psi}, X, (\alpha, \beta)\right)$

Ω	p1 0.5	p2 0.6	p3 0.7
o1	{4, 2}	{2}	{2}
o2	{4, 3}	{4, 3}	{2}
o3	{1}	{3, 4}	{3}
o4	{2, 4, 3}	{4, 2, 3}	{2}

Table 13. The scores $\Gamma\left(h_j(o_i)\right)$ with u_i

Ω	p1 0.5	p2 0.6	p3 0.7	u_i
o1	3	2	2	4.1
o2	3.5	3.5	2	5.25
o3	1	3.5	3	4.7
o4	4.5	4.5	2	**6.35**

7. Comparison Results

In this present sec., we first present the Fatimah-Alcantud method (Fatimah and Alcantud, 2021) for solving MFNSS based DMPs. Fatimah and Alcantud (2021) proposed the MFNSS based approach as follows:

Algorithm 2 (Fatimah and Alcantud, 2021):

Step1: Enter a nonempty universe $\Omega = \{o_1, o_2, ..., o_n\}$, a set of parameters $P = \{p_1, p_2, ..., p_m\}$, an MFNSS (Ψ, P) of dimension q, a vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$

of threshold, and a weight $w=(w_1,w_2,...,w_m)$, where $\alpha_j, w_j \in [0,1]$ associated with each attribute $p_j \in P$

Step2: Obtain the *HNSS* $\left(H_{\Psi}^{\alpha}, P, N\right)$ in its tabular form.

Step3: Obtain the scores $\Gamma\left(h_j(o_i)\right)$ of all the HNTs $h_j(o_i)$ in $\left(H_{\Psi}^{\alpha}, P, N\right)$ by taking any operation (say, arithmetic mean, geometric mean), $\forall o_i \in \Omega$, and j=1,2,3,….,m

Step4: Compute $u_i = \sum_{j=1}^{m} w_j \times \Gamma\left(h_j(o_i)\right),\ \forall o_i \in \Omega$

Step5. The best optimal choice is to select o_s if u_s is maximized.

Step6. If o_s has many values, any of o_s may be selected.

In this sense, it is impossible to compare the proposed GDMM in Sec. 5 to the Fatimah-Alcantud method because it is the first method suggested in connection to FPIMFNSS in an intuitionistic fuzzy environment. However, if the simulated problem's uncertainties are reduced, the approach can be compared to the Fatimah-Alcantud method in a substructure MFNSS. As a result, we reduce the FPIMFNSS to MFNSS by considering only the membership-values of the objects in the FPIMFNSS and eliminating their non-membership values.

Let us consider that four candidates, denoted by $\Omega=\left\{o_1,o_2,o_3,o_4\right\}$, applied for a single vacant position advertised by an IT firm. The selection of a candidate in this firm is based on star ratings and gradings given by a selection board comprised of three experts: a Director, a Subject Specialist, and a Chairman. Then, the selection board of the firm has firstly determined a parameter set $P=\left\{p_1,p_2,p_3\right\}$, such that p_1 = experience, p_2 = knowledge of foreign language, and p_3 = knowledge of software. Suppose, the three experts observations (IF5SSs) are in Tables 14, 15, and 16 respectively and let $X=\left\{p_1^{0.5},p_2^{0.6},p_3^{0.7}\right\}$ be an FS over *P*. Then the results of combining the three experts observations, we have the resultant FPIMF5SS (Ψ, X) of dimension 3 as shown in Table 17.

If we consider the FPIMF5SS $\left(\Psi, X\right)$ as shown in Table 17, then we may get its reduced FMNSS $\left(\Psi', P\right)$ as in shown Table 18. We consider weight $w=(w(p_1)=0.5, w(p_2)=0.6, w(p_3)=0.7),$ same as our FS $X=\left\{p_1^{0.5},p_2^{0.6},p_3^{0.7}\right\}$ over P. Let (α,β) = {(0.5, 0.2), (0.5, 0.3), (0.6, 0.3)} be a fixed threshold for our proposed GDMM and α = {0.5, 0.5, 0.6} be a fixed threshold for Fatimah-Alcantud method.

Table 14. Director's observation

Ω	p_1	p_2	p_3
o_1	(3,0.4,0.5)	(3,0.4,0.5)	(4,0.5,0.5)
o_2	(3,0.5,0.4)	(2,0.5,0.4)	(3,0.5,0.3)
o_3	(2,0.5,0.5)	(3,0.5,0.3)	(4,0.5,0.3)
o_4	(2,0.5,0.4)	(4,0.6,0.2)	(4,0.5,0.4)

Table 15. Subject specialist's observation

Ω	p_1	p_2	p_3
o_1	(4,0.6,0.2)	(4,0.5,0.4)	(3,0.5,0.4)
o_2	(4,0.5,0.3)	(4,0.6,0.3)	(4,0.5,0.3)
o_3	(4,0.4,0.4)	(2,0.5,0.3)	(2,0.4,0.5)
o_4	(4,0.6,0.4)	(2,0.6,0.3)	(2,0.6,0.2)

Table 16. Chairman's observation

Ω	p_1	p_2	p_3
o_1	(2,0.6,0.3)	(2,0.6,0.3)	(2,0.6,0.3)
o_2	(2,0.6,0.2)	(3,0.5,0.3)	(2,0.6,0.3)
o_3	(1,0.5,0.2)	(1,0.6,0.4)	(3,0.6,0.3)
o_4	(3,0.5,0.2)	(3,0.6,0.3)	(3,0.4,0.3)

Table 17. The FPIMF5SS of dimension 3 (Ψ, X)

Ω	p_1 0.5	p_2 0.6	p_3 0.7
o_1	(3,0.4,0.5) (4,0.6,0.2) (2,0.6,0.3)	(3,0.4,0.5) (4,0.5,0.4) (2,0.6,0.3)	(4,0.5,0.5) (3,0.5,0.4) (2,0.6,0.3)
o_2	(3,0.5,0.4) (4,0.5,0.3) (2,0.6,0.2)	(2,0.5,0.4) (4,0.6,0.3) (3,0.5,0.3)	(3,0.5,0.3) (4,0.5,0.3) (2,0.6,0.3)
o_3	(2,0.5,0.5) (4,0.4,0.4) (1,0.5,0.2)	(3,0.5,0.3) (2,0.5,0.3) (1,0.6,0.4)	(4,0.5,0.3) (2,0.4,0.5) (3,0.6,0.3)
o_4	(2,0.5,0.4) (4,0.6,0.4) (3,0.5,0.2)	(4,0.6,0.2) (2,0.6,0.3) (3,0.6,0.3)	(4,0.5,0.4) (2,0.6,0.2) (3,0.4,0.3)

Table 18. The reduced MFNSS (Ψ', P) of the FPIMF5SS (Ψ, X)

Ω	s_1	s_2	s_3
o_1	(3,0.4)(4,0.6)(2,0.6)	(3,0.4)(4,0.5)(2,0.6)	(4,0.5)(3,0.5)(2,0.6)
o_2	(3,0.5)(4,0.5)(2,0.6)	(2,0.5)(4,0.6)(3,0.5)	(3,0.5)(4,0.5)(2,0.6)
o_3	(2,0.5)(4,0.4)(1,0.5)	(3,0.5)(2,0.5)(1,0.6)	(4,0.5)(2,0.4)(3,0.6)
o_4	(2,0.5)(4,0.6)(3,0.5)	(4,0.6)(2,0.6)(3,0.6)	(4,0.5)(2,0.6)(3,0.4)

Now, we apply the suggested GDMM as well as the Fatimah-Alcantud method to the FPIMF5SS (Ψ, X) and its reduced MFNSS (Ψ', P), as shown in Tables 17 and 18, respectively. The decision sets and the ranking orders of the approaches within their own structures are given in Table 19. Table 19 shows that according to the Fatimah-Alcantud method three candidates o_1, o_2, and o_4 are eligible for a single vacant position, but according to our suggested GDMM, only one candidate o_1 is eligible. In this example, Fatimah-Alcantud method is unable to determine the best candidate for a single vacant position, whereas we are able to do so. As a result, the proposed strategy has been successfully applied to a problem with additional uncertainty.

Table 19. The decision sets and ranking orders of the proposed GDMM and Fatimah-Alcantud method

DMMs (Algorithms)	Decision sets	Ranking
Algorithm-1 (Proposed GDMM)	$\{(o_1, 5.2), (o_2, 4.5), (o_3, 4.1), (o_4, 4.7)\}$	$o_1 \succ o_4 \succ o_2 \succ o_3$
Algorithm-2 (Fatimah and Alcantud, 2021)	$\{(o_1, 4.7), (o_2, 4.7), (o_3, 4.05), (o_4, 4.7)\}$	$o_1 = o_2 = o_4 \succ o_3$

8. Conclusions

IFSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by DMs. In comparison to FSs, IFSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this research work, we offer an approach for solving GDMPs with FPIMFNSSs by extending the MFNSS (Fatimah and Alcantud, 2021) based GDMM. In this study, we use the proposed GDMM to solve a real-life GDMP involving candidate eligibility for a single vacant position advertised by an IT firm. We also compare the ranking performances of the proposed GDMM with the Fatimah-Alcantud method and we have shown that the Fatimah-Alcantud method is unable to determine the best candidate for a single vacant position, whereas we are able to do so. We hope that this proposed model would provide the most up-to-date viewpoint and motion for dealing with real-world GDMPs.

In a future study, we will extend this proposed GDMM to other real-life applications in the field of pattern recognition and medical diagnostics.

Abbreviations:

DM	Decision maker
DMM	Decision making method
DMP	Decision making problem
FS	Fuzzy set
FSS	Fuzzy soft set
GDMM	Group decision-making method
GDMP	Group decision-making problem
HFS	Hesitant fuzzy set
HFSS	Hesitant fuzzy soft set
HNFSS	Hesitant N-fuzzy soft set
HNT	Hesitant N tuples
IF	Intuitionistic fuzzy
IFS	Intuitionistic fuzzy set
IFSS	Intuitionistic fuzzy soft set
IFNSS	Intuitionistic fuzzy N-soft set
IVFSS	Interval valued fuzzy soft set
IMFS	Intuitionistic multi fuzzy set
MCDM	Multi criteria decision making
MCGDM	Multi criteria group decision making
MFNSS	Multi-fuzzy *N*- soft set
MFS	Multi fuzzy set
MFSS	Multi fuzzy soft set
NSS	N-soft set
SST	Soft set theory

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References

Abdulkareem, K. H., Arbaiy, N., Zaidan, A. A., Zaidan, B. B., Albahri, O. S., Alsalem, M. A., & Salih, M. M. (2021). A new standardisation and selection framework for real-time image dehazing algorithms from multi-foggy scenes based on fuzzy Delphi and hybrid multi-criteria decision analysis methods. Neural Computing and Applications, 33, 1029-1054. https://doi.org/10.1007/s00521-020-05020-4

Abdulkareem, K. H., Arbaiy, N., Zaidan, A. A., Zaidan, B. B., Albahri, O. S., Alsalem, M. A., & Salih, M. M. (2020). A novel multiperspective benchmarking framework for selecting image dehazing intelligent algorithms based on BWM and group VIKOR techniques. International Journal of Information Technology & Decision Making, 19(3), 909–957

Akram, M., & Adeel, A. (2019). TOPSIS approach for MAGDM based on interval-valued hesitant fuzzy N-soft environment. International Journal of Fuzzy Systems, 21(3), 993–1009

Akram, M., Adeel, A., & Alcantud, J. C. R. (January, 2019). Group decision-making methods based on hesitant N-soft sets. Expert Systems with Applications, 115, 95–105

Akram, M., Adeel, A., Alcantud, J. C. R. (2018). Fuzzy N-soft sets: a novel model with applications., Journal of Intelligent & Fuzzy Systems, 35(4), 4757– 4771

Akram, M., Adeel, A., Alcantud, J. C. R. (2019). Hesitant fuzzy N-soft sets: A new model with applications in decision-making. Journal of Intelligent & Fuzzy Systems, 36(6), 6113–6127

Akram, M., Ali, G., Alcantud, J. C. R. (2019). New decision-making hybrid model: intuitionistic fuzzy N-soft rough sets., Soft Computing, 23(20), 9853–9868

Akram, M., Ali, G., Alcantud, J. C. R., Fatimah, F. (2021). Parameter reductions in N-soft sets and their applications in decision-making. Expert Systems, 38(1), 28-42. https://doi.org/10.1111/exsy.12601

Alcantud, J. C. R. (2020). Soft open bases and a novel construction of soft topologies from bases for topologies., Mathematics, 8(5), 672

Alcantud, J. C. R., & Mathew, T. J. (2017). Separable fuzzy soft sets and decision making with positive and negative attributes. Applied Soft Computing, 59(2), 586–595.

Alcantud, J. C. R., & Santos-García, G. (2017). A new criterion for soft set based decision-making problems under incomplete information. International Journal of Computational Intelligence Systems, 10, 394–404.

Alcantud, J. C. R., & Torra, V. (2018). Decomposition theorems and extension principles for hesitant fuzzy sets. Information Fusion, 41, 48–56.

Alcantud, J. C. R., & Torrecillas Muñoz, M. J. (2017). Intertemporal choice of fuzzy soft sets. Symmetry, 9, 253

Alcantud, J. C. R., Cruz-Rambaud, S., Torrecillas, M. J., Muñoz (2017). Valuation fuzzy soft sets: a flexible fuzzy soft set based decision making procedure for the valuation of assets. Symmetry, 9(11), 253, https://doi.org/10.3390/sym9110253

Alcantud, J. C. R., Feng, F., Yager, R. R. (2020). An N-soft set approach to rough sets. IEEE Transactions on Fuzzy Systems, 28(11), 2996–3007

Ali, M. I., Feng, F., Liu, X. Y, Min, W. K, & Shabir, M. (2009). On some new operations in soft set theory. Computer & Mathematics with Applications, 57(9), 1547–1553

Al-Qudah, Y., & Hassan, N. (2017). Operations on complex multi-fuzzy sets. Journal of Intelligent & Fuzzy Systems, 33, 1527–1540.

Al-Qudah, Y., & Hassan, N. (2018). Complex multi-fuzzy soft set: its entropy and similarity measure. IEEE Access 6, 65002–65017.

Atanassov, K. T. (1986). Intuitionistic fuzzy sets., Fuzzy Sets Systems, 20(1), 87–96.

Azam, M., & Bouguila, N. (2019). Bounded generalized Gaussian mixture model with ICA., Neural Processing Letters, 49, 1299–1320.

Azam, M., Bouguila, N. (2020). Multivariate bounded support Laplace mixture model. Soft Computing, 24, 13239–13268.

Bhardwaj, N., & Sharma, P. (2021). An advanced uncertainty measure using fuzzy soft sets: application to decision-making problems. Big Data Mining and Analytics, 4(2), 94–103.

Çağman, N., Çitak, F., & Enginoğlu, S. (2010). Fuzzy parameterized fuzzy soft set theory and its applications., Turkish Journal of Fuzzy Systems, 1(1), 21-35.

Çağman, N., Çitak, F., & Enginoğlu, S. (2011). FP-soft set theory and its applications, Annals of Fuzzy Mathematics and Informatics, 2(2), 219- 226.

Chen, Y., Liu, J., Chen, Z., & Zhang, Y. (2020). Group decision-making method based on generalized vague N-soft sets, In: Chinese Control And Decision Conference (CCDC), 4010–4015.

Dalkılıç, O. (2021). A novel approach to soft set theory in decision-making under uncertainty. International Journal of Computer Mathematics, 98(10), 1935-1945. https://doi.org/10.1080/00207160.2020.1868445

Dalkiliç, O. (2021). On topological structures of virtual fuzzy parametrized fuzzy soft sets. Complex & Intelligent Systems, https://doi.org/10.1007/s40747-021-00378-x

Dalkılıç, O., & Demirtaş, N. (2021). Bipolar fuzzy soft D-metric spaces. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 70(1), 64–73

Das S., & Kar S. (2015). The Hesitant Fuzzy Soft Set and Its Application in Decision-Making. In: Chakraborty M.K., Skowron A., Maiti M., Kar S. (eds) Facets of Uncertainties and Applications. Springer Proceedings in Mathematics & Statistics, vol 125. Springer, New Delhi. https://doi.org/10.1007/978-81-322-2301-6_18

Das, S., & Kar, S. (2013). Intuitionistic multi fuzzy soft set and its application in decision making. In: Maji P., Ghosh A., Murty M.N., Ghosh K., Pal S.K. (eds) Pattern Recognition and Machine Intelligence. PReMI 2013. Lecture Notes in Computer Science, vol 8251. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-45062-4_82.

Das, S., & Kar, S. (2014). Group decision making in medical system: An intuitionistic fuzzy soft set approach, Applied Soft Computing, 24, 196-211, https://doi.org/10.1016/j.asoc.2014.06.050.

Das, S., Kar, M. B., Kar, S., & Pal, T. (2018). An approach for decision making using intuitionistic trapezoidal fuzzy soft set, Annals of Fuzzy Mathematics and Informatics, 16, 99–116, https://doi.org/10.30948/afmi.2018.16.1.99

Das, S., Kar, M. B., Pal, T., & Kar, S. (2014). Multiple attribute group decision making using interval-valued intuitionistic fuzzy soft matrix, *2014 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, 2222-2229, https://doi.org/10.1109/FUZZ-IEEE.2014.6891687.

Das, S., Malakar, D., Kar, S. et al. (2019) Correlation measure of hesitant fuzzy soft sets and their application in decision making. Neural Computing & Application, 31, 1023–1039. https://doi.org/10.1007/s00521-017-3135-0

Dey, A., & Pal, M. (2015). Generalised multi-fuzzy soft set and its application in decision making. Pacific Science Review A: Natural Science and Engineering, 17(1), 23–28.

Fatimah, F., & Alcantud, J. C. R. (2018). Expanded dual hesitant fuzzy sets, In: International Conference on Intelligent Systems (IS), 102–108. https://doi.org /10.1109/IS.2018.8710539.

Fatimah, F., & Alcantud, J. C. R. (2021). The multi-fuzzy N-soft set and its applications to decision-making, Neural Computing and Applications, *33*(17), 11437-11446. https://doi.org/10.1007/s00521-020-05647-3

Fatimah, F., Rosadi, D., & Hakim, R. B. F. (2018). Probabilistic soft sets and dual probabilistic soft sets in decision making with positive and negative parameters. Journal of Physics: Conference Series, 983(1), 12-28.

Fatimah, F., Rosadi, D., Hakim, R. B. F., & Alcantud, J. C. R. (2019). Probabilistic soft sets and dual probabilistic soft sets in decision-making. Neural Computing and Applications, 31, 397–407.

Fatimah, F., Rosadi, D., Hakim, R. B. F., & Alcantud, J. C. R. (2018). N-soft sets and their decision-making algorithms. Soft Computing, 22(12), 3829–3842.

Feng, F., Jun, Y.B., Liu, X., Li, L. (210). An adjustable approach to fuzzy soft set based decision making. Journal of Computational & Applied Mathematics, 234, 10-20.

Gao, R., & Wu, J. (2021). Filter with its applications in fuzzy soft topological spaces. AIMS Mathematics, 6(3), 2359–2368.

Kamacı, H., & Petchimuthu, S. (2020). Bipolar N-soft set theory with applications., Soft Computing, 24, 16727–16743.

Krishankumar, R., Subrajaa, L. S., Ravichandran, K. S., Kar, S., & Saied, A. B. (2019). A framework for multi-attribute group decision-making using double hierarchy hesitant fuzzy linguistic term set. International Journal of Fuzzy Systems. 21, 1130–1143 https://doi.org/10.1007/s40815-019-00618-w

Krishankumar, R., Ravichandran, K. S., Gandomi, A. H. & Kar, S. (2021a). Interval-valued probabilistic hesitant fuzzy set-based framework for group decision-making with unknown weight information, Neural Computing and Applications, 33, 2445–2457

Krishankumar, R., Ravichandran, K. S., Liu, P., Kar, S. & Gandomi, A. H. (2021). A decision framework under probabilistic hesitant fuzzy environment with probability estimation for multi-criteria decision making, Neural Computing and Applications, 33, 8417–8433.

Kushwaha, D. K., Panchal, D., & Sachdeva, A. (2020). Risk analysis of cutting system under intuitionistic fuzzy environment. *Reports in Mechanical Engineering*, *1*(1), 162-173. https://doi.org/10.31181/rme200101162k.

Lathamaheswari, M., Nagarajan, D., & Kavikumar, J. (2020). Triangular interval type-2 fuzzy soft set and its application. Complex & Intelligent Systems, 6, 531–544.

Liu, J., Chen, Y., Chen, Z., & Zhang, Y. (2020). Multi-attribute decision making method based on neutrosophic vague N-soft sets. Symmetry, 12, 853.

Liu, P., & Zhang, L. (2017). Multiple criteria decision-making method based on neutrosophic hesitant fuzzy Heronian mean aggregation operators., Journal of Intelligent and Fuzzy Systems, 32(1), 303–319.

Liu, P., Zhang, L. (2017). An extended multiple criteria decisionmaking method based on neutrosophic hesitant fuzzy information., Journal of Intelligent and Fuzzy Systems, 32(6), 4403–4413.

Liu, X., Kim, H., Feng, F., & Alcantud, J. C. R. (2018). Centroid transformations of intuitionistic fuzzy values based on aggregation operators. Mathematics, 6(11), 215.

Ma, X., Liu, Q., Zhang, J. (2017). A survey of decision-making methods based on certain hybrid soft set models. Artificial Intelligence Review, 47(4), 507– 530.

Maji, P. K., Biswas, R., Roy, A. R. (2001). Fuzzy soft sets., Journal of Fuzzy Mathematics, 9(3), 589–602.

Maji, P. K., Biswas, R., Roy, A. R. (2001). Intuitionistic fuzzy soft sets., Journal of Fuzzy Mathematics, 9(3), 677–692.

Maji, P. K., Biswas, R., Roy, A. R. (2002). An application of soft sets in decision-making problem., Computers & Mathematics with Applications, 44(8–9), 1077–1083.

Maji, P. K., Biswas, R., Roy, A. R. (2003). Soft set theory. Computers & Mathematics with Applications, 45(4–5), 555–562.

Maji, P. K., Roy, A. R., Biswas, R. (2004). On intuitionistic fuzzy soft sets. Journal of Fuzzy Mathematics, 12(3), 669–683.

Močkoř J., Hurtik P. (2021) Fuzzy Soft Sets and Image Processing Application. In: Aliev R.A., Kacprzyk J., Pedrycz W., Jamshidi M., Babanli M., Sadikoglu F.M. (eds) 14th International Conference on Theory and Application of Fuzzy Systems and Soft Computing – ICAFS-2020. ICAFS 2020. Advances in Intelligent Systems and Computing, vol 1306. Springer, Cham. https://doi.org/10.1007/978-3-030-64058-3_6.

Mohammed, M. A., Abdulkareem, K. H. (2020). Benchmarking methodology for selection of optimal COVID-19 diagnostic model based on entropy and TOPSIS methods. IEEE Access, 8, 99115– 99131.

Molodtsov, D. (1999). Soft set theory-first results. Computers & Mathematics with Applications, 37(4–5), 19–31

Paik, B., & Mondal, S. K. (2021). A distance-similarity method to solve fuzzy sets and fuzzy soft sets based decision-making problems. Soft Computing 24, 5217–5229.

Paik, B., & Mondal, S. K. (2021). Representation and application of Fuzzy soft sets in type-2 environment. *Complex & Intelligent Systems*, *7*(3), 1597-1617.

Peng, X, & Li, W. (2019) Algorithms for hesitant fuzzy soft decision making based on revised aggregation operators, WDBA and CODAS. Journal of Intelligent & Fuzzy Systems, 36(6):6307–6323.

Peng, X. D., & Garg, H. (2018). Algorithms for interval-valued fuzzy soft sets in emergency decision-making based on WDBA and CODAS with new information measure., Computers & Industrial Engineering,119, 439–452.

Peng, X. D., & Liu, C. (2017). Algorithms for neutrosophic soft decision making based on EDAS, new similarity measure and level soft set. Journal of Intelligent & Fuzzy Systems, 32(1), 955–968.

Peng, X. D., & Yang, Y. (2017). Algorithms for interval-valued fuzzy soft sets in stochastic multi-criteria decision-making based on regret theory and prospect theory with combined weight. Applied Soft Computing, 54, 415–430.

Peng, X., & Dai, J. (2017). Hesitant fuzzy soft decision-making methods based on WASPAS, MABAC and COPRAS with combined weights. Journal of Intelligent & Fuzzy Systems, 33(2), 1313–1325.

Petchimuthu, S., Garg, H., Kamacı, H., & Atagün, A. O. (2020). The mean operators and generalized products of fuzzy soft matrices and their applications in MCGDM. Computational and Applied Mathematics, 39(2), 1–32.

Riaz, M., Çagman, N., Zareef, I., & Aslaam, M. (2019). N-soft topology and its applications to multi-criteria group decision making. Journal of Intelligent & Fuzzy Systems, 36(6), 6521–6536.

Riaz, M., Naeem, K., Zareef, I., & Afzal, D. (2020). Neutrosophic N-soft sets with TOPSIS method for multiple attribute decision making. Neutrosophic Sets and Systems, 32, 1–23.

Roy, A. R., & Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems, Journal of Computational & Applied Mathematics, 203, 412-418.

Sebastian, S., & Ramakrishnan, T. V. (2011). Multi-fuzzy sets: an extension of fuzzy sets. Fuzzy Information & Engineering, 1, 35–43.

Shinoj, T. K., & John, S. J. (2012). Intuitionistic fuzzy multisets and its application in medical diagnosis. World Academy of Science, Engineering and Technology, 61, 1178–1181.

Terepeta, M. (2019). On separating axioms and similarity of soft topological spaces., Soft Computing, 23(3), 1049–1057.

Torra, V. (2010). Hesitant fuzzy sets. International Journal of Intelligent Systems, 25(6), 529–539.

Torra, V., & Narukawa, Y. (2009). On hesitant fuzzy sets and decisions., IEEE International Conference on Fuzzy Systems, 1–3, 1378–1382.

Xia, M. M., & Xu, Z. S. (2011). Hesitant fuzzy information aggregation in decision-making. International Journal of Approximate Reasoning, 52, 395–407.

Yang, Y., Tan, X., & Meng, C. (2013). The multi-fuzzy soft set and its application in decision making., Applied Mathematical Modelling, 37, 4915–4923.

Youssef, M. I., & Webster , B. (2022). A multi-criteria decision making approach to the new product development process in industry. *Reports in Mechanical Engineering*, *3*(1), 83-93. https://doi.org/10.31181/rme2001260122y.

Zadeh, L. A. (1965). Fuzzy sets. Information & Control, 8(3), 338–353 34.

Zhan, J., & Alcantud, J. C. R. (2019). A survey of parameter reduction of soft sets and corresponding algorithms. Artificial Intelligence Review, 52(3), 1839–1872.

Zhu, B., Xu, Z. S., & Xu, J. P. (2014). Deriving a ranking from hesitant fuzzy preference relations under group decision-making. IEEE Transactions on Cybernetics, 44(8), 1328–119.

Zhu, K., & Zhan, J. (2016). Fuzzy parameterized fuzzy soft sets and decision making, International Journal of Machine Learning and Cybernetics, 7, 1207–1212.